FISCAL POLICY MULTIPLIERS: THE ROLE OF MONOPOLISTIC COMPETITION, DISTORTIONARY TAXATION, AND FINITE LIVES

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ABSTRACT

In this paper a dynamic macroeconomic model of monopolistic competition is developed for the closed economy. Forward looking consumers may have finite lives, demand goods, supply labour, and save part of their income in the form of shares and government bonds. Competitive producers manufacture final goods by using differentiated intermediate inputs. These inputs are themselves produced in a monopolistically competitive sector using labour and capital. The model is used to investigate analytically the short-run, transition, and long-run effects of fiscal policy under various financing methods. The policy experiments are conducted under both the Ramsey-Barro case of infinite horizons, and the Blanchard case of finite horizons. Comparisons with the conventional case of perfect competition and with the static literature are also made. Simple expressions for Keynesian multipliers are derived.

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CONTENTS

1. Introduction

Keynes showed in his *General Theory* that in case of deficient demand on labour and goods markets the government can step in and increase its consumption to raise national income and employment. During the 1970s models were built which operationalised Keynes' insights by assuming perfect competition on goods and labour markets and postulating fixed (or sticky) wages and prices. More recently, however, a number of authors have stressed that large government-induced output and employment effects can also be obtained in models with monopolistic competition, explicit price setting, and clearing labour and goods markets (see Silvestre (1993) and Dixon and Rankin (1994) for excellent surveys of this literature).

Static models in this vein typically rely on an endogenous labour supply response in order to generate positive output and employment effects of public consumption. For a fixed number of firms, a lump-sum tax-financed increase in public consumption makes agents poorer which prompts them to decrease consumption and increase labour supply. The resulting increase in firm profits mitigates the reduction in household wealth and thus sets in motion a multiplier process. Heijdra and Van der Ploeg (1996) have shown within the context of a static model that free entry of firms may have important productivity-enhancing effects. In their model, entry of firms eliminates profits but reduces the true price index of composite consumption thus increasing labour productivity and the real wage.

During the last few years, a number of authors working in the real business cycle (RBC) tradition have started to develop (stochastic) dynamic general equilibrium models in which the number of firms similarly plays a vital role (see Chatterjee and Cooper (1993), Devereux et al. (1993, 1996a-b), Heijdra (1994), and Bénassy (1996b)). Chatterjee and Cooper not only emphasize the role of entry and exit of firms in business fluctuations but also provide convincing empirical evidence that net business formation is strongly pro-cyclical (1993, pp. 2-3). They show with the aid of stochastic numerical simulations that a model with free exit/entry of firms exhibits a slower speed of adjustment (and thus more persistence) than both a model without entry/exit and a perfectly competitive model (see also Devereux et al. (1993) and Bénassy (1996b) on a related point). Since the lack of a quantitatively significant propagation mechanism is widely considered to be an important weakness of existing RBC models (Stadler, 1994, p. 1769), monopolistic competition is thus shown to be potentially useful.

The main objective of this paper is therefore to study theoretically to what extent monopolistically competitive models yield different predictions than more traditional models based on perfectly competitive behaviour. We follow the most recent literature by reserving a central role for exit/entry of firms. Whilst Chatterjee and Cooper (1993) study both productivity and preference shocks and Devereux et al. (1996a) consider productivity shocks, the present paper is more closely related to the static monopolistic competition literature by focusing on the effects of changes in government consumption. This makes it possible to relate our findings to the static literature mentioned above, to the RBC paper by Baxter and King (1993), and to the recent study of Devereux et al. (1996b).

The basic dynamic monopolistic competition model, variants of which were developed and studied independently by Heijdra (1994), Heijdra and Van der Ploeg (1996), and Devereux et al. (1996b), possesses strong RBC-style properties. A permanent unanticipated increase in government consumption which is financed by means of lump-sum taxes gives rise to a negative wealth effect which boosts labour supply. The saving-investment accelerator magnifies the shock and thus helps to explain a positive effect on output and employment. Crucial to this mechanism is the intertemporal substitution effect in labour supply. Free exit/entry of firms leads to a magnification of this labour supply effect so that even relatively small scale economies can make a major difference to the predicted output effect of government spending.

We extend the existing literature, as exemplified by Devereux et al. (1996b), in a number of different directions. First, we are able to provide a full analytical characterisation of the basic case they study. In order to emphasize the main economic mechanisms as clearly as possible, a deterministic model is developed. By making use of the Laplace transform techniques pioneered by Judd (1982) and assuming perfect foresight, a log-linearized version of the model can be solved for the impact, transition, and long-run effects. The theoretical impulse response functions can be shown to depend in a simple way on structural parameters. In contrast, Devereux et al. (1996b) only obtain analytical results for the long-run effects, and resort to numerical simulations to compute the impact and transitional effects.

Second, unlike Devereux et al. (1996b) who only consider lump-sum taxation, we also study explicitly the issue of financing with the aid of our model. If government consumption needs to be financed with an output tax, as in Baxter and King (1993), the scope for a positive output effect is significantly reduced as there are strong crowding-out effects on consumption and the capital stock in the long run. In this setting, increasing returns due to exit/entry of firms help to explain a large *negative* output effect of government consumption. Intuitively, the use of output taxation critically affects the core supply-side mechanism which is operative in the model. It thus leads to a reversal of the ‘Keynesian' conclusions obtained under lump-sum taxation. Numerical evidence furthermore confirms and extends Chatterjee and Cooper’s (1993) results, viz. output persistence is increasing both in the returns-to-scale parameter and in the initial output tax rate.

Third, we also go beyond Devereux et al. (1996b) by introducing overlapping generations of mortal households, thus allowing for Ricardian non-equivalence and a meaningful role for bond financing (which is studied in an appendix). In this setting, consumption and the capital stock may be crowded out in the long run due to a generational-turnover effect, even if lump-sum taxes are used. If households have finite lives and labour supply is fixed, as in Blanchard (1985), the model predicts a negative long-run output effect of government consumption. Again, as with output taxation, the existence of increasing returns magnifies this negative effect on output. Numerical results suggest that output persistence decreases as the deviation from Ricardian equivalence becomes more significant, i.e. as the birth rate increases and the expected planning horizon becomes shorter.

The remainder of this paper is organised as follows. In section 2, the basic theoretical model is developed. In section 3 a useful solution approach is suggested for analyzing the dynamic properties of the model. In section 4 the representative agent version of the model is used to study the efficacy of fiscal policy under the different modes of financing. In section 5, the overlapping generations version of the model is studied. Finally, in section 6, the paper concludes with some suggestions for further research. In a brief appendix it is shown that deficit financing can give rise to non-monotonic impulse response functions in the overlapping generations version of the model. It is furthermore shown how anticipation and duration effects can be studied with the aid of the Laplace transform method.

2. A model of perpetual youth and monopolistic competition

2.1. Households

Following Blanchard (1985), there is a fixed population of agents each facing a given constant probability of death. During their entire life agents have a time endowment of unity which they allocate over labour and leisure. The utility functional at time t of the representative agent born at time v is denoted by $\Lambda(v,t)$:

$$\Lambda(v,t) \equiv \int_t^{\infty} \left[\varepsilon_C \log C(v,\tau) + (1-\varepsilon_C) \log\left[1 - L(v,\tau)\right]\right] e^{(\alpha+\beta)(t-\tau)} d\tau, \tag{2.1}$$

where $C(v,\tau)$ and $L(v,\tau)$ are, respectively, consumption of a homogeneous good and labour supply in period τ by an agent born in period v, α is the pure rate of time preference (α>0), and β is the probability of death ($\beta \geq 0$). The agent's budget restriction is:[1]

$$\dot{A}(v,\tau) = \left[r(\tau)+\beta\right] A(v,\tau) + W(\tau)L(v,\tau) - T(\tau) - C(v,\tau), \tag{2.2}$$

where $r(\tau)$ is the real rate of interest, $W(\tau)$ is the real wage rate (assumed age-independent for convenience), $T(\tau)$ are real net lump-sum taxes, and $A(v,\tau)$ are real financial assets. The price of the final good ($P_Y(\tau)$ defined in the text below (2.7)) is used as the numeraire.

The representative agent chooses a time profile for $C(v,\tau)$ and $L(v,\tau)$ in order to maximise (2.1) subject to (2.2) and a No Ponzi Game (NPG) solvency condition. The optimal time profile for consumption is:

$$\frac{\dot{C}(v,t)}{C(v,t)} = r(t) - \alpha, \tag{2.3}$$

and labour supply is linked to consumption according to:

$$1 - L(v,t) = \frac{(1-\varepsilon_C)\, C(v,t)}{\varepsilon_C W(t)}. \tag{2.4}$$

A crucial feature of the Blanchard (1985) model is the simple demographic structure, which enables the aggregation over all currently alive households. At each instance a large cohort of size βF is born and βF agents die. Normalising F to unity the size of the population is constant and equal to unity. Given this simple demographic structure, the aggregate variables can be calculated as the weighted integral of the values for the different generations. Aggregate financial wealth is, for example, $A(t) \equiv \int_{-\infty}^{t} A(v,t) \beta e^{\beta(v-t)} dv$, and the aggregate values for $C(t)$, $L(t)$, and $T(t)$ are defined in a similar fashion. It is shown in Heijdra (1997) that the main equations describing the behaviour of the aggregate household sector can be written as in equations (T1.2) and (T1.7) in Table 1. Equation (T1.2) is the aggregate Euler equation modified for the existence of overlapping generations of finitely-lived agents. It has the same form as the Euler equation for individual households (equation (2.3)) except for the correction term due to the distributional effects caused by the turnover of generations. Optimal consumption *growth* is the same for all generations (since they face the same interest rate) but older generations have a higher consumption *level* than younger generations (since the former generations are wealthier). Since existing generations are continually being replaced by newborns, who hold no financial wealth, aggregate consumption growth falls short of individual consumption growth. The correction term appearing in (T1.2) thus represents the difference in average consumption and consumption by newborns:[2]

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \alpha - \beta\varepsilon_C(\alpha+\beta)\left(\frac{A(t)}{C(t)}\right) = \frac{\dot{C}(v,t)}{C(v,t)} - \beta\varepsilon_C\left(\frac{C(t) - C(t,t)}{C(t)}\right). \tag{2.5}$$

Equation (2.5) shows that, since the aggregate stock of financial assets is positive ($A(t)>0$), the steady-state interest rate must exceed the rate of time preference, i.e. $r>\alpha$. The rising individual consumption profile that this implies (see (2.3)) ensures that financial wealth is transferred from old to young generations in the steady-state (see Blanchard, 1985).

2.2. Firms

We introduce monopolistic competition in the standard manner.[3] There are two production sectors in the economy. The final goods sector uses inputs produced by the intermediate goods sector to produce its output which is either consumed by households or the government, or invested by households to augment the aggregate capital stock. The final goods sector is characterized by perfect competition. The intermediate goods sector, on the other hand, is populated by a large number of monopolistically competitive firms who each rent capital and labour from the households in order to produce a ‘slightly unique' variety of a differentiated intermediate product.

Technology in the final goods sector is described by the following CES aggregate of all existing intermediate inputs:

$$Y(t) = N(t)^{\eta}\left[N(t)^{-1}\sum_{i=1}^{N(t)} Z_i(t)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}, \tag{2.6}$$

where $Y(t)$ is aggregate output of final goods, $Z_i(t)$ is the quantity of variety i of the differentiated intermediate good used as inputs in the production of final goods, $N(t)$ is the number of input varieties, η regulates the productivity effects of increased input variety (see below), and σ is the constant substitution elasticity between the different input varieties.[4]

The representative producer in the final goods sector minimizes the cost of producing a given quantity of final goods. It does so by choosing the optimal mix of different input varieties. This leads to a cost function of the form, $TC_Y(t) \equiv P(t)Y(t)$, where $P(t)$ is given by:

$$P(t) \equiv N(t)^{-\eta}\left[N(t)^{-\sigma}\sum_{i=1}^{N(t)} P_i(t)^{1-\sigma}\right]^{1/(1-\sigma)}, \tag{2.7}$$

and where $P_i(t)$ is the price of input variety i. Marginal cost pricing implies that the price of the final good equals $P_Y(t)=P(t)/[1-t_Y(t)]$, where $t_Y(t)$ is a tax levied on the output of the final goods sector.[5] The input demand functions are obtained by applying Shephard’s Lemma to (2.7):

$$Z_i(t) \equiv \frac{\partial TC_Y(t)}{\partial P_i(t)} = Y(t)\frac{\partial P(t)}{\partial P_i(t)} = N(t)^{-(\sigma+\eta)+\sigma\eta}\, Y(t)\left(\frac{P_i(t)}{P(t)}\right)^{-\sigma}. \tag{2.8}$$

The formulation of the final goods sector implies *external* economies of scale due to increasing diversity provided $\eta>1$. This is the basic Ethier (1982) insight: more diversity in the differentiated goods sector enables final goods producers to use a more ‘roundabout' production process and hence lower unit cost. Of course, these scale economies only become effective if the number of firms is allowed to change.

In the intermediate goods sector there are $N(t)$ identical firms that each produce a single variety of the differentiated input. The typical firm i rents capital, $K_i(t)$, and labour, $L_i(t)$, from the households and its gross production function, $F(.)$, exhibits non-decreasing returns to scale ($\lambda \geq 1$):

$$Z_i(t) \;+\; \Phi \;=\; F(L_i(t),K_i(t)) \;\equiv\; L_i(t)^{\lambda\varepsilon_L} K_i(t)^{\lambda(1-\varepsilon_L)}, \tag{2.9}$$

where $0<\varepsilon_L<1$, $Z_i(t)$ is net marketable production of input variety i, and Φ is fixed cost modelled in terms of firm i's own output. The cost function associated with (2.9) is given by:

$$TC_i(t) \equiv \left[\frac{W^N(t)}{\varepsilon_L}\right]^{\varepsilon_L} \left[\frac{P_Y(t)\left[r(t)+\delta\right]}{1-\varepsilon_L}\right]^{1-\varepsilon_L} \left[Z_i(t)+\Phi\right]^{1/\lambda}, \tag{2.10}$$

where $W^N(t) \equiv W(t)P_Y(t)$ is the nominal wage rate and $P_Y(t)[r(t)+\delta]$ is the nominal rental charge on capital.

Each firm in the imperfectly competitive intermediate goods sector faces a constant-elasticity demand for its own input variety, the expression for which is given in (2.8). As a result, the price of input i is set equal to a constant markup times marginal cost:

$$P_i(t) \;=\; \mu\left(\frac{\partial TC_i(t)}{\partial Z_i(t)}\right) \;=\; \left(\frac{\mu}{\lambda\rho_i(t)}\right)\left(\frac{TC_i(t)}{Z_i(t)}\right), \tag{2.11}$$

where $\mu \equiv \sigma/(\sigma-1)>1$ and $\rho_i(t) \equiv [Z_i(t)+\Phi]/Z_i(t)>1$ measures (local) internal increasing returns to scale due to the existence of fixed costs (Rotemberg and Woodford, 1995, p. 253). Furthermore, the factor demands by firm i are determined by the usual marginal productivity conditions for labour and capital:

$$\frac{\partial Z_i(t)}{\partial L_i(t)} \;=\; \mu\left(\frac{W^N(t)}{P_i(t)}\right), \qquad \frac{\partial Z_i(t)}{\partial K_i(t)} \;=\; \mu\left(\frac{P_Y(t)}{P_i(t)}\right)\left[r(t)+\delta\right]. \tag{2.12}$$

Under Chamberlinian monopolistic competition exit and entry is free, so that the zero pure profit condition holds in the intermediate goods sector. As a result the price equals average cost:

$$P_i(t) \;=\; \frac{TC_i(t)}{Z_i(t)}. \tag{2.13}$$

By combining (2.11) and (2.13), we obtain $\mu=\lambda\rho_i(t)$ which implies a simple expression for the equilibrium firm size in the intermediate goods sector:

$$Z_i(t) = \bar{Z} \equiv \frac{\lambda\Phi}{\mu-\lambda}, \tag{2.14}$$

where $\mu>\lambda$ is required for the equilibrium to exist.

Since free exit/entry eliminates all excess profit, the market value of claims on the capital stock (i.e. 'shares') is equal to the replacement value of the capital stock. As a result $K(t)$ measures both the physical capital stock and the real value of shares.

2.3. Government

The government consumes $G(\tau)$ units of the final good and its periodic budget identity is given in (T1.3), where $B(\tau)$ is real government debt at time τ. The government can finance its expenditure on goods plus debt services by issuing more debt ($\dot{B}(\tau)$), or by changing one or both of its tax instruments, viz. the lump-sum tax ($T(\tau)$) or the tax rate on output of the final goods sector ($t_Y(\tau)$). Since the government must remain solvent, the NPG condition is $\lim_{\tau\to\infty}B(\tau)\times\exp[-\int_t^\tau r(s)ds]=0$, so that (T1.3) can be integrated forward to derive the government budget restriction:

$$B(t) = \int_t^\infty \left[T(\tau) + t_Y(\tau)\,Y(\tau) - G(\tau)\right]\exp\left[-\int_t^\tau r(s)ds\right]d\tau. \tag{2.15}$$

Solvency of the government implies that the level of government debt must equal the present value of present and future primary surpluses.

2.4. Symmetric equilibrium

The model is symmetric and can thus be expressed in aggregate terms. Equation (2.14) shows that all existing firms in the intermediate sector are of equal size, $\bar{Z}$, and hence (by (2.11)) charge the same price and (by (2.12)) demand the same amounts of capital and labour, i.e. $K_i(t)=\bar{K}(t)$ and $L_i(t)=\bar{L}(t)$. Equation (2.6) yields the expression for aggregate output in the final goods sector, i.e. $Y(t)=N(t)^\eta\bar{Z}$. Hence, aggregate output of final goods is an iso-elastic function of the number of input varieties, $N(t)$.

The main equations of the model are reported in Table 1. The aggregate physical capital stock evolves according to (T1.1), which shows that net investment equals gross investment minus replacement of the worn-out capital stock. The movement of consumption is described by equation (T1.2), which is the aggregate Euler equation corrected for the turnover of generations. We have used the fact that financial wealth is the sum of shares and government bonds. The government budget identity is given in (T1.3). The aggregate demands for labour and capital (under free entry/exit) are given by (T1.4) and (T1.5), respectively. The equilibrium condition for the final goods market is given in (T1.6), and aggregate labour supply is given in (T1.7). The equilibrium number of firms and the aggregate production function for the final goods sector are given by (T1.8). There are constant returns to scale in the aggregate production factors for the equilibrium number of product varieties, but increasing returns to scale for aggregate output.[6]

In a seminal paper, Weitzman (1994) has recently demonstrated that a generalised aggregator function such as (2.6) can be interpreted as a reduced form of a spatial model of monopolistic competition on the circle, provided each firm can choose its own level of specialisation. In his model, an increase in the supply of production factors increases the number of firms *and* decreases the incentive of individual firms to capture a wide segment of the market. Instead, each firm produces a larger quantity of a more specialised product at a lower price. As a result, the number of firms and aggregate output depend on aggregate factor supplies as in equation (T1.8).[7]

3. Model properties

3.1. Stability

The local stability of the model can be studied by log-linearized it around an initial steady state. The main expressions are given in Table 2. The state variables are the aggregate capital stock (which is predetermined) and consumption (which is a jump variable). By using labour demand (T2.4), labour supply (T2.7) and the aggregate production function (T2.8), a useful 'quasi-reduced form' expression for aggregate output is obtained:

$$\tilde{Y}(t) = \eta\phi(1-\varepsilon_L)\tilde{K}(t) - (\phi-1)\left[\tilde{C}(t) + \tilde{t}_Y(t)\right], \tag{3.1}$$

where $\tilde{Y}(t) \equiv dY(t)/Y$, $\tilde{K}(t) \equiv dK(t)/K$, $\tilde{C}(t) \equiv dC(t)/C$, $\tilde{t}_Y(t) \equiv dt_Y(t)/(1-t_Y)$, and ϕ is a crucial parameter representing the intertemporal labour supply elasticity as magnified by the diversity effect η:

$$\phi \equiv \frac{1+\omega_{LL}}{1+\omega_{LL}(1-\eta\varepsilon_L)} \geq 1, \tag{3.2}$$

where ω_{LL} ($\equiv(1-L)/L\geq 0$) is the ratio between leisure and labour, which also represents the intertemporal substitution elasticity of labour supply. Note that $\phi=1$ if labour supply is exogenous (since $L=1$ implies that $\omega_{LL}=0$). Since $\omega_{LL}\geq 0$, the sign restriction on ϕ is automatically implied if

$\eta\varepsilon_L \le 1$. If $\eta\varepsilon_L<1$, ϕ is a concave function of ω_{LL} with a positive asymptote of $(1-\eta\varepsilon_L)^{-1}$ as $\omega_{LL}\to\infty$, and if $\eta\varepsilon_L=1$, $\phi=1+\omega_{LL}$. If $\eta\varepsilon_L>1$, ϕ has a vertical asymptote at $\omega_{LL}=(\eta\varepsilon_L-1)^{-1}$, and for $0<\omega_{LL}<(\eta\varepsilon_L-1)^{-1}$, ϕ is a convex and increasing function of ω_{LL} exceeding unity. In order to cover this remaining case, we make the following convenient assumption regarding the range of admissible values for the intertemporal substitution elasticity of labour supply.

ASSUMPTION 1: *If* $\eta\varepsilon_L>1$ *it is assumed that* $0\le\omega_{LL}<(\eta\varepsilon_L-1)^{-1}$.

By using (3.1), (T2.5), and (T2.6) in (T2.1)-(T2.2), the dynamical system can be written as:

$$\begin{bmatrix} \dot{\tilde{K}}(t) \\ \dot{\tilde{C}}(t) \end{bmatrix} = \begin{bmatrix} (\delta/\omega_I)\left[\eta\,\phi\,(1-\varepsilon_L)-\omega_I\right] & -(\delta/\omega_I)(\omega_C+\phi-1) \\ -(r-\alpha)-(r+\delta)\left[1-\eta\,\phi\,(1-\varepsilon_L)\right] & (r-\alpha)-(r+\delta)(\phi-1) \end{bmatrix} \begin{bmatrix} \tilde{K}(t) \\ \tilde{C}(t) \end{bmatrix} - \begin{bmatrix} \gamma_K(t) \\ \gamma_C(t) \end{bmatrix} \tag{3.3}$$

where $\dot{\tilde{K}}(t)\equiv d\dot{K}(t)/K$, $\dot{\tilde{C}}(t)\equiv d\dot{C}(t)/C$, $\gamma_K(t)$ and $\gamma_C(t)$ are (potentially time-varying) forcing terms, and ω_C, ω_I and ω_G are, respectively, the share of consumption, investment and government consumption in national income ($\omega_C+\omega_I+\omega_G=1$). Saddle-point stability holds provided the determinant of the Jacobian matrix on the right-hand side of (3.3) is negative. Proposition 1 summarizes some stability results for the benchmark model that will prove useful in the discussion of policy shocks.

PROPOSITION 1: *The model satisfies the following properties:* ***(1) Infinite horizons*** ($\beta=0$): *If (i) labour supply is exogenous* ($\phi=1$) *or (ii) if labour supply is endogenous* ($\phi>1$) *and initial government spending is zero* (ω_G), *a necessary and sufficient condition for saddle-point stability is that* $\xi\equiv 1-\eta(1-\varepsilon_L)>0$; *(iii) If labour supply is endogenous* ($\phi>1$) *and government spending is positive* ($\omega_G>0$), $\xi>0$ *is a sufficient condition for saddle-point stability.* ***(2) Finite horizons*** ($\beta>0$): *(iv)* $\xi>\omega_G/\phi$ *is a sufficient condition for saddle-point stability.* ***(3)*** *In the stable case, the characteristic roots are* $r^*>0$ *and* $-h^*<0$. *The unstable root satisfies the inequality* $r^*>r-\alpha+\omega_C(r+\delta)$. PROOF: See Heijdra (1997).

The intuition behind the requirement $\xi>0$ is that there be diminishing returns to the aggregate capital stock (see (T1.8)). If lives are infinite, labour supply is elastic and government spending is positive, the negative wealth effect of the capital stock on labour supply ensures that the marginal product of capital is diminishing even if $\xi=0$. As agents get wealthier, they consume more leisure and thus supply less labour. This reduces the marginal productivity of capital. With both finite horizons and elastic labour supply, the necessary and sufficient condition depends on the various parameters. To simplify the discussion we simply assume that the sufficient condition holds:

ASSUMPTION 2: $\xi \equiv 1-\eta(1-\varepsilon_L) > \omega_G/\phi \geq 0$.

This condition is very mild. For example, taking typical values ω_G=0.2, η=1.30, and ε_L=0.7, we obtain ξ=0.61, which easily satisfies the sufficient condition for exogenous labour supply (ϕ=1), and *a fortiori* for endogenous labour supply (ϕ>1).[8]

3.2. Graphical Apparatus

In order to facilitate the discussion of the model, it is first summarised graphically by means of two schedules plotted in Figure 1. The *IS curve* represents all points for which the goods market is in equilibrium with a constant capital stock ($\dot{\tilde{K}}(t)=0$). The *MKR curve* is the Modified Keynes-Ramsey rule, which represents the steady-state aggregate Euler equation augmented for the turnover of generations ($\dot{\tilde{C}}(t)=0$). The IS curve is obtained by rewriting the first equation in (3.3) in steady-state form, and is unambiguously upward sloping in ($\tilde{C}(t),\tilde{K}(t)$) space:[9]

$$\tilde{C}(t) = \left(\frac{\eta\phi(1-\varepsilon_L)-\omega_I}{\omega_C+\phi-1}\right)\tilde{K}(t) - \left(\frac{\omega_I}{\delta(\omega_C+\phi-1)}\right)\gamma_K(t). \tag{3.4}$$

The dynamic forces operating on the economy off the IS curve are obtained from the first equation of (3.3). Points above the IS curve are associated with a falling capital stock over time because both goods consumption is too high and labour supply (and hence production) is too low. Consequently investment is too low to be able to replace the depreciated part of the capital stock. The opposite is the case for points below the IS curve.

The MKR curve is obtained by using the steady-state version of the second equation of (3.3):

$$\tilde{C}(t) = \left(\frac{r-\alpha+(r+\delta)\left[1-\eta\phi(1-\varepsilon_L)\right]}{(r-\alpha)-(r+\delta)(\phi-1)}\right)\tilde{K}(t) - \left(\frac{1}{-(r-\alpha)+(r+\delta)(\phi-1)}\right)\gamma_C(t). \tag{3.5}$$

The slope of the MKR curve is ambiguous because it is determined by two effects which work in opposite directions, viz. the *generational turnover* (GT) effect and the *labour supply* (LS) effect. The intuition behind these two effects can best be explained by looking at the two polar cases.

3.2.1. Labour supply effect with infinite lives

The pure LS effect is isolated by studying the model with endogenous labour supply and infinitely-lived agents, i.e. $\phi>1$ and $\beta=0$. In that case the MKR curve represents points for which the real interest rate equals the rate of time preference, $r[C,K,t_Y]=\alpha$, so that the slope of MKR depends on the partial derivatives $\partial r/\partial C$ and $\partial r/\partial K$. In order to explain the intuition behind these partial derivatives, Figure 2 depicts the situation on the markets for production factors. A useful expression for the (inverse) demand for capital (K^D) is obtained by combining (T2.5) and (T2.8):

$$\left(\frac{r}{r+\delta}\right)\tilde{r}(t) = \eta\varepsilon_L \tilde{L}(t) - \left[1-\eta(1-\varepsilon_L)\right]\tilde{K}(t) - \tilde{t}_Y(t), \tag{3.6}$$

where $\tilde{r}(t)\equiv \mathrm{d}r(t)/r$, $\tilde{L}(t)\equiv \mathrm{d}L(t)/L$. In terms of Figure 2(a), K^D is downward sloping in view of Assumption 2, and an increase in employment shifts K^D to the right. For a given stock of capital, the real interest rate clears the rental market for capital, and in the infinite horizon model the long-run supply curve of capital is horizontal and coincides with the dashed line in Figure 2(a).

By using (T2.4) and (T2.8), the (inverse) demand for labour (L^D) can be written as:

$$\tilde{W}(t) = (\eta\varepsilon_L - 1)\tilde{L}(t) + \eta(1-\varepsilon_L)\tilde{K}(t) - \tilde{t}_Y(t), \tag{3.7}$$

where $\tilde{W}(t)\equiv \mathrm{d}W(t)/W$. In terms of Figure 2(b), an increase in the stock of capital shifts L^D up, but the slope of L^D is ambiguous and depends on the strength of the diversity effect. If $\eta\varepsilon_L<(=,>)1$, L^D is downward sloping (horizontal, upward sloping). Labour supply, L^S, is upward sloping and shifts to the left if consumption rises (see (T2.7)). This is the wealth effect in labour supply, as consumption is itself proportional to total wealth. Finally, Assumption 1 ensures that the labour supply curve is steeper than the labour demand curve.

An increase in consumption (from C_0 to C_1) shifts the labour supply curve to the left, say from $L^S(W,C_0)$ to $L^S(W,C_1)$ in Figure 2(b), and for a given capital stock, employment falls from L_0 to L_1. This reduces the marginal product of capital, shifts the demand for capital to the left, say from $K^D(r,L_0)$ to $K^D(r,L_1)$ in Figure 2(a), and causes a fall in the interest rate. This explains why $\partial r/\partial C<0$.

An increase in the capital stock (from K_0 to K_1) has two effects. First, the *direct* effect leads to a rightward shift of the capital supply curve which, for a given level of employment, leads to a reduction in the rental price of capital. In terms of Figure 2(a) the direct effect is represented by the move from E_0 to B. There is also an *indirect* effect, because the increase in the capital stock boosts labour demand, say from $L^D(W,K_0)$ to $L^D(W,K_1)$ in Figure 2(b). For a given level of consumption, this leads to an expansion of employment from L_0 to L_2, represented by the move from E_0 to B. This increase in employment, in turn, boosts the demand for capital, say from

$K^D(r,L_0)$ to $K^D(r,L_2)$ in Figure 2(a). The indirect effect thus represents the move from point B to point C directly above it. For a moderate value of the intertemporal substitution elasticity of labour supply ($\omega_{LL}\approx 0$) or a weak diversity effect ($\eta\approx 1$), the labour supply parameter is small ($1<\phi<\bar{\phi}\equiv 1/[\eta(1-\varepsilon_L)]$), and the direct effect of the capital stock dominates the employment-induced effect, so that the rate of interest depends negatively on the capital stock, $\partial r/\partial K<0$, and the MKR curve in Figure 1 is downward sloping (as $dC/dK\equiv-(\partial r/\partial K)/(\partial r/\partial C)<0$ in that case). Points to the left of the curve are associated with a low capital stock, a high rate of interest, and a rising full consumption profile.

For a high enough value of the labour supply parameter ($\phi=\bar{\phi}$), however, the rate of interest does not depend on the capital stock as the two effects exactly cancel. In terms of Figure 2(a), the employment expansion shifts the demand for capital all the way to intersect supply in point D. In that case the MKR curve is horizontal. For points above the MKR curve, consumption is too high, and both labour supply and the rate of interest are too low (i.e. $r<\alpha$). As a result, the consumption profile is downward sloping. For an even higher value of the labour supply parameter ($\phi>\bar{\phi}$), the employment-induced effect dominates the direct effect, capital demand shifts all the way to intersect supply in point E, the rate of interest depends positively on the capital stock, $\partial r/\partial K>0$, and the MKR curve is upward sloping. Points to the left of the MKR curve are now associated with a low rate of interest, and a falling consumption profile.

3.2.2. Generational-turnover effect with exogenous labour supply

The pure GT effect is isolated by studying the model with exogenous labour supply and finitely-lived agents, for which $\phi=1$ and $\beta>0$. In that case the MKR curve represents points for which the tilt to the consumption profile of individual households is precisely sufficient to ensure the turnover of financial assets across generations, $r[K,t_Y]-\alpha=\beta(\alpha+\beta)K/C$, where r now does not depend on C because labour supply is exogenous. From Figure 2 it is clear that with a fixed supply of labour, only the direct effect of a change in K remains so that $\partial r/\partial K<0$.

The MKR curve is upward-sloping because of the turnover of generations. Its slope can be explained by appealing directly to equation (2.5) (with $\varepsilon_C=1$ and $A=K$) and Figure 4. Suppose that the economy is initially on the MKR curve, say at point E_0. Now consider a lower level of consumption, say at point B. With the same capital stock, both points feature the same rate of interest. Accordingly, individual consumption growth, $\dot{C}(v,t)/C(v,t)$ $[=r-\rho]$, coincides in the two points. Expression (2.5) indicates, however, that aggregate consumption growth depends not only on individual growth but also the *proportional* difference between average consumption and consumption by a newly born generation, i.e. $[C(t)-C(t,t)]/C(t)$. Since newly-born generations start without any financial capital, the *absolute* difference between average consumption and consumption of a newly-born household depends on the average capital stock and is thus the same in the two points. Since the level of aggregate consumption is lower in B, this point features a larger proportional difference between average and newly-born consumption, thereby decreasing aggregate consumption growth. In order to restore zero growth of aggregate consumption, the capital stock must fall (to point E_1). The smaller capital stock not only raises individual consumption growth by increasing the rate of interest but also lowers the drag on aggregate consumption growth due to the turnover of generations because a smaller capital stock narrows the gap between average wealth (i.e. the wealth of the generations that pass away) and wealth of the newly born.

For points to the left of the MKR curve, the capital stock is low and, consequently, the interest rate is higher than the rate of time preference so that the consumption profile is rising. The opposite holds for points to the right of the MKR curve. In terms of Figure 4, steady-state equilibrium is attained at the intersection of IS and MKR in point E_0. Given the configuration of arrows, it is clear that this equilibrium is saddle-point stable, and that the saddle path, SP_0, is upward sloping and steeper than the IS curve.

4. Fiscal policy in the representative-agent model

4.1. Introduction

The vast majority of studies on the intertemporal optimisation approach to macroeconomics is based on the notion of a representative infinitely-lived agent (see, for example, Baxter and King (1993), Chatterjee and Cooper (1993), Heijdra and Van der Ploeg (1996), and Devereux et al. (1996b)). In order to facilitate the comparison with that literature, this section studies the effects of fiscal policy under infinite lives, i.e. the birth rate is set equal to zero (β=0). This also implies that the steady-state interest rate equals the rate of time preference (r=α).

4.2. Lump-sum tax financing

The model can now be used to study the effects of fiscal policy on the economy, both in the short-run, on the transition path, and in the long-run. The first case to be studied concerns an unanticipated impulse at time t=0 to government consumption which is financed by means of lump-sum taxes. Since $r[K,C,t_Y]=\alpha$ along the MKR curve, its position is unaffected by government spending because lump-sum taxes are used. This implies that the forcing term in (3.5) is zero, i.e. $\gamma_C(t)\equiv 0$ for all $t\geq 0$.

The IS curve shifts down and to the right as a result of the increase in government spending as the forcing term in (3.4) is equal to $\gamma_K(t) \equiv (\delta\omega_G/\omega_I)\tilde{G} > 0$ for all $t \geq 0$, where $\tilde{G} \equiv dG/G$. Increasing government consumption withdraws resources from the economy. In order to maintain the same capital stock in equilibrium, goods consumption must fall. Depending on the magnitude of ϕ, four cases can be distinguished that are all consistent with saddle point stability.[10]

4.2.1. Exogenous labour supply.

The effects of the fiscal impulse are first illustrated, in Figure 1, for the case of inelastic labour supply (with $\phi=1$), in which case MKR is vertical (not drawn). The IS curve and the saddle path (both not drawn) shift down by the amount of the shock to intersect the MKR curve in point B. Since the capital stock is predetermined in the short run, the economy moves from E_0 to B, and consumption decreases by the full amount of the fiscal impulse. Intuitively, the representative agent observes the permanently higher level of lump-sum taxes needed to finance the additional government consumption, feels poorer as a result, and cuts back consumption once and for all, i.e. $dC(t)/dG=-1$ for all $t \geq 0$. Since the capital stock is unchanged and labour supply is fixed, the marginal products of capital and labour, and hence the interest rate and the wage, are unchanged also, i.e. $\tilde{r}(t)=\tilde{W}(t)=0$ for all $t \geq 0$. Furthermore, output and investment remain unchanged also, i.e. $\tilde{Y}(t)=\tilde{I}(t) \equiv dI(t)/I=0$ for all $t \geq 0$. There is thus no transitional dynamics in the model if labour supply is exogenous and the shock is unanticipated. This conclusion holds regardless of the assumed industrial structure. Despite the fact that there are increasing returns in the intermediate sector ($\eta>1$ in (T2.8)) factor supplies do not change and the diversity effect plays no role. Hence, as is the case for static models of monopolistic competition, an endogenous labour supply response is important to obtain non-zero multipliers.

4.2.2. Endogenous labour supply.

If labour supply is endogenous the MKR curve is no longer vertical. We illustrate the intuition behind the effects of the fiscal shock for the case of moderately elastic labour supply ($1<\phi<\bar{\phi}$) in Figure 1, but the analytical expressions given below hold for all cases with elastic labour supply. As before, the effect of the boost in public spending is to shift the IS curve (and the corresponding saddle path) down and to the right. Since the capital stock is predetermined in the impact period, the economy moves from E_0 to A on the saddle path SP_1. The higher level of taxes needed to finance the increase in government consumption again makes the agent feel poorer. The agent reacts by cutting both consumption and *spending* on leisure at impact:

The wealth effect in labour supply is thus unambiguously positive. There is also a substitution effect in labour supply, however, as the relative price of leisure (the wage) may change. It is easy

$$\tilde{C}(0) \;=\; \tilde{W}(0) \;-\; (1/\omega_{LL})\tilde{L}(0) \;=\; -\frac{\left[r^{*}+(\alpha+\delta)(\phi-1)\right]\omega_G \tilde{G}}{r^{*}(\phi+\omega_C-1)} \;<\; 0. \tag{4.1}$$

to show, however, that with lump-sum taxation the combination of the two effects must produce an increase in labour supply and employment at impact:

$$\tilde{L}(0) \;=\; -\left(\frac{\phi-1}{\eta\varepsilon_L}\right)\tilde{C}(0) \;>\; 0. \tag{4.2}$$

The intuition behind this result can be illustrated with the aid of Figure 2(b) which depicts the labour market. The fall in consumption shifts labour supply to the right, say from $L^S(W,C_0)$ to $L^S(W,C_2)$. The wealth effect thus corresponds to the move from E_0 to C (recall that consumption is proportional to total wealth). Labour demand is given in (3.7) and Assumption 1 ensures that it is less steep than labour supply. As capital is predetermined, the position of the labour demand curve is unaffected, so that employment rises at impact regardless of the magnitude of $\eta\varepsilon_L$. The substitution effect thus corresponds to the move from point C to D. The impact effect on the wage rate is ambiguous as can be seen by using (3.7): $\tilde{W}(0)=(\eta\varepsilon_L-1)\tilde{L}(0)$. In view of (4.2) and (4.3), this implies that the wage is countercyclical (if $\eta\varepsilon_L<1$), constant (if $\eta\varepsilon_L=1$), or procyclical (if $\eta\varepsilon_L>1$).

The boost to labour supply causes an expansion in aggregate output and an increase in the marginal product of capital, and hence the interest rate, despite the fact that the capital stock is fixed in the short run:

$$\tilde{Y}(0) \;=\; \left(\frac{\alpha}{\alpha+\delta}\right)\tilde{r}(0) \;=\; -(\phi-1)\,\tilde{C}(0) \;>\; 0. \tag{4.3}$$

The increase in the real interest rate not only results in an upward sloping time profile in consumption but also creates a boom in saving-investment by the representative household:

$$\tilde{I}(0) \;=\; \frac{(\alpha+\delta)(\phi-1)\,\omega_G \tilde{G}}{r^{*}\omega_I} \;>\; 0. \tag{4.4}$$

Hence, both consumption and the capital stock start to rise over time:

$$\tilde{C}(t) \;=\; \tilde{C}(0)e^{-h^{*}t} \;+\; \tilde{C}(\infty)\left(1-e^{-h^{*}t}\right), \qquad \tilde{K}(t) \;=\; \tilde{K}(\infty)\left(1-e^{-h^{*}t}\right), \tag{4.5}$$

where h^* is (minus) the stable characteristic root of the dynamical system (3.3) which represents the adjustment speed in the economy, and the long-run effects on consumption and the capital stock are given by:

In terms of Figure 1 equations (4.5)-(4.6) describe the smooth transition from point A to the new steady-state equilibrium at point E_1. The long-run effect on the capital stock is positive but the

$$\tilde{C}(\infty) = -\frac{(\alpha+\delta)\left[1-\eta\phi(1-\varepsilon_L)\right]\omega_G\tilde{G}}{r^*h^*(\omega_I/\delta)}, \quad \tilde{K}(\infty) = \frac{(\alpha+\delta)(\phi-1)\,\omega_G\tilde{G}}{r^*h^*(\omega_I/\delta)} > 0. \tag{4.6}$$

effect on consumption is ambiguous. For the moderately elastic case illustrated in Figure 1, $\phi<\bar{\phi}$ and consumption falls in the long run. Hence there is still some crowding out of private consumption in this case, though to a lesser extent than with exogenous labour supply, i.e. $-1<dC(\infty)/dG<0$. The reason for this is that agents react to the fiscal shock by accumulating a larger capital stock and supplying more labour, which in the steady-state gives rise to a higher level of aggregate output. Since there is no long-run effect on the interest rate, $\tilde{r}(\infty)=0$, (T2.5) ensures that there is no long-run effect on the capital-output ratio either. Furthermore, capital stock equilibrium ensures that the investment-capital ratio is unchanged in the long run also:

$$\tilde{Y}(\infty) = \tilde{I}(\infty) = \tilde{K}(\infty) > 0. \tag{4.7}$$

The long-run effect on the labour market can be illustrated with the aid of Figure 2(b), which is drawn under the assumption that labour demand is downward sloping (i.e. $\eta\varepsilon_L<1$). The short-run equilibrium in the labour market is at point D. Since consumption rises over time (following its initial drop), $\tilde{C}(\infty)>\tilde{C}(0)$ and the labour supply curve shifts to the left, say from $L^S(W,C_2)$ to $L^S(W,C_3)$. At the same time, however, the increase in the capital stock raises the marginal productivity of labour which leads to an increase in the demand for labour, say from $L^D(W,K_0)$ to $L^D(W,K_1)$. The new equilibrium is at E_1 and both employment and the wage rise:

$$\tilde{W}(\infty) = \tilde{K}(\infty) - \tilde{L}(\infty) = \left(\frac{\eta-1}{\eta\varepsilon_L}\right)\tilde{K}(\infty) > 0, \tag{4.8}$$

$$\tilde{L}(\infty) = \left(\frac{1-\eta(1-\varepsilon_L)}{\eta\varepsilon_L}\right)\tilde{K}(\infty) > 0. \tag{4.9}$$

With perfect competition in the intermediate sector ($\eta=1$) there is no long-run effect on the wage rate. In that case the constant steady-state real interest rate ($r=\alpha$) uniquely determines the optimal capital-labour ratio as a function of preference and technology parameters α, ε_L, and δ. This capital-labour ratio also fully determines the marginal product of labour and the real wage rate in that case. Under monopolistic competition, on the other hand, the capital-labour ratio and hence the real wage both rise with the capital stock and employment (see (4.8)-(4.9)).

If labour supply is highly elastic ($\phi>\bar{\phi}$) there is an increase in long-run consumption (see the first expression in (4.6)). There exists ‘crowding-in' of private by public consumption, d$C(\infty)$/dG>0, so that the real output multiplier is guaranteed to exceed unity (since both steady-state investment and government spending also rise). This suggests that the diversity effect plays an important role in the *size* of the long-run output multiplier as ϕ is increasing in η (see (3.2)). After some manipulation, the long-run output effect (given in (4.7)) can be written in a multiplier format:

$$\frac{\mathrm{d}Y(\infty)}{\mathrm{d}G} \equiv \frac{1}{1 - \omega_I + \omega_C CO} > 0, \tag{4.10}$$

where CO is the consumption crowding-out term which can be written as:

$$CO \equiv \frac{1}{\omega_{LL}}\left[1 - \left(\frac{\eta - 1}{\eta}\right)\left(\frac{1 + \omega_{LL}}{\varepsilon_L}\right)\right]. \tag{4.11}$$

Equation (4.11) shows that the crowding-out term is decreasing in the diversity effect, $\partial CO/\partial\eta<0$, and hence attains its maximum value of $CO=1/\omega_{LL}$ under perfect competition ($\eta=1$). This explains why the output multiplier is larger under monopolistic competition than under perfect competition. Indeed, the multiplier derived recently by Baxter and King (1993) is obtained as a special case of (4.10)-(4.11) by imposing perfect competition (i.e. setting the diversity effect η equal to unity). In order to obtain a large output multiplier under perfect competition, the consumption crowding-out effect must not be too strong. This is only possible, for a given share of consumption, if the intertemporal substitution effect in labour supply is very strong (i.e. ω_{LL} is high). Under monopolistic competition, on the other hand, crowding-out is less severe so that a smaller intertemporal substitution effect suffices to explain a given multiplier.

A further implication of (4.11) is that the crowding-out term is always non-negative under perfect competition, whereas it may be negative under monopolistic competition provided the diversity effect is sufficiently strong. Hence, the highly elastic case for which there is crowding-in of consumption is not possible under perfect competition. In terms of the diversity parameter, there exists ‘crowding-in' of consumption if $\eta>(1+\omega_{LL})/(1+\omega_{LL}-\varepsilon_L)>1$.[11]

4.2.3. Relationship to static literature

Before turning to alternative financing methods, it is useful to compare the results obtained from the dynamic model to the ones obtained from a static model like the one suggested by Heijdra and Van der Ploeg (1996). In doing so, the role of saving and the accelerator mechanism[12] are further clarified. In the static model, capital is a fixed factor of production and in order to assure a well-defined static equilibrium it is assumed that depreciation is zero ($\delta=\omega_I=0$). Households have no means by which to transfer resources across time, so that saving is zero and income is exhausted on consumption and taxes. In terms of Table 2, the model consists of (T2.3) to (T2.8), with capital exogenous, i.e. $\tilde{K}(t)=\tilde{K}_0=0$. If lump-sum taxes are used, the government budget identity can be ignored and the model can be reduced to the following two equations:

$$\tilde{Y} = \eta\phi(1-\varepsilon_L)\tilde{K}_0 - (\phi-1)\tilde{C}, \tag{4.12}$$

$$\tilde{Y} = \omega_C\tilde{C} + \omega_G\tilde{G}, \tag{4.13}$$

where the time index is left out because the model is static. Equation (4.12) is an aggregate supply function and is obtained by solving (T2.4), (T2.7) and (T2.8) for aggregate output. The negative effect of consumption on aggregate output in (4.12) represents the income effect in labour supply. Equation (4.13) is the goods market clearing condition. By solving (4.12) and (4.13) and setting $\tilde{K}_0=0$, the equilibrium effects on aggregate output and consumption are obtained:

$$0 < \frac{\mathrm{d}Y}{\mathrm{d}G} = \frac{\phi-1}{\phi-1+\omega_C} < 1, \qquad -1 < \frac{\mathrm{d}C}{\mathrm{d}G} = -\frac{\omega_C}{\phi-1+\omega_C} < 0. \tag{4.14}$$

These expressions generalize the results of Heijdra and Van der Ploeg (1996, pp. 1290-92) by including a fixed factor of production. In the absence of capital accumulation, consumption is crowded out and the output multiplier is less than unity. The increase in public consumption raises the rental rate on fixed capital, as $\tilde{r}=\tilde{Y}>0$, employment expands, as $\eta\varepsilon_L\tilde{L}=\tilde{Y}>0$, but the effect on the wage is ambiguous, since $\tilde{W}=[(\eta\varepsilon_L-1)/\eta\varepsilon_L]\tilde{Y}$.[13] Despite this ambiguity, household full income, which is defined as $W+rK_0-T$, falls. This explains the mechanism by which both consumption and leisure are cut back (and labour supply expands). In the static model the accelerator mechanism, which plays a vital magnifying role in the dynamic model, is absent. This explains in what sense the static and dynamic long-run multipliers differ.

4.3. Output tax financing

Up to this point, it has been assumed that the policy maker is able to finance the additional government spending by means of (non-distorting) lump-sum taxes. This was done not for reasons of realism, but rather in order to concentrate on the basic mechanisms underlying the multiplier. In this section, we study what happens to the effects discussed above if the policy maker can only expand government spending by raising the distorting output tax, i.e. $\dot{\tilde{B}}(t)=\tilde{B}(t)=\tilde{T}(t)=0$ and $\tilde{t}_Y(t)\neq 0$ for all $t\geq 0$. The government budget identity (T2.3) can then be written as follows:

The first term on the right-hand side of (4.15) represents the tax-rate effect whilst the second term is the tax-base effect. By substituting (4.15) into (3.1), the following 'quasi-reduced form'

$$\tilde{t}_Y(t) = \left(\frac{\omega_G}{1-t_Y}\right)\tilde{G} - \left(\frac{t_Y}{1-t_Y}\right)\tilde{Y}(t). \tag{4.15}$$

expression for output is obtained:

$$\tilde{Y}(t) = \eta\phi(1-\varepsilon_L)\Delta_Y\tilde{K}(t) - (\phi-1)\Delta_Y\left[\tilde{C}(t) + \frac{\omega_G\tilde{G}}{1-t_Y}\right], \tag{4.16}$$

where $\Delta_Y \equiv [1-(\phi-1)t_Y/(1-t_Y)]^{-1} = (1-t_Y)/(1-\phi t_Y)$. We assume that the economy operates on the upward sloping section of the Laffer curve ($t_Y<1/\phi$), which implies that $\Delta_Y>1$ (if $t_Y=0$ initially there is no erosion of the tax base and $\Delta_Y=1$).

By using (4.16) and following the standard solution approach, the appropriate expressions for the IS and MKR curves are obtained:

$$\tilde{C}(t) = \left(\frac{\eta\phi(1-\varepsilon_L)\Delta_Y - \omega_I}{\omega_C + (\phi-1)\Delta_Y}\right)\tilde{K}(t) - \left(\frac{\phi\Delta_Y\omega_G}{\omega_C + (\phi-1)\Delta_Y}\right)\tilde{G}, \tag{4.17}$$

$$\tilde{C}(t) = -\left(\frac{1-t_Y-\eta\phi(1-\varepsilon_L)\Delta_Y}{(\phi-1)\Delta_Y}\right)\tilde{K}(t) - \left(\frac{\phi\Delta_Y\omega_G}{(\phi-1)\Delta_Y}\right)\tilde{G}. \tag{4.18}$$

As before, the IS curve is upward sloping and the MKR curve is downward (upward) sloping for the moderately (highly) elastic case with $\phi\Delta_Y<(>)\bar{\phi}(1-t_Y)$. We illustrate the intuition behind the results with the aid of Figure 3 which depicts the moderately elastic case.

Ceteris paribus consumption and the capital stock, an increase in the output tax has a direct negative effect on labour demand (see (3.7)), and thus also on output, the rate of interest, and investment. As a result, an increase in government spending shifts both the IS and MKR curves. The IS curve shifts down and to the right both because of the increased government spending (as before in section 4.2) *and* because of the adverse tax effect on investment. The MKR curve shifts because of the adverse effect of the tax on labour demand and hence the marginal product of capital and the rate of interest. In view of (4.17) and (4.18), the MKR curve shifts down by more than the IS curve.

The representative agent feels poorer as a result of the higher level of public spending and reacts at impact by reducing consumption and spending on leisure:

where the sign follows because $(1-t_Y)r^*>\omega_C(\alpha+\delta)$ also in the presence of output taxation (see Heijdra, 1997). In terms of Figure 3, the economy jumps at impact from the initial equilibrium at E_0 to point A on the saddle path. The reduction of consumption does not automatically imply,

$$\tilde{C}(0) \;=\; \tilde{W}(0) \;-\; (1/\omega_{LL})\tilde{L}(0) \;=\; -\frac{\phi\Delta_Y\left[r^*(1-t_Y) \;-\; \omega_C(\alpha+\delta)\right]\omega_G\tilde{G}}{r^*(1-t_Y)\left[\omega_C+(\phi-1)\Delta_Y\right]} \;<\; 0, \qquad (4.19)$$

however, that employment increases at impact. This is because, in contrast to the situation with lump-sum taxes (see section 4.2), it is now possible that the positive wealth effect on labour supply is more than offset by the negative substitution effect due to the increase in the output tax, even if the initial output tax is zero. In terms of a diagram like Figure 2(b), the wealth effect shifts labour supply to the right but the output tax shift labour demand downwards, leaving the net effect on employment and hence on output ambiguous:

$$\eta\varepsilon_L\tilde{L}(0) \;=\; \tilde{Y}(0) \;=\; -\frac{(\phi-1)\Delta_Y\left[r^*(t_Y-1+\omega_C)+\phi\Delta_Y\omega_C(\alpha+\delta)\right]\omega_G\tilde{G}}{(1-t_Y)\left[\omega_C+(\phi-1)\Delta_Y\right]r^*}. \qquad (4.20)$$

If the wealth effect in labour supply dominates (is dominated by) the substitution effect caused by the shift in labour demand, employment and output increase (decrease). A positive employment and output response is less likely the higher the initial output tax. Despite the ambiguity of the output effect, the impact effect on the tax rate is unambiguously positive:

$$\tilde{t}_Y(0) \;=\; \frac{\omega_G\tilde{G}}{1-t_Y} \;-\; \left(\frac{t_Y}{1-t_Y}\right)\tilde{Y}(0) \;>\; 0. \qquad (4.21)$$

Since labour supply shifts to the right and labour demand shifts down, the impact effect on the wage rate is ambiguous in general:

$$\tilde{W}(0) \;=\; \frac{(1-\eta\varepsilon_L)\omega_{LL}\tilde{C}(0) \;-\; \tilde{t}_Y(0)}{1+\omega_{LL}(1-\eta\varepsilon_L)}. \qquad (4.22)$$

If $\eta\varepsilon_L<(=)1$, however, labour demand is downward sloping (horizontal) so that the wage must fall.[14]

The demand for capital shifts down because of the output tax (see (3.6)). Even if the impact response of employment is positive, this direct effect dominates the employment-induced effect on capital demand so that the interest rate falls at impact:

$$\tilde{r}(0) \;=\; -\frac{(\alpha+\delta)\omega_C\phi\Delta_Y\left[r^*(1-t_Y) \;+\; (\phi-1)\Delta_Y(\alpha+\delta)\right]\omega_G\tilde{G}}{\alpha r^*(1-t_Y)^2\left[\omega_C+(\phi-1)\Delta_Y\right]} \;<\; 0, \qquad (4.23)$$

Furthermore, by using (4.19) and (4.20) in (T2.6), the impact effect on saving-investment is obtained:

$$\tilde{I}(0) = -\frac{(\alpha+\delta)\,\omega_C\phi\Delta_Y\,\omega_G\tilde{G}}{(1-t_Y)\,\omega_I r^*} < 0. \tag{4.24}$$

The fall in the interest rate gives rise to a downward sloping consumption profile. Similarly, (4.24) shows that investment falls at impact so that the capital stock starts to fall over time. The transition path from point A to E_1 in Figure 3 has the same form as (4.5) but with $\tilde{C}(0)$ given by (4.19) and the long-run effects on consumption and the capital stock, respectively, by:

$$\tilde{K}(\infty) = \tilde{I}(\infty) = \left(\frac{\omega_C}{1-t_Y-\omega_I}\right)\tilde{C}(\infty) = -\frac{(\alpha+\delta)\,\omega_C\phi\,\Delta_Y\,\omega_G\tilde{G}}{(1-t_Y)\,(\omega_I/\delta)\,r^*h^*} < 0, \tag{4.25}$$

where the sign of the long-run consumption effect follows from the fact that $1\text{-}t_Y\text{-}\omega_I=\omega_A+t_Y(1\text{-}\varepsilon_L)>0$ (see Table 2). Both capital and consumption are crowded out in the long run, a result which stands in stark contrast to the lump-sum financing case (see section 4.2. above).

The long-run effect on employment is determined by the interplay of demand and supply on the labour market. The increase in the output tax and the reduction of the capital stock both cause a downward shift in labour demand, but the fall in consumption prompts a rightward shift in labour supply. The net effect on employment is:

$$\tilde{L}(\infty) = \frac{\omega_T(\alpha+\delta)\,(\phi-1)\,\Delta_Y\,\omega_G\tilde{G}}{(1-t_Y)\,(\omega_I/\delta)\,r^*h^*}, \tag{4.26}$$

where $\omega_T \equiv T/Y$ is the initial share of lump-sum taxes in output ($\omega_T=\omega_G\text{-}t_Y$). If $\omega_T=0$, the wealth effect in labour supply is exactly offset by the substitution effect caused by the decline in labour demand, so that there is no long-run effect on equilibrium employment. If $\omega_T>0$ (<0) the wealth effect in labour supply dominates (is dominated by) the substitution effect so that employment expands (contracts). The long-run effect on the wage rate is similarly dependent on ω_T

$$\tilde{W}(\infty) = -\frac{\Delta_Y(\alpha+\delta)\left[(\phi-1)\,\omega_T+\eta\,\varepsilon_L\,\omega_C\phi\right]\omega_G\tilde{G}}{\eta\,\varepsilon_L(1-t_Y)\,(\omega_I/\delta)\,r^*h^*}. \tag{4.27}$$

Unless ω_T is very negative, the wage falls in the long run.

By combining the expressions for employment and the capital stock (viz. (4.25) and (4.26)) the long-run effect on output can be written in the familiar multiplier format:

$$\frac{\mathrm{d}Y(\infty)}{\mathrm{d}G} = \frac{(\phi-1)\,\omega_T-\omega_C\phi\eta\,(1-\varepsilon_L)}{(\phi-1)\,\omega_T + \omega_C\phi\left[1-t_Y-\eta\,(1-\varepsilon_L)\right]}, \tag{4.28}$$

where the denominator of (4.28) is positive due to saddle-point stability. If $\omega_T \leq 0$, output falls as both employment and the capital stock fall. A positive output effect is thus only possible if there are pre-existing lump-sum taxes.

As was the case for lump-sum taxation, there is a simple relationship between the dynamic and static models of monopolistic competition (see also section 4.2.3 above). In the static model non-depreciating capital is a fixed factor of production. If output taxes are used to finance additional government consumption, the static aggregate supply expression is obtained from (4.16):

$$\tilde{Y} = -(\phi - 1)\Delta_Y \left[\tilde{C} + \frac{\omega_G \tilde{G}}{1 - t_Y} \right], \tag{4.29}$$

where we have already used the fact that $\tilde{K}(t) = \tilde{K}_0 = 0$. By solving (4.13) and (4.29), the expressions for consumption and output are obtained:

$$\frac{\mathrm{d}Y}{\mathrm{d}G} = \frac{\omega_T [\phi \Delta_Y - 1]}{\omega_C + (\phi - 1)\Delta_Y}, \qquad \frac{\mathrm{d}C}{\mathrm{d}G} = -\frac{\omega_C \phi \Delta_Y}{\omega_C + (\phi - 1)\Delta_Y} < 0. \tag{4.30}$$

Consumption falls but the output effect is ambiguous. If $\omega_T = 0$, the income effect in labour supply is exactly offset by the substitution effect caused by the decline in labour demand, so that there is no effect on employment and output. A similar result was obtained by Molana and Moutos (1992) in a static model with a fixed number of firms. If $\omega_T > 0$ (<0) the income effect in labour supply dominates (is dominated by) the substitution effect so that employment and output expand (contract). A notable feature of the output tax is that the rental rate on fixed capital falls, $\tilde{r} = -\omega_C \phi \Delta_Y / [\omega_C + (\phi-1)\Delta_Y] < 0$. Hence, whereas capital owners gain as a result of fiscal policy under lump-sum taxation (see section 4.2.3), they unambiguously lose out under output taxation. As before, the effect on the wage rate is ambiguous.

4.4. Some numerical evidence

In order to illustrate the quantitative significance of returns to scale and the mode of financing, this section presents a calibrated example of the model. Since we wish to study the effects of the intertemporal substitution elasticity in labour supply (ω_{LL}), the pre-existing output tax (t_Y), and the diversity parameter (η) on the various output multipliers, the model is calibrated in such a way that these parameters can be freely varied. In terms of Table 3, the parameters that are held fixed throughout the simulations are the rate of pure time preference ($\alpha = 0.03$), the rate of depreciation of the capital stock ($\delta = 0.07$), the share of labour income ($\varepsilon_L = 0.7$), and the share of government spending ($\omega_G = 0.2$). In panel (a), $t_Y = 0$ and ω_{LL} is varied and in panel (b), $\omega_{LL} = 2$ and t_Y is varied. Panels (a) and (b) both refer to the case of infinitely-lived agents ($\beta = 0$). Once these coefficients are set, all other information regarding shares can be obtained (see Heijdra, 1997).

In Table 3(a), the impact and long-run output multipliers ($dY(0)/dG$ and $dY(\infty)/dG$) as well as the adjustment speed of the economy (h^*) are reported for different values of ω_{LL} across columns and different values for η across rows. The first row of Table 3(a) refers to the perfectly competitive case (η=1). Both the impact and long-run multipliers are increasing in ω_{LL}. The higher the willingness of the representative household to substitute leisure across time, the larger the effect on output. For a high enough value of ω_{LL}, the impact multiplier exceeds the long-run multiplier and output overshooting takes place. The adjustment speed of the economy is also increasing in ω_{LL}. A high value of ω_{LL} thus reduces the persistence of output to shocks. The same pattern is observed for the other rows in Table 3(a).

Going down the columns in Table 3(a) reveals that both the impact and long-run multipliers are increasing in the level of η. Especially if the intertemporal substitution elasticity is high, even mildly increasing returns can have a significant effect on the size of the multiplier. For example, if ω_{LL}=5 the impact and long-run multipliers are respectively 1.15 and 1.10 if η=1 and 1.59 and 1.48 if η=1.3.

In Table 3(b) we study the financing issue in more detail. Across columns different pre-existing values for the initial output tax are considered. Since $\omega_T \equiv \omega_G - t_Y$, a higher value for t_Y implies a lower value for ω_T. Four major conclusions emerge from Table 3(b). First, both the impact and long-run output multipliers and the adjustment speed in the economy are decreasing in the initial output tax rate. Second, for the given calibration both impact and long-run multipliers are negative even if the initial tax rate is zero (see the first column). Hence, despite the fact that ω_T is positive if $t_Y<\omega_G$, and employment rises in the long run (see (4.26)), the crowding-out effect of government consumption on the capital stock dominates and the long-run output multiplier is negative. Third, glancing down the columns in Table 3(b) one observes that the impact and long-run output multipliers are decreasing in the diversity parameter η. The knife thus cuts both ways in the sense that the diversity effect magnifies the distortionary effect of the output tax on output. Fourth, the adjustment speed of the economy is decreasing in η, suggesting that the diversity effect can help explain output persistence (see also Bénassy (1996b) on this point).

5. Fiscal policy with finitely-lived agents

5.1. Introduction

Up to this point attention has been focused on the Barro-Ramsey case of infinitely lived households. It is straightforward to analyze the complications that occur when finite horizons are assumed. In order to focus on the pure GT effect, this section deals with the case of exogenous labour supply (ϕ=1). Apart from the existence of monopolistic competition in the goods market, this is also the case studied by Blanchard (1985).

The relevant MKR curve is obtained by rewriting the second equation in (3.3) in steady-state form and imposing ϕ=1:

$$\tilde{C}(t) = \left(\frac{r - \alpha + (r + \delta)\left[1 - \eta\,(1 - \varepsilon_L)\right]}{r - \alpha} \right) \tilde{K}(t) . \tag{5.1}$$

The GT effect ensures that the MKR curve is upward sloping (see section 3.2.2 and Figure 4). Points to the left of the MKR curve are associated with a low capital stock, a high rate of interest, and a rising aggregate consumption profile.

5.2. Lump-sum tax financing

The effects of fiscal policy are illustrated with the aid of Figure 4. The unanticipated increase in government spending shifts the IS curve down and to the right. As capital is predetermined, the economy jumps from E_0 to A on the saddle path, after which consumption and the capital stock smoothly fall towards the new equilibrium in E_1. The intuition is as follows. The higher level of government spending and lump-sum taxes causes a negative wealth effect on all existing generations who as a result cut consumption by the same absolute amount. This implies that aggregate consumption falls at impact:

$$\tilde{C}(0) = -\frac{\left[r^* - (r - \alpha)\right]\omega_G \tilde{G}}{r^* \omega_C} < 0, \tag{5.2}$$

where we have used the fact that $r^* > r - \alpha$ (see Proposition 1). This corresponds with the shift from E_0 to A in Figure 4. Since capital is predetermined. labour supply is exogenous, and lump-sum taxes are used, nothing happens to aggregate output, wages and the rate of interest at impact, i.e. $\tilde{Y}(0)=\tilde{W}(0)=\tilde{r}(0)=0$. The reduction in aggregate consumption does not fully compensate for the higher level of government spending, so that $-1<dC(0)/dG<0$, and saving-investment falls at impact:

$$\tilde{I}(0) = -\frac{(r - \alpha)\,\omega_G \tilde{G}}{r^* \omega_I} < 0. \tag{5.3}$$

In terms of Figure 4, point A lies to the left of the new IS curve so that the capital stock starts to fall over time:

The gradual decrease in the capital stock exerts upward pressure on the interest rate:

$$\tilde{K}(t) = \left(1 - e^{-h^* t}\right)\tilde{K}(\infty), \qquad \tilde{K}(\infty) = -\frac{(r-\alpha)\,\omega_G \tilde{G}}{r^* h^* (\omega_I/\delta)} < 0. \tag{5.4}$$

$$\tilde{r}(t) = \left(1 - e^{-h^* t}\right)\tilde{r}(\infty), \qquad \tilde{r}(\infty) = \frac{(r-\alpha)(r+\delta)\left[1-\eta\,(1-\varepsilon_L)\right]\omega_G \tilde{G}}{r\,r^* h^* (\omega_I/\delta)} > 0. \tag{5.5}$$

At the same time, the decline in the capital stock reduces the importance of the GT effect because the difference between aggregate and new-born consumption falls (see section 3.2.2). This means that the generations that pass away are replaced by newly born generations that are less wealthy. As a result aggregate consumption starts to fall:

$$\tilde{C}(t) = \tilde{C}(0)\,e^{-h^* t} + \left(1 - e^{-h^* t}\right)\tilde{C}(\infty), \qquad \tilde{C}(\infty) = -\frac{\left[(r-\alpha) + (r+\delta)\left[1-\eta\,(1-\varepsilon_L)\right]\right]\omega_G \tilde{G}}{r^* h^* (\omega_I/\delta)} < 0. \tag{5.6}$$

In terms of Figure 4, (5.4) and (5.6) describe the gradual movement from point A to the new equilibrium E_1. Capital stock equilibrium implies that $\tilde{I}(\infty)=\tilde{K}(\infty)<0$ and, since labour supply is exogenous, the long-run effect on wages and output is fully explained by the decline in the capital stock, $\tilde{W}(\infty)=\tilde{Y}(\infty)=\eta(1-\varepsilon_L)\tilde{K}(\infty)<0$.

The long-run output effect can be written in the following multiplier format:

$$\frac{\mathrm{d}Y(\infty)}{\mathrm{d}G} = -\frac{(r-\alpha)\,\eta\,(1-\varepsilon_L)}{\left(1-\eta\,(1-\varepsilon_L)\right)\left[\omega_C (r+\delta) + r - \alpha\right] - \omega_G (r-\alpha)} < 0, \tag{5.7}$$

where the denominator is positive by saddle point stability. In Heijdra (1997) it is shown that this multiplier is decreasing in the diversity effect, *i.e.* $\partial[\mathrm{d}Y(\infty)/\mathrm{d}G]/\partial\eta<0$. Hence, the existence of diversity effects amplifies the crowding out effect of fiscal policy.

5.3. Finite horizons and endogenous labour supply

In the most general form of the model, horizons are finite and labour supply is endogenous. From the discussion in sections 4.2 and 5.2 it is clear that the LS effect ensures a positive long-run output multiplier under lump-sum taxation, whereas the GT effect works in the opposite direction. It is not surprising therefore that the sign of the long-run output multiplier for the most general case is ambiguous. As labour supply becomes more and more elastic (ϕ rises), the LS effect starts to dominate the GT effect and the multiplier becomes positive.

The intuition built up with the case of exogenous labour supply suggests that the effect of finite lives is to reduce the size of the multiplier. Finitely-lived agents do not feel the full burden of the taxes needed to finance the additional government spending, and as a result do not cut back consumption to the same extent as under infinite lives. As a result, the labour supply and saving responses are also smaller, and the long-run effect on the capital stock is smaller. Proposition 2 shows that it is indeed possible to derive that the long-run output effect is decreasing in the birth rate β.

PROPOSITION 2: *The long-run output multiplier is a decreasing function of the instantaneous probability of death* β. PROOF: See Heijdra (1997).

5.4. Some numerical evidence

In Table 3(c)-(d) the quantitative significance of the birth rate is analyzed numerically. Panel (c) reports the impact and long-run multipliers for different combinations of the birth rate β and the intertemporal substitution elasticity of labour supply ω_{LL}. As is to be expected from the theoretical results, these multipliers are increasing in ω_{LL} and decreasing in β. Interestingly, the magnitude of ω_{LL} is much more important to the size of the multiplier than β. For example, even for a very high birth rate, say β=0.5, a relatively modest value of ω_{LL} suffices to explain a positive long-run multiplier.

In panel (d) the interaction between the birth rate and the diversity effect is studied. The results suggest that the diversity effect exerts a much more pronounced effect on the multipliers than the birth rate. Tables 3(c)-(d) demonstrate that the adjustment speed of the economy is increasing in the birth rate. Hence, an economy populated with finitely-lived agents shows much less output persistence than an economy populated with infinitely-lived representative agents.

6. Conclusions

The paper demonstrates the crucial role of increasing returns to scale in a monopolistically competitive world. Under Chamberlinian monopolistic competition, in which excess profits are always zero due to instantaneous entry or exit of firms, a large long-run output multiplier is obtained even for modest Ethier-style productivity effects, provided (in order of quantitative importance) (1) the intertemporal substitution effect in labour supply is strong, (2) lump-sum taxes are available, and (3) the generational turnover effect is relatively weak (and Ricardian equivalence holds approximately). If any one (or a combination) of these conditions is violated, positive output multipliers become unlikely. The scale economies then prove to be a ‘double-edged sword,' in the sense that they help explain larger crowding-out effects and thus more negative output multipliers than under the standard perfectly competitive case. The Keynesian quest for large multipliers thus has not come to an end with the macroeconomic literature on monopolistic competition.

Appendix. Further results

A.1. Debt financing

Under finite horizons, Ricardian equivalence fails and government debt has real effects, both in the long run and along the transition path. Consequently, the third and final fiscal policy experiment to be analyzed concerns a permanent unanticipated increase in government spending which is financed by means of government debt. The notion of debt financing is modelled as follows. The path of lump-sum taxes is:

$$\tilde{T}(t) = \left[1 - e^{-\xi_T t}\right]\tilde{T}(\infty), \qquad \xi_T > 0, \tag{A.1}$$

which implies that a deficit is opened up at impact, which is gradually being closed over time. Under the assumption that $t_Y = \tilde{t}_Y(t) = 0$, the government solvency condition can be written in general terms as $\mathcal{L}\{\tilde{T},r\} = \mathcal{L}\{\tilde{G},r\}$,[15] which implies that the long-run increase in lump-sum taxes equals $\tilde{T}(\infty) = [(r+\xi_T)/\xi_T]\tilde{G}$. In the long run, lump-sum taxes must rise by enough to cover the additional government spending on goods *plus* the interest payments on the public debt that is accumulated during the transition period. Hence, the path of lump-sum taxes is tilted, with relatively low taxes soon after the shock gradually rising to a level above what is needed to pay for public consumption. By substituting the expression for $\tilde{T}(\infty)$ into (A.1) and using (T2.3) (with $t_Y = \tilde{t}_Y(t) = 0$ imposed), the path for government debt is obtained:

$$\tilde{B}(t) = \left[1 - e^{-\xi_T t}\right]\tilde{B}(\infty), \qquad \tilde{B}(\infty) = (r/\xi_T)\,\omega_G\,\tilde{G}, \tag{A.2}$$

where $\tilde{B}(t) \equiv r\mathrm{d}B(t)/Y$. The government thus allows for a smooth build-up of government debt, from an initial position of zero to an exogenously given long-run level of $B(\infty)$. The lower ξ_T, the the slower is the adjustment in the lump-sum tax and the larger is the resulting long-run debt. Provided $\xi_T > 0$, however, the resulting debt process is stable.

The MKR curve is obtained by writing the second equation in (3.3) in it steady-state form.

$$\tilde{C}(t) = \left(\frac{(r-\alpha) + (r+\delta)\left[1 - \eta\,\phi(1-\varepsilon_L)\right]}{(r-\alpha) + (r+\delta)(1-\phi)}\right)\tilde{K}(t) + \left(\frac{1}{(r-\alpha) + (r+\delta)(1-\phi)}\right)\gamma_C(t). \tag{A.3}$$

Equation (A.3) generalizes (5.1) by allowing for an endogenous labour supply decision. As was explained in section 3.2, the slope of the MKR curve is explained by the combination of the labour supply and generational turnover effects. In Figure A.1, the MKR curve has been drawn under the assumption that the former dominates the latter effect, so that MKR slopes down. This is the case for $\hat{\phi}<\phi<\tilde{\phi}$, where $\hat{\phi}\equiv 1+(r-\alpha)/(r+\delta)$ and $\tilde{\phi}\equiv \hat{\phi}/[\eta(1-\varepsilon_L)]$. Since debt is only gradually accumulated, the MKR curve is gradually shifted down and to the left over time. In terms of (A.3), bond financing causes the shock term to be time-varying, i.e. $\gamma_C(t)\equiv[(r-\alpha)/\omega_A]\times \tilde{B}(t)$, where $\tilde{B}(t)$ is defined in (A.2).

The impact, transition, and long-run effects on all variables of an unanticipated permanent increase in government spending have been computed in Heijdra (1997). The intuition behind these results can de demonstrated with the aid of Figure A.1.

A.1.1. Long-run results

The long-run effect of the shock is to shift the IS curve from IS_0 to IS_1 and the MKR curve from MKR_0 to MKR_1. The steady-state equilibrium shifts from E_0 to E_1. If no debt financing would be used ($\xi_T\to\infty$), MKR would not shift. Hence, E' is the steady-state under pure lump-sum taxation. As is illustrated in Figure A.1, the long-run effect on the capital stock and consumption are unambiguously lower under bond-financing:

$$\left[\tilde{K}(\infty)\right]_B = \left[\frac{\left[-(r-\alpha)+(r+\delta)(\phi-1)\right]\omega_G\tilde{G}}{r^*h^*(\omega_I/\delta)}\right] - \frac{(r-\alpha)(\phi+\omega_C-1)r\omega_G\tilde{G}}{r^*h^*(\omega_I/\delta)\omega_A\xi_T}, \tag{A.4}$$

$$\left[\tilde{C}(\infty)\right]_B = -\left[\frac{\left[r-\alpha+(r+\delta)\left[1-\eta\phi(1-\varepsilon_L)\right]\right]\omega_G\tilde{G}}{r^*h^*(\omega_I/\delta)}\right] - \frac{(r-\alpha)\left[\eta\phi(1-\varepsilon_L)-\omega_I\right]r\omega_G\tilde{G}}{r^*h^*(\omega_I/\delta)\omega_A\xi_T}, \tag{A.5}$$

where $[.]_B$ denotes bond financing. The terms in square brackets on the right-hand side of (A.4) and (A.5) denote, respectively, the long-run capital and consumption effects under lump-sum tax financing. Government debt partially crowds out claims to physical capital in the portfolios of households, more so the lower is the value of ξ_T. In the long-run, output, investment, and the real wage rate are lower than under pure lump-sum taxation, and the interest rate is higher. Employment is lower provided initial government spending is low.

A.1.2. Impact and transition results

As in section 5.2, the increase in public consumption causes a negative wealth effect, because eventually lump-sum taxes are raised, so that consumption falls at impact:
where we have once again used the fact that $r^*>r-\alpha$ (see Proposition 1 and Heijdra (1997)). Output,

$$\left[\tilde{C}(0)\right]_B = -\left[\frac{\left[r^* - (r-\alpha) + (\phi-1)(r+\delta)\right]\omega_G \tilde{G}}{r^*(\phi+\omega_C-1)}\right] + \frac{(r-\alpha) r \omega_G \tilde{G}}{\omega_A (r^* + \xi_T)} < 0, \tag{A.6}$$

employment, and the interest rate rise at impact, as $\eta\varepsilon_L \tilde{L}(0)=[r/(r+\delta)]\tilde{r}(0)=\tilde{Y}(0)=-(\phi-1)\ \tilde{C}(0)$, the effect on wages depends on the strength of the diversity effect, as $\tilde{W}(0)=[(\eta\varepsilon_L-1)\ /(\eta\varepsilon_L)]\tilde{Y}(0)$, but the effect on investment is ambiguous:

$$\omega_I \tilde{I}(0) = -\left(\frac{(r-\alpha)(r+\delta)(\phi+\omega_C-1)}{(1-\varepsilon_L)(r^*+\xi_T)} + \left[r-\alpha-(\phi-1)(r+\delta)\right]\right)\frac{\omega_G \tilde{G}}{r^*}. \tag{A.7}$$

With a weak labour supply effect ($1\leq\phi<\hat{\phi}$) the term in square brackets on the right hand side is positive so that investment unambiguously falls regardless of the value of ξ_T. With a moderate labour supply effect ($\hat{\phi}<\phi<\tilde{\phi}$) the term in square brackets is negative and investment rises provided ξ_T is not too low. This is the case drawn in Figure A.1. The impact effect is a move from point E_0 to point B which lies below both IS_1 and MKR_0. This implies that the capital stock and consumption start to rise, say from point A to point B. Beyond point B the capital stock starts to fall again. As debt starts to accumulate during transition, the MKR shifts down and meets the stable trajectory at point C, after which consumption falls along with the capital stock towards the new equilibrium at point E_1.

An interesting implication of a debt-financed boost in government consumption is thus that adjustment may be non-monotonic. The general expressions for the transition path of consumption and the capital stock are given by:

$$\left[\tilde{C}(t)\right]_B = \left[\tilde{C}(0)\right]_B e^{-h^* t} + \left[\tilde{C}(\infty)\right]_B \left(1-e^{-h^* t}\right) + \Omega_C \left[\frac{e^{-\xi_T t} - e^{-h^* t}}{h^* - \xi_T}\right], \tag{A.8}$$

$$\left[\tilde{K}(t)\right]_B = \left(1-e^{-h^* t}\right)\left[\tilde{K}(\infty)\right]_B + \Omega_K \left[\frac{e^{-\xi_T t} - e^{-h^* t}}{h^* - \xi_T}\right], \tag{A.9}$$

where Ω_K and Ω_X are positive constants,[16] $[\tilde{C}(0)]_B$, $[\tilde{C}(\infty)]_B$, and $[\tilde{K}(\infty)]_B$ are given above, and the term in square brackets on the right-hand side of (A.8)-(A.9) is a non-negative bell-shaped (temporary) transition term. This term is zero for t=0 and as $t\rightarrow\infty$. Equations (A.8)-(A.9) confirm what the heuristic derivation in Figure A.1 already suggested: adjustment in consumption and the capital stock need not be monotonic. The capital stock may overshoot its new steady-state level, especially if the time profile of the debt path is flat (ξ_T low). Similarly, consumption may rise (following the initial decline) during part of the transition period, before falling again.

The intuition behind the impact results is straightforward. Since lump-sum taxes under bond financing are initially lower than under pure lump-sum financing, finitely-lived agents anticipate that they may not live long enough to face the higher taxes later. As a result, they feel wealthier than under pure lump-sum financing, and cut back consumption to a lesser degree. This explains why the increases in investment and labour supply are smaller than under pure lump-sum taxation.

Our conclusions regarding the long-run effects on the capital stock and consumption thus appear to contradict those obtained by Marini and Van der Ploeg (1988, p. 783). They however adopt a stabilisation rule for debt which makes lump-sum taxes negatively dependent on the change in debt. This means that debt is reduced, rather than increased, in the long run in order to make room for the additional government consumption. In our model this stabilisation rule would actually imply that the government engages in surplus rather than deficit financing throughout the transition period.

A.2. Temporary and anticipated fiscal policy

One of the main themes in the study of fiscal policy is the difference between the effects of temporary and permanent policy. Baxter and King employ numerical methods to study to what extent the impact output multiplier depends on the duration that the fiscal policy impulse is in operation (1993, p. 315). In this section it is demonstrated that the method of Laplace transforms, pioneered in the context of perfect foresight models by Judd (1982), can be used to study this issue analytically. In the interest of brevity we only consider the impact effect on output under lump-sum tax financing, although we do allow for anticipation effects.

The shock to government spending is assumed to be positive in the time interval $t_I \le t \le t_E$, where t_I is the implementation time, $t_E \equiv t_I + \varepsilon$ is the end time (so that ε is the duration of the fiscal impulse), and the announcement time of the shock is $t_A=0$. This specification nests the four most commonly used shocks: there are anticipation effects if $t_I>0$, the impulse is temporary if ε is finite, and it is permanent if $\varepsilon \to \infty$. The Laplace transform of the shock to the IS curve is given by:

$$\mathfrak{L}\{\gamma_K(t), s\} \equiv \int_0^\infty \gamma_K(t) e^{-st} \mathrm{d}t = \left(\frac{e^{-t_I s}\left[1 - e^{-\varepsilon s}\right]}{s} \right) \left(\frac{\delta \omega_G}{\omega_I} \right) \tilde{G}, \tag{A.10}$$

where the Laplace transform $\mathfrak{L}\{\gamma_K(t),s\}$ can thus be interpreted as the present value of $\gamma_K(t)$ using s

as the discounting factor. It is shown in Heijdra (1997) that the impact output multiplier is in that case fully characterized by the Laplace transform of the shock evaluated at the unstable characteristic root r^*:

$$\frac{\mathrm{d}Y(0)}{\mathrm{d}G} = \left[\frac{(\phi - 1)\left[r^* - \left[r - \alpha + (r + \delta)(1 - \phi)\right]\right]}{r^*(\phi + \omega_C - 1)} \right] e^{-t_I r^*}\left[1 - e^{-\varepsilon r^*}\right] > 0. \tag{A.11}$$

The term in square brackets on the right-hand side of (A.11) is the positive short-run output multiplier under lump-sum tax financing for the permanent unanticipated shock, which is denoted by $[\mathrm{d}Y(0)/\mathrm{d}G]_T$. Equation (A.11) can be used to derive the duration and anticipation effects on the impact multiplier:

$$\frac{\partial}{\partial t_I}\left(\frac{\mathrm{d}Y(0)}{\mathrm{d}G}\right) = -r^*\left[\frac{\mathrm{d}Y(0)}{\mathrm{d}G}\right]_T e^{-t_I r^*}\left[1 - e^{-\varepsilon r^*}\right] < 0, \tag{A.12}$$

$$\frac{\partial}{\partial \varepsilon}\left(\frac{\mathrm{d}Y(0)}{\mathrm{d}G}\right) = r^*\left[\frac{\mathrm{d}Y(0)}{\mathrm{d}G}\right]_T e^{-(t_I + \varepsilon) r^*} > 0. \tag{A.13}$$

Equation (A.12) shows that the impact multiplier declines as the implementation time lies further into the future. Agents have more time over which to spread out the anticipated additional tax burden, and as a result the impact effect on output (and labour supply) is smaller. Furthermore, equation (A.13) says that the longer the duration of the shock, the larger is the impact multiplier. We thus confirm *analytically* the numerical results reported by Baxter and King (1993, pp. 324-326).

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Table 1: Short-run version of the model

$$\dot{K}(t) \;=\; I(t) \;-\; \delta K(t) \tag{T1.1}$$

$$\dot{C}(t) \;=\; \big[r(t)-\alpha\big]C(t) \;-\; \beta\varepsilon_C(\alpha+\beta)\big[K(t)+B(t)\big] \tag{T1.2}$$

$$\dot{B}(t) \;=\; r(t)B(t) \;+\; G(t) \;-\; T(\tau) \;-\; t_Y(\tau)\,Y(\tau) \tag{T1.3}$$

$$W(t)L(t) \;=\; \varepsilon_L\big[1-t_Y(t)\big]Y(t) \tag{T1.4}$$

$$\big[r(t)+\delta\big]K(t) \;=\; (1-\varepsilon_L)\big[1-t_Y(t)\big]Y(t) \tag{T1.5}$$

$$Y(t) \;=\; C(t) \;+\; I(t) \;+\; G(t) \tag{T1.6}$$

$$L(t) \;=\; 1 \;-\; \frac{(1-\varepsilon_C)\,C(t)}{\varepsilon_C\,W(t)} \tag{T1.7}$$

$$Y(t) \;=\; \left(\frac{\lambda\Phi}{\mu-\lambda}\right)N(t)^{\eta} \;=\; \Omega_0 L(t)^{\eta\varepsilon_L}K(t)^{\eta(1-\varepsilon_L)}, \qquad \Omega_0 \equiv \left[\frac{\lambda}{\mu}\right]^{\eta/\lambda}\left[\frac{\mu-\lambda}{\lambda\Phi}\right]^{(\eta-\lambda)/\lambda} \tag{T1.8}$$

Table 2: Log-linearized version of the model

$$\dot{\tilde{K}}(t) = \delta\left[\tilde{I}(t) - \tilde{K}(t)\right] \tag{T2.1}$$

$$\dot{\tilde{C}}(t) = (r-\alpha)\,\tilde{C}(t) + r\,\tilde{r}(t) - (r-\alpha)\left[\tilde{K}(t) + (1/\omega_A)\,\tilde{B}(t)\right] \tag{T2.2}$$

$$\omega_G\,\tilde{G}(t) + \tilde{B}(t) = r^{-1}\dot{\tilde{B}}(t) + \omega_T\,\tilde{T}(t) + (1-t_Y)\left[\tilde{t}_Y(t) + \left(\frac{t_Y}{1-t_Y}\right)\tilde{Y}(t)\right] \tag{T2.3}$$

$$\tilde{L}(t) = \tilde{Y}(t) - \tilde{W}(t) - \tilde{t}_Y(t) \tag{T2.4}$$

$$\tilde{K}(t) = \tilde{Y}(t) - \left(\frac{r}{r+\delta}\right)\tilde{r}(t) - \tilde{t}_Y(t) \tag{T2.5}$$

$$\tilde{Y}(t) = \omega_C\,\tilde{C}(t) + \omega_I\,\tilde{I}(t) + \omega_G\,\tilde{G}(t) \tag{T2.6}$$

$$\tilde{L}(t) = \omega_{LL}\left[\tilde{W}(t) - \tilde{C}(t)\right] \tag{T2.7}$$

$$\tilde{Y}(t) = \eta\,\tilde{N}(t) = \eta\left[\varepsilon_L\,\tilde{L}(t) + (1-\varepsilon_L)\,\tilde{K}(t)\right] \tag{T2.8}$$

Definitions:

ε_L	WL/Y.	Share of before tax wage income in real output.
ω_A	rK/Y.	Share of income from financial assets in real output, $\omega_A=\omega_C+\omega_T-(1-t_Y)\varepsilon_L$, and $\omega_A=(1-\varepsilon_L)(1-t_Y)-\omega_I \Leftrightarrow \omega_A+t_Y(1-\varepsilon_L)=1-t_Y-\omega_I>0$.
ω_G	G/Y.	Share of government spending in real output.
ω_C	C/Y.	Share of private consumption in real output
ω_I	I/Y.	Share of investment spending in real output, $\omega_C+\omega_I+\omega_G=1$.
ω_{LL}	$(1-L)/L$	Ratio between leisure and labour.
ω_T	T/Y.	Share of lump-sum taxes in real output, $\omega_G=\omega_T+t_Y$.
t_Y		Proportional tax rate on output levied on firms
η		Diversity effect.

Table 3. Fiscal policy multipliers

(a) The effect of diversity and intertemporal substitution in labour supply

		ω_{LL}=0.01	ω_{LL}=0.5	ω_{LL}=1	ω_{LL}=2	ω_{LL}=5
		Parameter values: α=0.03, β=0, δ=0.07, ε_L=0.7, ω_G=0.2, t_Y=0				
η=1	dY(0)/dG d$Y(\infty)$/dG h^*	0.012 0.017 0.104	0.406 0.508 0.127	0.626 0.725 0.143	0.867 0.922 0.164	1.148 1.101 0.191
η=1.1	dY(0)/dG d$Y(\infty)$/dG h^*	0.013 0.019 0.097	0.446 0.575 0.121	0.689 0.815 0.139	0.961 1.031 0.161	1.288 1.225 0.194
η=1.3	dY(0)/dG d$Y(\infty)$/dG h^*	0.015 0.025 0.084	0.526 0.721 0.110	0.815 1.009 0.129	1.149 1.261 0.156	1.589 1.482 0.200
η=1.5	dY(0)/dG d$Y(\infty)$/dG h^*	0.018 0.032 0.072	0.602 0.887 0.098	0.936 1.222 0.119	1.337 1.507 0.149	1.933 1.752 0.209

(b) The effect of diversity and output tax financing

		t_Y=0	t_Y=0.05	t_Y=0.1	t_Y=0.2	t_Y=0.3
		Parameter values: α=0.03, β=0, δ=0.07, ε_L=0.7, ω_G=0.2, ω_{LL}=2				
η=1	dY(0)/dG d$Y(\infty)$/dG h^*	-0.097 -0.165 0.164	-0.146 -0.239 0.164	-0.203 -0.331 0.164	-0.353 -0.600 0.159	-0.584 -1.131 0.144
η=1.1	dY(0)/dG d$Y(\infty)$/dG h^*	-0.104 -0.185 0.161	-0.158 -0.270 0.161	-0.222 -0.376 0.159	-0.390 -0.702 0.151	-0.648 -1.402 0.130
η=1.3	dY(0)/dG d$Y(\infty)$/dG h^*	-0.118 -0.226 0.156	-0.181 -0.335 0.153	-0.255 -0.477 0.149	-0.452 -0.951 0.133	-0.739 -2.224 0.094
η=1.5	dY(0)/dG d$Y(\infty)$/dG h^*	-0.127 -0.270 0.149	-0.197 -0.408 0.144	-0.279 -0.594 0.137	-0.488 -1.286 0.110	-0.751* -3.778 0.057

* The model is unstable for this combination of t_Y and η as $\Delta_Y \to \infty$ and $h^* \to 0$ as $\eta \to 1.5$. The figures reported refer to the case of η=1.49 in which case the model is stable.

(c) The effect of the birth rate and intertemporal substitution in labour supply

		ω_{LL}=0	ω_{LL}=0.5	ω_{LL}=1	ω_{LL}=2	ω_{LL}=5
		Parameter values: α=0.03, δ=0.07, ε_L=0.7, ω_G=0.2, η=1.3, t_Y=0				
β=0.01	dY(0)/dG	0.000	0.520	0.809	1.145	1.586
	d$Y(\infty)$/dG	-0.020	0.706	0.998	1.253	1.479
	h^*	0.086	0.112	0.131	0.157	0.200
β=0.05	dY(0)/dG	0.000	0.484	0.769	1.109	1.564
	d$Y(\infty)$/dG	-0.127	0.612	0.920	1.197	1.448
	h^*	0.100	0.126	0.144	0.168	0.208
β=0.10	dY(0)/dG	0.000	0.441	0.714	1.051	1.520
	d$Y(\infty)$/dG	-0.222	0.506	0.818	1.109	1.390
	h^*	0.124	0.152	0.170	0.192	0.226
β=0.50	dY(0)/dG	0.000	0.342	0.562	0.845	1.270
	d$Y(\infty)$/dG	-0.380	0.291	0.576	0.840	1.105
	h^*	0.347	0.410	0.442	0.470	0.479

(d) The effect of diversity and the birth rate

		β=0.01	β=0.05	β=0.1	β=0.5	β=1
		Parameter values: α=0.03, δ=0.07, ε_L=0.7, ω_G=0.2, ω_{LL}=2, t_Y=0				
η=1	dY(0)/dG	0.863	0.830	0.776	0.597	0.554
	d$Y(\infty)$/dG	0.916	0.871	0.802	0.596	0.553
	h^*	0.165	0.176	0.199	0.467	0.826
η=1.1	dY(0)/dG	0.956	0.922	0.866	0.675	0.629
	d$Y(\infty)$/dG	1.024	0.976	0.900	0.673	0.625
	h^*	0.162	0.173	0.197	0.468	0.830
η=1.3	dY(0)/dG	1.145	1.109	1.051	0.845	0.793
	d$Y(\infty)$/dG	1.253	1.197	1.109	0.840	0.783
	h^*	0.157	0.168	0.192	0.470	0.840
η=1.5	dY(0)/dG	1.332	1.300	1.244	1.037	0.982
	d$Y(\infty)$/dG	1.499	1.435	1.336	1.027	0.959
	h^*	0.151	0.162	0.187	0.472	0.852

Footnotes

1. A dot above a variable designates the derivative with respect to time, e.g. $\dot{A}(v,\tau)\equiv dA(v,\tau)/d\tau$. Heijdra (1997) contains all derivations for the present paper, and is available from the author upon request.

2. We use the fact that $C(t)=\varepsilon_C(\alpha+\beta)[A(t)+H(t)]$ and $C(t,t)=\varepsilon_C(\alpha+\beta)H(t)$ in the second step, where $H(t)$ is 'full' human wealth. i.e. the after-tax value of the household's time endowment:

$$H(t) \equiv \int_t^\infty \left[W(\tau) - T(\tau)\right] \exp\left[\int_t^\tau -\left[r(s)+\beta\right] ds\right] d\tau.$$

3. This approach was suggested by Ethier (1982, p. 391) and has been used by Hornstein (1993), Farmer (1993, p. 129), Bénassy (1996a-b), and Devereux et al. (1996a-b). This approach has some notational advantages but otherwise yields a model of production that is isomorphic to the model discussed in Heijdra (1994) and Heijdra and Van der Ploeg (1996) provided a common diversity effect is imposed in the various aggregator functions and the variety substitution elasticities are the same.

4. Our specification (2.6) is more general than the one used by Hornstein (1993) and Devereux et al. (1996b) in that the diversity and price-elasticity effects are parameterized separately. Ethier (1982), Heijdra (1994), Heijdra and Van der Ploeg (1996), Devereux et al. (1996a), and Bénassy (1996a-b) also explicitly distinguish the two conceptually different effects.

5. The output tax is equivalent to a uniform tax on labour and capital income. See Baxter and King (1993, p. 318) and below.

6. Foreshadowing the discussion on short-run output multipliers somewhat, equation (T1.8) shows clearly that, as capital is predetermined, output effects occur at impact only if there is a labour supply response.

7. Interestingly, his analysis suggests a close relationship between the markup and the diversity effect, which in our notation amounts to $\eta=(\mu+1)/2$. See also Bénassy (1996b) and Chatterjee and Cooper (1993, p. 7). Bénassy (1996a, p. 46) suggests that in the working paper version of Dixit and Stiglitz (1977) the diversity and price-elasticity effects were actually parameterised separately, as in (2.6).

8. Note that our stability discussion generalizes the remarks by Devereux et al. (1996, p. 242).

9. From the information on steady-state shares it is clear that $\omega_A=(1-\varepsilon_L)(1-t_Y)-\omega_I$, where $\omega_A=rK/Y>0$ is the gross income share of capital. Hence, we can derive that $\eta\phi(1-\varepsilon_L)-\omega_I=(t_Y+\eta\phi-1)(1-\varepsilon_L)+\omega_A>0$ since $\eta\geq1$, $t_Y\geq0$, and $\phi\geq1$.

10. These case are $\phi=1$, $1<\phi<\bar{\phi}$, $\phi=\bar{\phi}$, and $\phi>\bar{\phi}$. Saddle point stability ensures that the IS curve is steeper than the MKR curve. The second case is drawn in Figure 1.

11. After this paper was completed I became aware of Devereux et al. (1996b, p. 244) who compute long-run elasticities of employment, the capital stock, output, the real wage, and consumption with respect to the output *share* of government consumption. It can be shown that their equations (28)-(32) and my expressions (4.6)-(4.9) are in fact equivalent.

12. Note that (4.4) and the second expression in (4.6) imply $\tilde{I}(0)=(h^*/\delta)\tilde{K}(\infty)$, which represents a simple analytical expression for the accelerator mechanism alluded to by Baxter and King (1993, p. 321). The same relationship between $\tilde{I}(0)$ and $\tilde{K}(\infty)$ holds for (4.24)-(4.25) and (5.3)-(5.4) below.

13. If only labour is used in production, $\varepsilon_L=1$, and the wage unambiguously rises. See Heijdra and Van der Ploeg (1996, pp. 1290-92).

14. Only if the diversity effect is strong ($\eta\varepsilon_L>1$) and the wealth effect in labour supply is large enough to render the output effect positive ($\tilde{Y}(0)>0$), is it possible that the wage rises at impact. Note that under perfect competition ($\eta=1$) the wage must fall at impact.

15. $\mathfrak{L}\{x,s\}$ is the Laplace transformation of $x(t)$ evaluated at s:

$$\mathfrak{L}\{x,s\} \equiv \int_0^\infty x(t)e^{-st}\mathrm{d}t.$$

Intuitively, $\mathfrak{L}\{x,s\}$ represents the present value of $x(t)$ using s as the discount factor.

16. Ω_K and Ω_C are defined as follows:

$$\Omega_K \equiv \frac{\delta(r-\alpha)(\phi+\omega_C-1)\tilde{B}(\infty)}{\omega_A\omega_I(r^*+\xi_T)}>0, \qquad \Omega_C \equiv \frac{(r-\alpha)\left[(\delta/\omega_I)(\eta\phi(1-\varepsilon_L)-\omega_I)+\xi_T\right]\tilde{B}(\infty)}{(r^*+\xi_T)\,\omega_A}>0.$$

Figure 1. Fiscal policy under lump-sum taxation: Endogenous labour supply and infinite lives

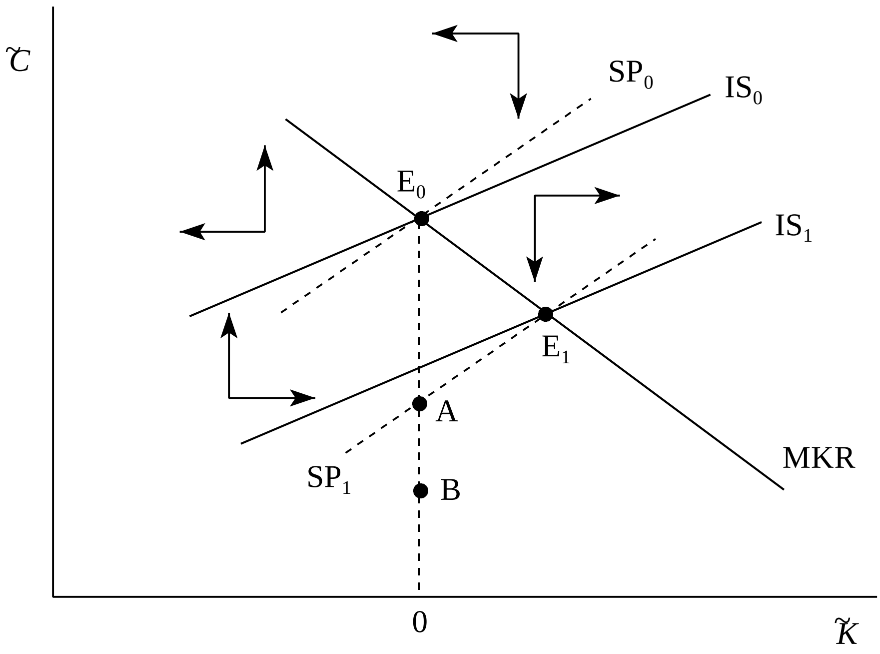


Figure 3. Fiscal policy under output taxation

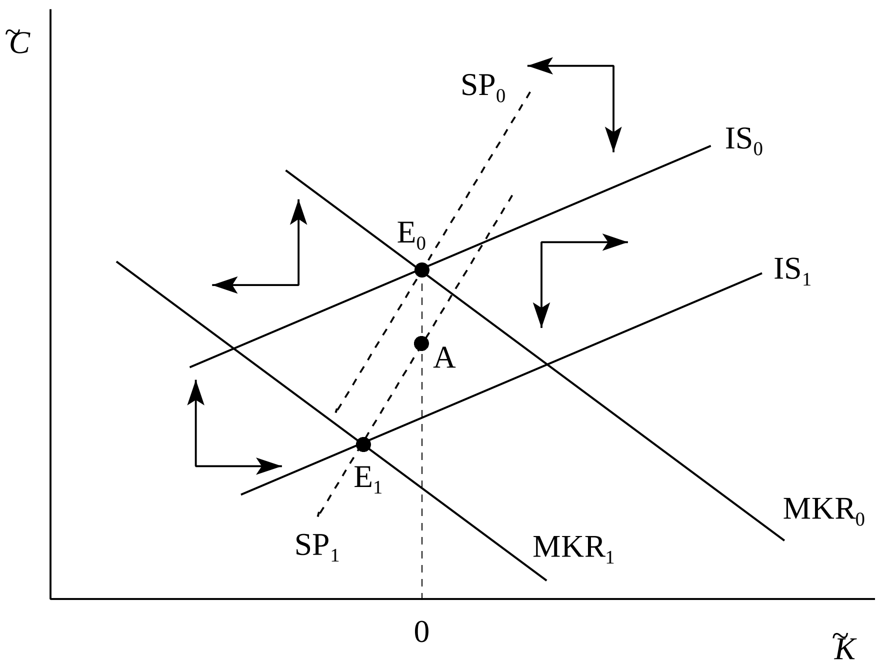
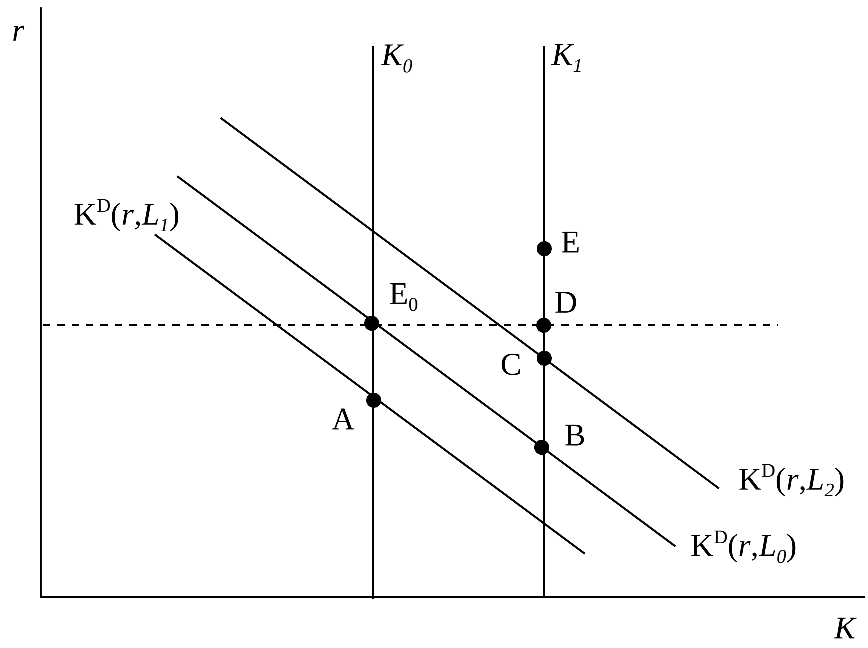


Figure 2. The markets for capital and labour

Panel (a): The rental market for capital



Panel (b): The labour market

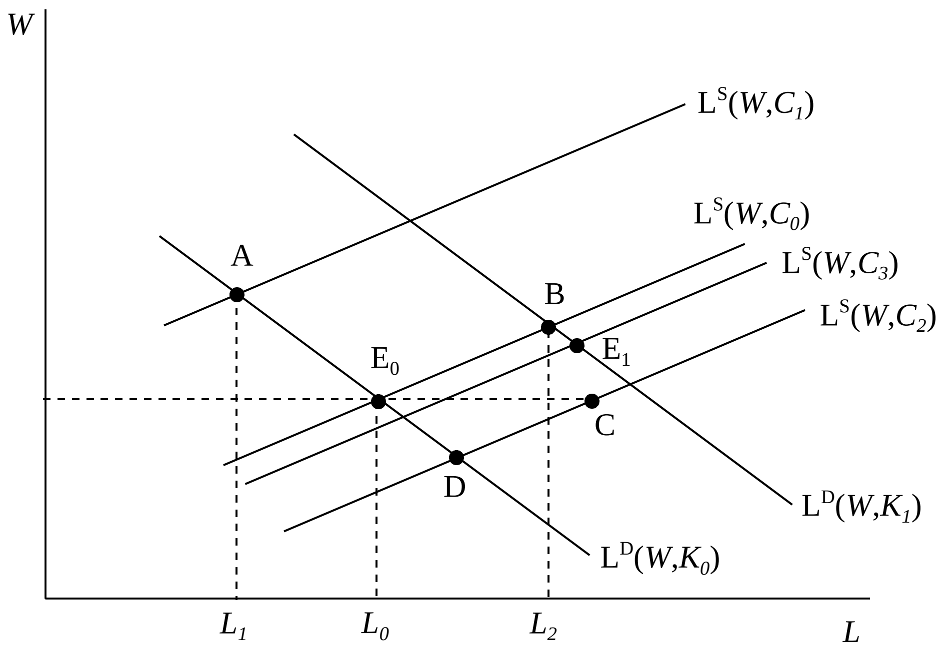


Figure 4. Fiscal policy under lump-sum taxation: Exogenous labour supply and finite lives

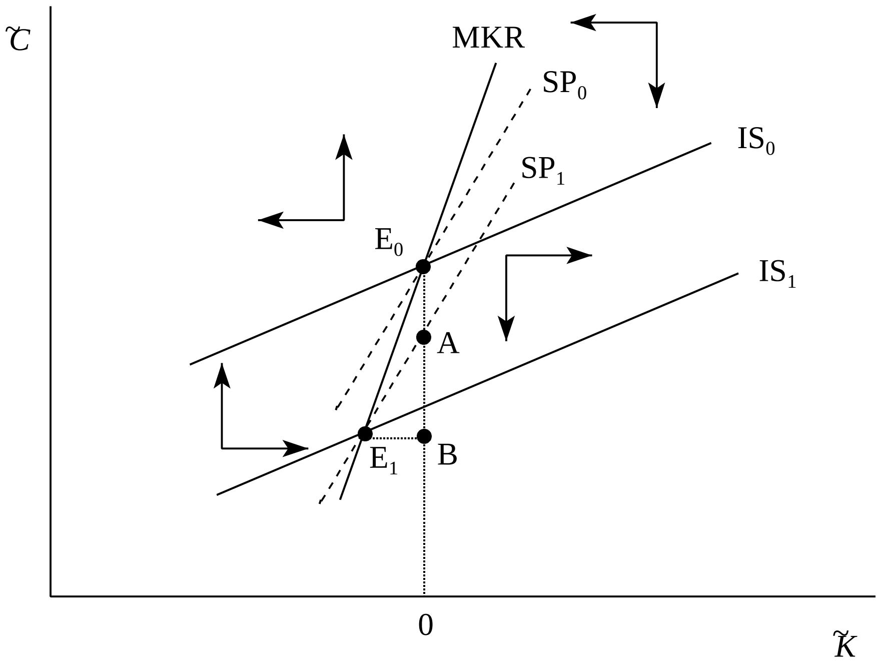


Figure A.1. Fiscal policy under bond financing