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ABSTRACT

In this paper a dynamic macroeconomic model of monopolistic competition is developed for the closed economy. Forward looking consumers demand differentiated goods and supply labour. They save part of their income in the form of shares and government bonds. Forward looking producers manufacture the differentiated goods by using labour and capital. Investment is subject to external adjustment costs. The model is used to investigate analytically the short-run, transition, and long-run effects of fiscal policy under alternative government financing regimes. The policy experiments are conducted under both the Ramsey-Barro case of infinite horizons, and the Blanchard case of finite horizons. Comparisons with the conventional case of perfect competition are also made. Simple expressions for Keynesian multipliers are derived. The sensitivity of the multipliers to finite lives, Ethier productivity effects, intratemporal and intertemporal substitution effects, and restricted entry are also studied. The quantitative significance of the assumption of an imperfectly competitive market structure is demonstrated.

JEL codes: E12, E63, L16

Keywords: monopolistic competition, love of variety, returns to scale, capital accumulation, Keynesian multipliers, labour supply, overlapping generations

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1. Introduction

Following the influential paper by Oliver Hart (1982), a number of authors have used the assumption of imperfect competition in order to provide credible microeconomic foundations for New Keynesian macroeconomics. Prominent examples include Ng (1982, 1986), Blanchard and Kiyotaki (1987), Mankiw (1988), and Dixon (1987). The results obtained by some of these New Keynesian theorists seem quite spectacular. Mankiw (1988), for example, shows that the balanced-budget multiplier is certainly positive and even approaches unity (the traditional Keynesian Cross result) as the mark-up of price over marginal costs rises.[1] Blanchard and Kiyotaki (1987) use a version of the Dixit-Stiglitz (1977) model to show that movements in demand can affect output and welfare, provided small menu costs are present. Again a very Keynesian conclusion derived from an imperfectly competitive framework.

The imperfect competition approach has not remained unchallenged in its attempt to assume the position once held by the IS-LM model in macroeconomic theory and policy. The overlapping generations model of Blanchard (1985) has managed to take centre stage in the last few years. Extensions by Buiter (1988) and Marini and van der Ploeg (1988) for the closed economy and Matsuyama (1987) and Giovannini (1988) for the open economy show this model to be both practical and extremely flexible. Marini and van der Ploeg (1988, p. 779) have shown that tax-financed fiscal policy is at best impotent (if agents have infinite planning horizons) or leads to more than full crowding-out (if horizons are finite).

Proponents of the imperfect competition approach may argue that the overlapping generations model yields these Classical conclusions exactly because one of the corner stones of the Keynesian creed has been left out, namely increasing returns to scale. Indeed, in an influential article Weitzman (1982) has argued that increasing returns to scale should occupy the central position in any plausible theory of the macroeconomic process. It is therefore the purpose of this paper to investigate the efficacy of fiscal policy in a model in which the basic insight from both schools of thought have been synthesised. From the imperfect competition approach we borrow the notion of increasing returns to scale and differentiated products, whilst the demographic structure has been taken from the overlapping generations approach.

In our view this synthesis enriches both approaches. The imperfect competition approach is significantly extended by including explicit dynamics into the model. The dynamic linkages are formed by capital accumulation, overlapping generations, and by the government budget restriction. This enables an evaluation of various types of fiscal policy within the same unified framework, taking into account the various financing methods at the disposal of the government. The assumption of perfect foresight furthermore allows for a meaningful distinction between the short-run, the transition period, and the long-run. The overlapping generations model of Blanchard (1985), Buiter (1988), and Marini and van der Ploeg (1988) is extended as well. First, the model is extended by incorporating an endogenous labour supply decision by households. Second, due to the increasing returns to scale assumption and the explicit price-setting by imperfectly competitive producers, the model is potentially capable of yielding Keynesian conclusions.

[1]Not all authors reach such vintage Keynesian conclusions. Ng (1986, pp. 157-159), for example, argues that the balanced-budget multiplier is likely to be negative. See also the discussion by Heijdra and van der Ploeg (1993).

In addition to achieving a synthesis between two highly influential approaches in modern macroeconomics, the paper also enables us to raise a number of further issues. First, by building the model on firm microeconomic foundations, the relationship between New Keynesian and real business cycle (RBC) theories is further clarified. One of the crucial elements of the model developed in this paper is the intertemporal labour-leisure choice, a feature it shares with the RBC model. It avoids, however, the Panglossian aspect of RBC theory that was forcefully ridiculed by Krugman (1994, p. 204). Indeed, because of the existence of the distortion of monopolistic competition, the world described in this paper is inherently second-best, and government intervention can improve matters. This point has been largely ignored in much of the static literature on the macroeconomics of monopolistic competition.[2]

A second, related, point is that the model could be potentially useful in explaining an empirical puzzle. It is well known that empirical studies find significant output effects of government spending (See, *e.g.*, Barro, 1981; Aschauer, 1988; Garcia-Milà, 1989). In the perfectly competitive model these output effects can only be explained by appealing to unrealistically large intertemporal and intratemporal substitution elasticities. The results in this paper indicate that part of this empirical tension could be attributed to the existence of scale economies. Even relatively small scale economies can make a major difference to the predicted output effect of government spending.

Third, the benchmark version of the model developed in this paper can be analyzed with the aid of simple phase diagrams. Unlike Baxter and King (1993), who resort to numerical simulation of their RBC model, we can demonstrate the effects of permanent or temporary and anticipated or unanticipated policy shocks both analytically and in very simple diagrams. This simple didactic style may make the model useful for teaching purposes.

Fourth, by basing the model on monopolistic competition and overlapping generations, the model should be useful to issues of trade policy and industrial policy as well. Indeed, it could be viewed as a contribution to the integration of macroeconomics, public finance and industrial policy. Due to the existence of overlapping generations and the availability of bond financing, the possibility of Pareto-improving industrial or trade policy can be usefully analyzed with the aid of the model.

The paper is organised as follows. In section 2, the basic theoretical model is developed. In section 3 a useful solution approach is suggested for analyzing the dynamic properties of a benchmark version of the model. This approach is used to examine the efficacy of fiscal policy under three types of financing: financing by means of lump-sum taxes, financing by means of a distortionary tax on labour income, and bond-financing. The efficacy of fiscal policy is shown to depend critically on the assumed market structure and on the length of the agents' planning horizon. In section 4, the effects on the magnitude of the long-run multiplier of various extensions to the benchmark model are discussed in turn. Furthermore, some additional properties of the model are illustrated by means of numerical simulations of a plausibly calibrated version of the model. Finally, in section 5, the paper concludes with some suggestions for further research.

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[2]A discussion of the intergenerational welfare effects of fiscal policy is found in a companion paper (Heijdra, 1994). In Heijdra and van der Ploeg (1993) we discuss the optimal level of government spending in a static model of monopolistic competition.

2. A Model of Perpetual Youth and Imperfect Competition

2.1. Individual Households

The basic model of household behaviour builds on the work of Blanchard (1985). Our extension to his model lies in the explicit recognition of an investment goods sector, an endogenous labour supply decision, and in the introduction of diversified consumption goods into the utility function of the agents. This in turn opens the scope for imperfectly competitive behaviour on the part of producers, which forms the major innovation of this paper.

In this model there is a fixed population of agents each facing a given constant probability of death. During their entire life agents have a time endowment of unity which they allocate over labour and leisure. The utility functional of the representative agent born at time s is assumed to have the following form.

$$\underset{\{C(s,v),\,1-L(s,v)\}}{\text{Max}} \int_t^\infty \frac{1}{1-1/\xi}\left(\Omega(C(s,v),1-L(s,v))^{1-1/\xi} - 1\right) e^{(\alpha+\beta)(t-v)} dv \tag{2.1}$$

Where $C(s,v)$ and $L(s,v)$ are, respectively, consumption of the composite diversified good and labour supply in period v of an agent born in period s, α is the pure rate of time preference ($\alpha > 0$), β is the probability of death ($\beta \geq 0$), $\Omega(.)$ is the instantaneous utility function, and $\xi(>0)$ is the intertemporal substitution elasticity.[3] The agent's budget restriction is given by the following.[4]

$$\dot{A}(s,t) = [r(t)+\beta]A(s,t) + [1-t_L(t)]W(t)L(s,t) - Z(t) - C(s,t) \tag{2.2}$$

Where $r(t)$ is the real rate of interest on government bonds, $W(t)$ is the real wage rate (assumed age-independent for convenience), $t_L(t)$ is the proportional tax rate on labour income, $Z(t)$ are real net lump-sum taxes, and $A(s,t)$ are real financial assets. Total consumption is defined as the sum of goods consumption and the opportunity cost of leisure consumption.

$$X(s,t) \equiv C(s,t) + [1-t_L(t)]\,W(t)(1-L(s,t)) \tag{2.3}$$

The economy consists of imperfectly competitive firms that each produce a single variety of a diversified good. These goods are close but imperfect substitutes in consumption. Following Spence (1976) and Dixit and Stiglitz (1977) the diversified goods can be aggregated over existing varieties $(1,2,..,N(t))$ in order to obtain $C(s,t)$.

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[3]As $\xi \rightarrow 1$ the utility function converges to $\log[\Omega(.,.)]$.

[4]Underlying the budget restriction is the Keynesian assumption that claims on physical capital and government bonds are perfect substitutes ensuring equalisation of their *ex ante* rate of return. A dot above a variable designates the derivative with respect to time, *e.g.*, $\dot{A}(s,t) \equiv dA(s,t)/dt$.

$$C(s,t) \equiv \left[\sum_{i=1}^{N(t)} C_i(s,t)^{\frac{\sigma_C - 1}{\sigma_C}} \right]^{\frac{\sigma_C}{\sigma_C - 1}} \qquad \sigma_C > 1 \tag{2.4}$$

Where $C_i(s,t)$ is the consumption of a good of variety i in period t by an agent born in period s, and σ_C is the elasticity of substitution between the differentiated goods. The restriction on σ_C ensures that the individual agents exhibit a love of variety and that all existing varieties will in fact be demanded.

The consistent aggregate price deflator for the differentiated commodities associated with (2.4) is defined as follows.

$$P(t) \equiv \left[\sum_{i=1}^{N(t)} P_i(t)^{1-\sigma_C} \right]^{\frac{1}{1-\sigma_C}} . \tag{2.5}$$

In order to keep the model as simple as possible, the instantaneous utility function is assumed to be of the simple modified CES form.

$$\Omega[.,.] \equiv \left[\gamma_C^{\frac{1}{\sigma_{CM}}} C(s,t)^{\frac{\sigma_{CM}-1}{\sigma_{CM}}} + (1-\gamma_C)^{\frac{1}{\sigma_{CM}}} \left(1 - L(s,t)\right)^{\frac{\sigma_{CM}-1}{\sigma_{CM}}} \right]^{\frac{\sigma_{CM}}{\sigma_{CM}-1}} \tag{2.6}$$

Where σ_{CM} is the substitution elasticity between consumption and leisure. We assume that $0<\gamma_C\leq 1$ and $\sigma_{CM}\geq 0$. The representative agent chooses a time profile for $C_i(s,v)$, $L(s,v)$, $C(s,v)$, and $X(s,v)$ in order to maximise (2.1) subject to (2.2)-(2.6), and a No Ponzi Game (NPG) solvency condition. The solutions are derived in the Appendix, where a three-stage solution method is used to solve the model. For period t the solutions for $X(s,t)$, $L(s,t)$, $C(s,t)$, and $C_i(s,t)$ are as follows.

$$X(s,t) = \left[\Delta(t)\right]^{-1}[A(s,t) + H(s,t)] \tag{2.7a}$$

$$1 - L(s,t) = \frac{(1-\gamma_C)[(1-t_L(t))W(t)]^{-\sigma_{CM}}}{\left[P_U(t)^{1-\sigma_{CM}}\right]} X(s,t) \tag{2.7b}$$

$$C(s,t) = \frac{\gamma_C}{\left[P_U(t)^{1-\sigma_{CM}}\right]} X(s,t) \tag{2.7c}$$

$$C_i(s,t) = \left[\frac{P_i(t)}{P(t)} \right]^{-\sigma_C} C(s,t) \tag{2.7d}$$

Where $P_U(t)$ is the true price index involving the after tax wage rate and preference parameters.

$$P_U(t) \equiv \left[\gamma_C + (1-\gamma_C)[(1-t_L(t))W(t)]^{1-\sigma_{CM}}\right]^{\frac{1}{1-\sigma_{CM}}} \tag{2.7e}$$

Expected life-time human wealth ($H(s,t)$) and the inverse propensity to consume out of total wealth ($\Delta(t)$) are defined as follows.

$$H(s,t) \equiv \int_t^\infty [(1-t_L(v))W(v) - Z(v)]e^{-\int_t^v [r(\mu)+\beta]d\mu} dv \tag{2.7f}$$

$$\Delta(t) \equiv P_U(t)^{\xi-1} \int_t^\infty e^{-(1-\xi)\int_t^v [r(\mu)+\beta]d\mu} P_U(v)^{1-\xi} e^{\xi(\alpha+\beta)(t-v)} dv \tag{2.7g}$$

Equation (2.7a) relates total consumption to total wealth, which is the sum of financial and human wealth. Equation (2.7b-c) express leisure and the composite differentiated consumption good respectively as shares of total consumption. Finally, equation (2.7d) defines the demands for the different varieties of the diversified good.

2.2. The Aggregate Households Sector

A crucial feature of the Blanchard (1985) model (and all models deriving from it) is the simple demographic structure, which enables the aggregation over all currently alive households. Assume that at each instance a large cohort of size βF is born and that βF agents die. Normalising F to unity the size of the population is constant and equal to unity (Blanchard, 1985).[5] Given this simple demographic structure, the aggregated variables can be calculated as the weighted sum of the values for the different generations. For example, aggregate financial wealth, $A(t)$, is calculated as follows.

$$A(t) \equiv \int_{-\infty}^t A(s,t)\beta e^{\beta(s-t)} ds \tag{2.8}$$

The aggregated values for $X(t)$, $L(t)$, $C(t)$, $Z(t)$, $H(t)$, and $C_i(t)$ can be obtained in the same fashion. The main equations describing the behaviour of the aggregated household sector are the following.

$$\dot{A}(t) = r(t)A(t) + [1-t_L(t)]W(t) - Z(t) - X(t) \tag{2.9a}$$

$$X(t) = [\Delta(t)]^{-1}[A(t)+H(t)] \tag{2.9b}$$

[5]The model can be easily extended to a (traditional) growth setting by adopting the Buiter (1988) extensions.

$$L(t) = 1 - \frac{(1-\gamma_C)[(1-t_L(t))W(t)]^{-\sigma_{CM}}}{\left[P_U(t)^{1-\sigma_{CM}}\right]} X(t) \tag{2.9c}$$

$$C_i(t) = \frac{\gamma_C}{\left[P_U(t)^{1-\sigma_{CM}}\right]} \left[\frac{P_i(t)}{P(t)}\right]^{-\sigma_C} X(t) \tag{2.9d}$$

Equation (2.9a) is the aggregated version of (2.2). In the aggregation the Blanchard assumption of perfect insurance markets is used to eliminate the parameter β from the expression. As is explained by Blanchard (1985, p. 229), $\beta A(t)$ is a transfer from those who die to those who remain alive. Consequently it does not form part of aggregate wealth.

Finally, (2.7f-g) imply that aggregate human wealth and the inverse propensity to consume out of total wealth respectively evolve as follows.

$$\dot{H}(t) = [r(t)+\beta]H(t) - [(1-t_L(t))W(t) - Z(t)] \tag{2.9e}$$

$$\dot{\Delta}(t) = -1 + \left[(1-\xi)[r(t) - \dot{P}_U(t)/P_U(t) + \beta] + \xi(\alpha+\beta)\right]\Delta(t) \tag{2.9f}$$

where $\dot{P}_U(t)/P_U(t)$ is the rate of change in the true price index $P_U(t)$ (defined in (2.7e) above).

2.3. Individual Firms

There are $N(t)$ identical firms that each produce one variety of the differentiated product.[6] The typical firm's decision on how much inputs to use is based on dynamic profit maximisation. There are increasing returns to scale at the firm level and the gross production function is homogenous of degree λ (≥ 1) in the two production factors capital ($K_i(t)$) and labour ($L_i(t)$). The production function, $F(K_i(t),L_i(t))$, is assumed to be of the CES type with increasing returns to scale.

$$Y_i(t) + f = F(K_i(t),L_i(t)) \equiv \left[\epsilon_L L_i(t)^{\frac{\sigma_{KL}-1}{\sigma_{KL}}} + (1-\epsilon_L)K_i(t)^{\frac{\sigma_{KL}-1}{\sigma_{KL}}}\right]^{\frac{\lambda\sigma_{KL}}{\sigma_{KL}-1}} \tag{2.10}$$

Where σ_{KL} is the substitution elasticity between capital and labour ($\sigma_{KL}>0$) and $0<\epsilon_L<1$, $Y_i(t)$ is net output, and f is fixed cost.[7] For convenience, these costs are modelled in terms of the firm's

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[6]This is a feature of the underlying model by Dixit and Stiglitz (1977). There is no incentive for any firm to produce a copy of an already existing variety.

[7]This approach to introducing fixed costs was used in a different context by Blanchard and Kiyotaki (1987, p. 661). Ethier (1982) rationalises the approach by appealing to indivisibilities in production. Technically, the inclusion of positive fixed costs guarantees that the firm's cost function is not iso-elastic, and that there are increasing returns to scale even if $\lambda=1$. If no fixed costs are included and $\lambda>1$ then so-called fragile equilibria result (See, *e.g.*, Heijdra and Broer, 1993).

own output. In order to obtain a well-defined and tractable theory of investment we adopt a two-sector technology in the vein of Srinivasan (1964), Uzawa (1969), Foley and Sidrauski (1971) and Mussa (1977). This approach yields an investment supply function. It furthermore implies that the imperfectly competitive firms face external adjustment costs.[8]

The investment goods market is assumed to operate under constant returns to scale and perfect competition. Using the basic insight of Ethier (1982), it is assumed that investment goods are constructed by using all existing varieties of the differentiated good as inputs. The production function is assumed to be as follows.

$$Q(t) = N(t)^{\alpha_Q} \left[\sum_{i=1}^{N(t)} N(t)^{-1} \, I_i(t)^{\frac{\sigma_I - 1}{\sigma_I}} \right]^{\frac{\sigma_I}{\sigma_I - 1}} \tag{2.11}$$

Where $I_i(t)$ is the quantity of variety i of the differentiated good used as inputs in the production of new investment goods, $Q(t)$ is the total production of new investment goods, and σ_I is the substitution elasticity between the different varieties. The parameter $\alpha_Q \geq 1$ regulates the productivity effects of increased variety, as is explained below.

The cost function associated with (2.11) is denoted by $TC_I(t) \equiv P_Q(t)Q(t)$, where $P_Q(t)$ is the price of new investment goods:

$$P_Q(t) = N(t)^{-\alpha_Q} \left[\sum_{i=1}^{N(t)} N(t)^{-\sigma_I} \, P_i(t)^{1-\sigma_I} \right]^{\frac{1}{1-\sigma_I}} \tag{2.12}$$

The input demand functions are also obtained from (2.12) by applying Shephard's Lemma.

$$I_i(t) \equiv \frac{\partial TC_I(t)}{\partial P_i(t)} = N(t)^{-(\sigma_I + \alpha_Q) + \alpha_Q \sigma_I} \, Q(t) \left[\frac{P_i(t)}{P_Q(t)} \right]^{-\sigma_I} \tag{2.13}$$

Note that the chosen formulation of the investment goods sector implies *external* economies of scale due to increasing diversity provided $\alpha_Q > 1$. This is the basic Ethier (1982) insight: more diversity in the differentiated goods sector enables investment goods producers to use a more "roundabout" production process and hence lower per unit cost, *i.e.*, $\partial P_Q / \partial N < 0$ at given input prices (P_i).

In order to motivate the conditions characterising the optimal behaviour of firms in the differentiated sector, consider the following heuristic derivation. Each firm in the imperfectly competitive consumption goods industry faces a demand for its product from three sources: the households sector (C_i), the investment goods sector (I_i), and the government (G_{0i}). Representative firm i's optimisation problem is as follows.

[8]The alternative is to assume internal adjustment costs leading to an investment demand function (See Abel and Blanchard, 1983). We have chosen to use the external adjustment costs approach since it puts less strain on the symmetric equilibrium assumption employed below, especially when exit/entry is allowed.

$$\text{Max } V_i^n(t) = \int_t^{\infty} \left[P_i(v)Y_i^d(v) - W^n(v)L_i(v) - P_Q(v)Q_i(v)\right]e^{-\int_t^v R^*(\mu)d\mu} dv \tag{2.14}$$

Subject to the following restrictions.

$$Y_i^d(t) = C_i(t) + I_i(t) + G_{0i}(t) \tag{2.15a}$$

$$\dot{K}_i(t) = Q_i(t) - \delta^*(t)K_i(t) \tag{2.15b}$$

$$Y_i(t) + f = \mathrm{F}[K_i(t),L_i(t)] \tag{2.15c}$$

$$Y_i^d(t) = Y_i(t) \tag{2.15d}$$

Where $\delta^*(t) \equiv \delta + \dot{N}(t)/N(t)$ is the "depreciation rate" from the firm's point of view, $R^*(t) \equiv R(t)$-$\dot{N}(t)/N(t)$ is the rate of return per firm, and W^n is the nominal wage rate ($W^n \equiv WP$).[9] Equation (2.15a) formally defines the sources of demand facing firm i. Equation (2.15b) shows that firm i's capital stock is adjusted by purchasing new investment goods from the investment good sector. Equation (2.15c) is the production function. Since there is no possibility of inventory formation, (2.15d) formalizes the restriction that production must equal demand.

It is shown in the Appendix that the familiar marginal conditions for labour and capital can be obtained.

$$\frac{\partial Y_i}{\partial L_i} = \left[\frac{\epsilon_i}{\epsilon_i+1}\right]\left[\frac{W^n}{P_i}\right] \tag{2.16a}$$

[9]The intuition for the $\dot{N}/N$ terms in the discounting factor is as follows. The owners of the capital stock (households) demand a return on their aggregate capital stock equal to the nominal return on government bonds (R). The nominal value of the capital stock is defined as $V^n \equiv NV_i^n$. By renting out the capital stock to firms in the differentiated goods sector, the owners of capital receive the cash-flow of these firms, the aggregate of which equals $CF \equiv NCF_i$. The arbitrage equation for shares and bonds then reads as $(\dot{V}^n+CF)/V^n=R$, which can be rewritten in terms of individual firms as $(\dot{V}_i^n+CF_i)/V_i^n=R\text{-}\dot{N}/N \equiv R^*$. Integrating this latter expression yields (2.14).

For the depreciation term the intuition runs as follows. As the number of firms increases ($\dot{N}/N>0$), the capital stock per active firm must decline because the aggregate capital stock is predetermined in the short run. This withdrawal of physical capital shows up in the accelerated decline in the individual firm's capital stock. Since the individual firm has no control over the total number of firms in the differentiated sector (N), it takes $\delta^*(t)$ as given.

$$\frac{\partial Y_i}{\partial K_i} = \left[\frac{\epsilon_i}{\epsilon_i+1}\right]\left[\frac{P_Q}{P_i}\right]\left(R+\delta-\dot{P}_Q/P_Q\right) \tag{2.16b}$$

Where ϵ_i is the elasticity of the demand curve faced by firm i. Note that (2.16a) and (2.16b) imply that the firm sets its price equal to a mark-up (defined as $\mu_i \equiv \epsilon_i/(1+\epsilon_i)$) times marginal cost.

The value of the individual firm depends critically on the assumption made regarding exit and entry of firms. In this paper we focus attention on the case of pure *Chamberlinian monopolistic competition* (CMC). Entry or exit occurs instantaneously to ensure that each active firm makes zero excess profit; the well-known tangency solution. In terms of the present model this implies that N adjusts instantaneously to eliminate excess-profits.[10] Given the iso-elastic *gross* production function (2.10), this implies a simple relationship between the equilibrium mark-up (μ_i) and technology and scale parameters, *i.e.*, $\mu_i = \lambda(Y_i + f)/Y_i$. Under this situation the nominal value of the firm is equal to the replacement value of its capital stock, *i.e.*, $V_i^n = P_Q K_i$.[11]

2.4. The Government

The government is assumed to produce a public good, denoted by G_0, which is provided to all consumers on a non-excludable basis free of charge. In order to retain symmetry, the production function for the public good is assumed to be defined over all existing varieties of the differentiated good.[12]

$$G_0(t) = N(t)^{\alpha_G}\left[\sum_{i=1}^{N(t)} N(t)^{-1}\, G_{0i}(t)^{\frac{\sigma_G-1}{\sigma_G}}\right]^{\frac{\sigma_G}{\sigma_G-1}} \tag{2.17}$$

Where σ_G is the substitution elasticity ($\sigma_G \geq 0$), and α_G is the diversity parameter. The government is assumed to be efficient in the sense that it produces $G_0(t)$ at minimum cost. The government's outlays on the differentiated goods therefore coincides with its "cost function" $TC_G(t) \equiv P_G(t)G_0(t)$,

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[10]In the static literature without saving and capital formation, the distinction between the short run and the long run is modelled somewhat artificially by setting N fixed in the short-run and variable in the long run. See, for example, Startz (1989), Dixon and Lawler (1993), and Heijdra and van der Ploeg (1993).

[11]The other polar case to be studied briefly is that of no entry/exit at all. This case can be labelled *restricted monopolistic competition* (RMC). There is monopolistic competition in the sense that many differentiated products are produced and marketed, but the tangency solution is dropped, and the number of firms (N) is held fixed. In that situation there typically exist non-zero excess profits (unless, by chance, the number of firms happens to be at the level consistent with tangency), so that the value of individual firms is augmented by the present value of current and future excess profits. This case is studied briefly in section 4.4 below.

[12]Alternatively, (2.17) may be interpreted as a component of each agent's utility function. Provided G_0 enters the utility function in a separable manner, the analysis of this paper is unchanged. Of course, any welfare-theoretic exercises involving changes in G_0 are critically affected by the chosen interpretation of (2.17).

where $P_G(t)$ is a "price index" of the public good:

$$P_G(t) \equiv N(t)^{-\alpha_G} \left[\sum_{i=1}^{N(t)} N(t)^{-\sigma_G} P_i(t)^{1-\sigma_G} \right]^{\frac{1}{1-\sigma_G}} \tag{2.18}$$

The government's demands for the various products are then obtained by applying Shephard's Lemma.

$$G_{0i}(t) \equiv \frac{\partial TC_G(t)}{\partial P_i(t)} = G_0(t) N(t)^{-(\sigma_G+\alpha_G)+\alpha_G\sigma_G} \left[\frac{P_i(t)}{P_G(t)} \right]^{-\sigma_G} \tag{2.19}$$

Since the objective of this paper is to analyze a wide range of economic policies, the financing method employed by the government must be spelled out in detail. To that effect the budget restriction of the government must be formulated. The periodic budget restriction can be written as follows.

$$\dot{B}(t) = (P_G(t)/P(t))\, G_0(t) + r(t)B(t) - t_L(t)\, W(t)L(t) - Z(t) \tag{2.20}$$

Where $B(t)$ is outstanding real government debt at time t. The government can finance its expenditure on goods and interest on debt by issuing new debt ($\dot{B}(t)$), or by changing one or both of its tax instruments, *i.e.*, lump-sum taxes ($Z(t)$) and the proportional tax rate on labour ($t_L(t)$). It is assumed that the government is expected to remain solvent, so that the following NPG (or solvency) condition is relevant.

$$\lim_{\tau\to\infty} B(\tau) \exp\left[-\int_t^{\tau} r(v)\mathrm{d}v \right] = 0 \tag{2.21}$$

The government's budget restriction is obtained by integrating (2.20) forward subject to the NPG condition (2.21).

$$B(t) = \int_t^{\infty} \left[Z(\tau) + t_L(\tau)\, W(\tau)L(\tau) - (P_G(\tau)/P(\tau))\, G_0(\tau) \right] e^{-\int_t^{\tau} r(v)\mathrm{d}v} \mathrm{d}\tau \tag{2.22}$$

2.5. The Elasticity of Demand and the Mark-Up

The elasticity of demand for firm i's product can be calculated easily. Using (2.9d), (2.13) and (2.19) the elasticity of the demand function facing firm i is easily seen to be equal to the following.

$$\epsilon_i = -[\sigma_C\omega_C + \sigma_I\omega_Q + \sigma_G\omega_G] = \frac{\mu_i}{1-\mu_i} \tag{2.23}$$

Where $\omega_C \equiv C_i/Y_i^d$ is the demand (and output) share of private consumption, $\omega_Q \equiv I_i/Y_i^d$ is the demand share of the (input demand originating from the) investment good sector, and $\omega_G \equiv G_{0i}/Y_i^d$ is the demand share of the government. The second equality in (2.23) implicitly defines the mark-up as a function of parameters σ_C, σ_I, and σ_G, and the share parameters ω_C, ω_Q, and ω_G. Note that (2.15a) implies that these shares add to unity, $\omega_C+\omega_Q+\omega_G=1$, so that the government can attempt to influence these share parameters by choice of its own consumption.

2.6. Symmetric Equilibrium

As is conventional in the macroeconomic literature on imperfect competition, the attention is restricted to the symmetric equilibrium in which the following conditions are satisfied: $\mu_i=\mu$, $P_i=\bar{P}$, $C_i=\bar{C}$, $Y_i=\bar{Y}$, $L_i=\bar{L}$, $K_i=\bar{K}$, $I_i=\bar{I}$, and $Q_i=\bar{Q}$ $(i=1,2,..,N(t))$. Under this assumption the firms in the differentiated goods sector can be treated in the aggregate. Note that $Y(t)$ is a quantity index for total production in the differentiated sector, which is defined as follows.

$$Y(t) \equiv \frac{\sum_{i=1}^{N(t)} P_i(t)\,Y_i(t)}{P(t)} = \left[\sum_{i=1}^{N(t)} Y_i(t)^{\frac{\sigma_C-1}{\sigma_C}}\right]^{\frac{\sigma_C}{\sigma_C-1}} \tag{2.24}$$

The complete model is given in its most general form in Table 1. Turning to this table, the dynamic part of the model is given by equations (A)-(E). The aggregate physical capital stock evolves according to (A). The movement of real total consumption is described by equation (B). Note that financial wealth, equalling the real value of shares (V) plus government debt (B), has been substituted. Equation (C) describes the path of the inverse propensity to consume out of total wealth, Δ. If the intertemporal substitution elasticity ξ equals unity, then this equation is not needed and $\Delta^{-1} = (\alpha+\beta)$. Equation (D) is the government budget restriction. If debt financing is used, equation (E) is also needed in order to ensure that the government's solvency condition (2.21) is satisfied. If the government uses tax instruments to finance its expenditure then B is constant, (D) is static, and (E) is not needed.

The static part of the model is given by equations (F)-(Q). Equation (F) gives the expression for the real value of shares under Chamberlinian monopolistic competition. The aggregate demands for labour and capital are given by (G) and (H) respectively. The equilibrium condition for the market for differentiated goods is given in (I).[13] The aggregate private demand for the composite differentiated good is given by (J). The labour supply equation is given in (K). The aggregate production function for the differentiated goods sector is given by (L), which represents (2.10) re-expressed in terms of the aggregate quantity index Y and aggregate inputs K and L. The LHS(M) defines the absolute value of the price elasticity faced by each producer of the differentiated good, but written in terms of aggregate shares. Formally (M) is used to define the mark-up of price over marginal cost (μ). The zero pure profit condition is enforced by (N). Together with (G)-(H) it ensures that price equals average cost for each active firm in the

[13]Equilibrium in the labour market, the market for existing capital, and the market for new investment goods has already been imposed by substituting the equilibrium conditions $L=N\bar{L}$, $K=N\bar{K}$ and $Q=N\bar{Q}$ in the various expressions.

differentiated goods sector.[14] Equations (P) and (Q) give the definitions of the various price indexes. Finally, equation (R) states the relationship between the rental rate on existing capital, the real rate of interest, and the rate of change in the relative price of new capital goods.

In Table 2 the log-linearized version of the complete model is given. The following notational conventions are adopted for all flow variables and the capital stock.

$$\dot{\tilde{x}}(t) \equiv \frac{\dot{x}(t)}{x(0)} \qquad \tilde{x}(t) \equiv \frac{\mathrm{d}x(t)}{x(0)} \tag{2.25}$$

Hence, a variable with a tilde ("~") denotes the percentage rate of change in that variable, relative to the initial steady-state and a variable with a tilde and a dot, is the time rate of change in terms of the initial level. For government debt (B), shares (V), and the labour tax rate (t_L) the following conventions are used, respectively.

$$\dot{\tilde{x}}(t) \equiv \frac{r(0)\dot{x}(t)}{Y(0)} \qquad \tilde{x}(t) \equiv \frac{r(0)\,\mathrm{d}x(t)}{Y(0)} \qquad x \in \{B, V\} \qquad \tilde{t}_L(t) \equiv \frac{\mathrm{d}t_L(t)}{1-t_L(0)} \tag{2.26}$$

These conventions have several advantages. First, by multiplying the stocks by $r(0)$ these are converted into flows, that can then be expressed in terms of the initial flow of production $Y(0)$, so that readily interpretable income shares emerge. Second, the resulting expressions are meaningful even if the initial level of the variable is zero (*e.g.*, $B(0)$ or $t_L(0)$ may equal zero). Note that the time index for initial variables like $r(0)$ and $t_L(0)$ has been suppressed.

The model can now be used to discuss the effects of fiscal policy on the economy, both in the short-run and in the long-run, and both for finite and infinite horizons. Three different types of fiscal policy can be conducted with the model. They all involve an impulse to government spending (G_0) financed either by (i) lump-sum taxes (Z), by (ii) the distorting labour tax (t_L), or by (iii) debt (B).

3. The Effects of Fiscal Policy

3.1. A Benchmark Case

The linearized model given in Table 2 is too complicated to characterize *analytically* the effects of the different types of fiscal policy for the short run, the transition path, and the long run. For that reason attention is focused in this section on a benchmark case of the model with the following values for the elasticities and parameters:

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[14]It is straightforward to show that marginal cost of any active firm (MC_i) equals total cost (TC_i) divided by λ times gross production (Y_i+f), *i.e.*, $MC_i=TC_i/\lambda(Y_i+f)$. Equations (2.16a-b) imply that $P_i=\mu MC_i$. The tangency condition requires $P_i=AC_i=TC_i/Y_i$. Combining these conditions and rewriting in terms of the quantity index Y yields equation (N). Note that if the number of firms is fixed, then the following inequalities hold: $(\mu/\lambda)Y_i/(f+Y_i)$ > (=,<) 1 iff P_i < (=,>) AC_i. If there are "too few" firms, then existing firms will set price above average cost, or, $P_i > AC_i$, so that excess profits are made.

$$\sigma_C = \sigma_I = \sigma_G \qquad 1-\alpha_Q = 1-\alpha_G = \frac{1}{1-\sigma_C} \qquad \xi = \sigma_{CM} = \sigma_{KL} = 1 \tag{3.1}$$

The consequences of these assumptions are as follows. First, setting $\sigma_C=\sigma_I=\sigma_G$ implies that the source of demand facing each producer in the differentiated goods sector does not matter. Equation (M) in Table 1 then implies that the gross mark-up is constant and equal to $\mu=\sigma_C/(\sigma_C-1)>1$. The entry condition (N) then also implies that output per active firm is constant at $\bar{Y}=\lambda f/(\mu-\lambda)$ under Chamberlinian monopolistic competition. Hence, any aggregate output movement is in that case due to changes in the number of firms. Second, setting $\alpha_Q=\alpha_G=\sigma_C/(\sigma_C-1)=\mu$ implies that there are external scale economies (due to changing variety) in the production of new investment goods and government output. By setting this so-called Ethier effect equal to the mark-up, the relative prices P_G/P and P_Q/P are constant and equal to unity. Essentially, the model describes a one-good economy in that case. Third, the intertemporal substitution elasticity has been set equal to unity, implying that the income and substitution effects of real interest changes cancel. Fourth, setting the substitution elasticity between capital and labour equal to unity implies that the input demand equations do not depend separately on the number of firms and the mark-up. Fifth, setting the substitution elasticity between consumption goods and leisure equal to unity implies constant budget shares for consumption and leisure. Sixth, as already stated, attention is focused on pure Chamberlinian monopolistic competition with free entry and exit of firms. Seventh, in order to economize on notation we assume that the initial level of debt and the labour tax are zero ($t_L(0)=B(0)=0$). In section 4 of the paper we investigate the likely effects of relaxing some of these assumptions in turn. The log-linearized version of the benchmark model is given in Table 3.

3.2. Lump-Sum Tax Financing

The first fiscal policy experiment to be conducted consists of a *permanent unanticipated* increase in government spending on the differentiated good financed by means of lump-sum taxes. The dynamic system can be condensed to:

$$\begin{bmatrix} \dot{\tilde{K}} \\ \dot{\tilde{X}} \end{bmatrix} = \begin{bmatrix} (\delta/\omega_Q)(\mu\phi(1-\epsilon_L)-\omega_Q) & (\delta/\omega_Q)(1-\phi-\omega_C) \\ -(r-\alpha)+(r+\delta)[\mu\phi(1-\epsilon_L)-1] & (r-\alpha)+(r+\delta)(1-\phi) \end{bmatrix} \begin{bmatrix} \tilde{K} \\ \tilde{X} \end{bmatrix} + \begin{bmatrix} -\delta\omega_G/\omega_Q \\ 0 \end{bmatrix} \tilde{G}_0 \tag{3.2}$$

where ω_C, ω_Q and ω_G are the share of consumption, investment and government consumption in national income respectively, μ is the mark-up, ϵ_L is the share of labour income in national income, and ϕ is a crucial parameter representing the strength of the labour supply elasticity:

$$\phi \equiv \frac{1+\omega_{LL}}{1+\omega_{LL}(1-\mu\epsilon_L)} \geq 1 \tag{3.3}$$

where ω_{LL} is the ratio between leisure and labour, which equals the intertemporal substitution elasticity of labour supply in the benchmark model. It is straightforward to verify that $\partial\phi/\partial\omega_{LL}>0$ and $\partial\phi/\partial\mu>0$. Note that $\phi=1$ if labour supply is inelastic ($\omega_{LL}=0$). The characteristic roots of the Jacobian matrix (Δ_1) on the RHS(3.2) are denoted by r^* and $-h^*$ respectively. Saddle point stability implies that $|\Delta_1|$ is negative, *i.e.*, $|\Delta_1| \equiv -h^*r^*<0$. Hence, the characteristic roots alternate in sign. Let $r^*>0$ denote the unstable root and $-h^*<0$ the stable root. Then h^* characterizes the adjustment speed of the system, and r^* characterizes the extent of the impact effects.

The model can be reduced to two schedules. The first of these is the line along which the capital stock is in equilibrium (referred to hereafter as the IS curve), and the second is the Modified Keynes-Ramsey equation for the economy (the MKR curve). The IS curve is obtained by rewriting the first equation in (3.2) in steady-state form, and is unambiguously upward sloping in $(\tilde{X},\tilde{K})$ space:

$$\tilde{X} = \left[\frac{\omega_A + (\mu\phi - 1)(1-\epsilon_L)}{\omega_C + \phi - 1}\right]\tilde{K} - \left[\frac{\omega_G}{\omega_C + \phi - 1}\right]\tilde{G}_0 \tag{3.4}$$

Increasing government consumption withdraws resources from the economy. In order to maintain the same capital stock, private consumption must fall. Hence, an increase in government consumption shifts the IS curve down and to the right. This has been illustrated in Figure 1. The dynamic forces operating on the economy off the IS curve are obtained from the first equation of (3.2). Points above the IS curve represent consumption levels that are too high to maintain the present capital stock. Hence, the capital stock tends to fall. The opposite is the case for points below the IS curve.

The MKR curve is obtained by using the steady-state version of the second equation of (3.2):

$$(r-\alpha)(\tilde{X}-\tilde{K}) + (r+\delta)[(\mu\phi(1-\epsilon_L)-1)\tilde{K}+(1-\phi)\tilde{X}] = 0 \tag{3.5}$$

The MKR curve represents two distinct effects; the finite horizon effect and the labour supply effect. The finite horizon effect ensures that the steady-state rate of interest exceeds the pure rate of time preference ($r>\alpha$). In order to induce new generations to start accumulating claims to the existing capital stock their consumption profile must be tilted upwards, leading to low consumption early in life and high consumption later on.[15] The labour supply effect operates through the rate of interest.

3.2.1. Infinite Horizons

In order to further investigate the labour supply effect, it is useful to first discuss the Barro-Ramsey case of *infinite horizons*. In that case the steady-state rate of interest equals the rate of time preference ($r=\alpha$), and the MKR curve (3.5) can be simplified to yield:

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[15]See Blanchard (1985, p. 229) and Blanchard and Fischer (1989, p. 124).

$$\tilde{X} = \left[\frac{1-\mu\phi(1-\epsilon_L)}{1-\phi}\right]\tilde{K} \tag{3.6}$$

The MKR curve is unaffected by government spending because lump-sum taxes are used. Depending on the strength of the labour supply effect, the MKR curve may be (i) vertical (if $\phi=1$), (ii) downward sloping (if $\phi>1$ but $\mu\phi(1-\epsilon_L)<1$), or (iii) upward sloping (if $\phi\gg 1$ and $\mu\phi(1-\epsilon_L)>1$). All three cases are consistent with saddle point stability, and have been illustrated graphically in Figure 1.[16] The dynamic forces operating on the economy can be ascertained from the second equation of (3.2). In the first case, with inelastic labour supply, a low level of the capital stock implies a high marginal productivity of capital, a high rate of interest ($r>\alpha$), and an upward sloping profile for total consumption ($\tilde{X}$ rises over time).[17] In the second and third cases, the marginal product of capital depends both on the level of the capital stock *and* labour supply. In case two, a low level of the capital stock is still associated with a high rate of interest and a rising total consumption profile. In the third case, the marginal product of capital is increasing because of the strong intertemporal substitution effect exerted by the interest rate on labour supply. Hence, a low level of the capital stock is associated with a *low* rate of interest and a falling total consumption profile. The dynamic information has been illustrated in Figure 1 for the three cases, providing a graphical illustration of the saddle point stability. In all cases the saddle path is upward sloping.

The effects of the unanticipated permanent fiscal impulse ($\tilde{G}_0>0$) can now be illustrated. First, consider the case of inelastic labour supply, with $\phi=1$, as illustrated in Figure 1-a. As a result of the increase in public spending the IS curve and the saddle path both shift down by the amount of the shock. Since the capital stock is predetermined in the short run, the economy moves from E_0 to E_1, and total consumption $\tilde{X}$ falls by the full amount of the fiscal impulse. There is full crowding out of private consumption by public consumption, and there is no transitional dynamics (since the capital stock remains unchanged). This conclusion holds regardless of the assumed industrial structure. To summarize, the short-run and long-run multipliers coincide and are equal to the following:

$$\frac{dY(\infty)}{dG_0}\Big|_Z = \frac{dK(\infty)}{dG_0}\Big|_Z = 1+\frac{dC(\infty)}{dG_0}\Big|_Z = \frac{dL(\infty)}{dG_0}\Big|_Z = \frac{dQ(\infty)}{dG_0}\Big|_Z = 0 \tag{3.7}$$

Matters are slightly more complicated for the case of an *anticipated permanent* increase in the level of government spending, since there is some transitional dynamics in that case. At announcement total consumption $\tilde{X}$ again drops, but by less than for the previous case (say to

[16]Saddle point stability ensures that the IS curve is steeper than the MKR curve.

[17]This first case also suggests a natural assumption about the maximum strength of the scale economies, measured in the benchmark case by the mark-up, μ. Under monopolistic competition the model becomes unstable if the scale economies are so strong that $\mu(1-\epsilon_L)>1$, causing an increasing marginal product of capital. Hence, we assume that $\mu(1-\epsilon_L)<1$. In view of plausible values for the mark-up ($\mu\approx 1.3$) and the share of labour in national income ($\epsilon_L\approx 0.70$), this assumption is not very strong indeed.

point A in Figure 1-a), and between announcement and implementation the economy follows a smooth trajectory in south-easterly direction towards the new saddle path (from A to B), with consumption falling further and the capital stock rising. At implementation the economy has reached the new saddle path (point B) and both consumption and the capital stock fall until the final equilibrium E_1 is reached.

The second case we consider is that of a moderately elastic labour supply ($\phi > 1$ but $\mu\phi(1-\epsilon_L) < 1$). This case has been illustrated in Figure 1-b. The effects of a permanent unanticipated fiscal impulse are to shift the IS curve (and the saddle path) to the right. Since the capital stock is predetermined in the short run, the economy moves from E_0 to A, and total consumption $\tilde{X}$ falls. Now there is transitional dynamics, however, and in the medium run the economy moves from A to E_1 with both total consumption and the capital stock increasing smoothly. Total consumption $\tilde{X}$ and labour supply $\tilde{L}$ both overshoot in the short run. The additional work effort ensures that the higher capital stock is put in place. The long-run multipliers are:

$$\frac{dY(\infty)}{dG_0}\Big|_Z = \left[\frac{\alpha Y}{\omega_V}\right]\frac{dK(\infty)}{dG_0}\Big|_Z = \left[\frac{1}{\omega_Q}\right]\frac{dQ(\infty)}{dG_0}\Big|_Z = \left[\frac{1-\phi}{\omega_C[1-\mu\phi(1-\epsilon_L)]}\right]\frac{dC(\infty)}{dG_0}\Big|_Z = \left[\frac{1}{1-\omega_Q-\omega_C\left[\frac{1-\mu\phi(1-\epsilon_L)}{1-\phi}\right]}\right] > 0 \tag{3.8}$$

$$\frac{dX(\infty)}{dG_0}\Big|_Z = \left[\frac{X}{L}\right]\left[\frac{\mu\epsilon_L}{1-\phi}\right]\left[\frac{1-\mu\phi(1-\epsilon_L)}{1-\mu(1-\epsilon_L)}\right]\frac{dL(\infty)}{dG_0}\Big|_Z = \left[\frac{X}{Y}\right]\left[\frac{1-\mu\phi(1-\epsilon_L)}{1-\phi}\right]\frac{dY(\infty)}{dG_0}\Big|_Z \tag{3.9}$$

In the long run, aggregate output, the capital stock, investment, and employment all rise, and consumption falls. There is still some crowding out of private consumption, though less so than with inelastic labour supply, $-1 < dC(\infty)/dG_0 < 0$. The reason for this is that agents react to the fiscal shock by accumulating a larger capital stock, which in the steady-state gives rise to a higher level of (replacement) investment. This additional aggregate demand partially counteracts the negative effect on consumption.

In order to analyze the impact and transition effects of the permanent unanticipated fiscal impulse, it is useful to write the solution of the model as follows:

$$\tilde{X}(t) = \tilde{X}(0)e^{-h^* t} + \tilde{X}(\infty)\left(1-e^{-h^* t}\right) \tag{3.10a}$$

$$\tilde{K}(t) = \left(1 - e^{-h^{*}t}\right)\tilde{K}(\infty) \tag{3.10b}$$

$$\tilde{X}(0)\Big|_{Z} = \left[\frac{r^{*} - (r+\delta)(1-\phi)}{r^{*}(1-\phi-\omega_C)}\right]\omega_G \tilde{G}_0 < 0 \tag{3.10c}$$

where h^{*} is the adjustment speed of the economy (measured by *minus* the stable root of Δ_y in (3.2)) and r^{*} is the unstable root. Equations (3.10a) and (3.10b) describe the transition path of the economy, and (3.10c) gives the jump in total consumption that occurs on impact. As was already illustrated in Figure 1, total consumption falls on impact. Using (3.10c) and the fact that $\tilde{K}(0)=0$, the following impact effects can be derived for output, consumption, investment, and employment:

$$\tilde{Y}(0)|_{Z} = (1-\phi)\tilde{X}(0)|_{Z} > 0 \qquad \tilde{C}(0)|_{Z} = \tilde{X}(0)|_{Z} < 0$$

$$\tilde{Q}(0)|_{Z} = \left[\frac{(r+\delta)(\phi-1)(1-\phi-\omega_C)}{[r^{*} - (r+\delta)(1-\phi)]\,\omega_Q}\right]\tilde{X}(0)|_{Z} > 0 \qquad \tilde{L}(0)|_{Z} = \frac{(1-\phi)}{\mu\epsilon_L}\tilde{X}(0)|_{Z} > 0 \tag{3.11}$$

Consumption of the composite differentiated good falls on impact as agents start to save by accumulating capital (since there are no government bonds), so that the demand for new investment goods is increased. Leisure falls, so that labour supply increases. The additional labour enables an increase in output even though capital is fixed in the short run. The policy-induced investment boom plus the direct effect of government spending together ensure that consumption is not fully crowded out in the short run.

The transition path for real national income can be expressed as the weighted average of the short-run and long-run effects:

$$\tilde{Y}(t)|_{Z} = \left(1 - e^{-h^{*}t}\right)\tilde{Y}(0)|_{Z} + e^{-h^{*}t}\tilde{Y}(\infty)|_{Z} \tag{3.12}$$

Both the short-run and long-run effects on output are positive, but it is not unambiguous which effect is largest. If $\mu\phi(1\text{-}\epsilon_L)$ is sufficiently low, it is possible that the long-run effect is smaller than the short-run effect, so that output overshoots its long-run steady-state along the transition path.[18]

The third case we consider is that of a highly elastic labour supply ($\phi > 1$ and $\mu\phi(1\text{-}\epsilon_L) > 1$). This case has been illustrated in Figure 1-c. An unanticipated permanent increase in government spending ($\tilde{G}_0 > 0$) has a negative impact effect on total consumption ($\tilde{X}(0) < 0$) as the economy jumps from E_0 to A. The rate of interest rises, thereby prompting a positive labour

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[18]The difference between the long-run and short-run effects can be written as follows:

$$\tilde{Y}(\infty) - \tilde{Y}(0) = \frac{\delta(r+\delta)\omega_G(\phi-1)}{\omega_Q r^{*} h^{*}}\left[1 + \frac{h^{*}}{h^{*}+\delta_{22}}[\mu\phi(1-\epsilon_L) - 1]\right]$$

where $\delta_{22}=(r+\delta)(\phi\text{-}1)<0$. Clearly, $r^{*}+\delta_{11}=h^{*}+\delta_{22}$, and it is possible to derive that $\text{sgn}(h^{*}+\delta_{22})=\text{sgn}[1\text{-}\mu\phi(1\text{-}\epsilon_L)]$. Hence, for $\mu\phi(1\text{-}\epsilon_L)$ relatively close to unity we have $\tilde{Y}(0) < \tilde{Y}(\infty)$.

supply response. In the transition period the additional work effort leads to the construction of a higher capital stock and a steadily rising level of total consumption. The long-run multipliers are the ones reported in (3.8)-(3.9). The long-run employment and consumption multipliers are now both positive. Now there exists "crowding-in" of private consumption by public consumption, *i.e.*, $dC(\infty)/dG_0>0$, so that the real output multiplier is guaranteed to exceed unity.

In order to determine the long-run effects on the factor prices r and W, it suffices to look at the factor price frontier. It can be shown that this has the following form:

$$\frac{\tilde{Y}(t)}{\sigma_C} = \omega_A \tilde{r}(t) + \epsilon_L \tilde{W}(t) \tag{3.13}$$

With infinite horizons the steady-state real rate of interest equals the rate of time preference ($\tilde{r}(\infty)=0$). Under perfect competition, moreover, the substitution elasticity between different varieties is infinite ($1/\sigma_C=0$), so that the steady-state real wage rate is fixed as well ($\tilde{W}(\infty)=0$). Under monopolistic competition the steady-state real wage rate can rise provided output rises.

3.2.2. Discussion

The real output multipliers derived in this section are remarkably simple in form, and bear a striking resemblance to the textbook IS-LM multipliers. The multiplier derived recently by Baxter and King (1993) is a special case obtained by imposing perfect competition (*i.e.*, setting the mark-up μ equal to unity). Note that the multiplier is increasing in the degree of monopoly in the economy. The higher the mark-up, the more severe is the aggregate demand externality that exists under monopolistic competition, and the higher is the multiplier. The following numerical example adapted from Baxter and King (1993, p. 320) illustrates the *quantitative* importance that the assumed market structure exerts on the magnitude of the various multipliers. Baxter and King calibrate one version of their model with $r=0.065$, $\omega_{LL}=4$, $\epsilon_L=0.58$, $\delta=0.10$, $\omega_G=0.2$, and $\omega_Q=0.255$. In that case, the following multipliers are obtained: $dY/dG_0=1.135$, $dQ/dG_0=0.289$, $dC/dG_0=-0.154$, $dK/dG_0=2.89$. If we introduce relatively modest scale economies leading to a mark-up of $\mu=1.3$ (and keep the rest of the economy unchanged), these multipliers are increased dramatically: $dY/dG_0=1.763$, $dQ/dG_0=0.4495$, $dC/dG_0=0.313$, $dK/dG_0=4.50$. Hence, adding imperfect competition results in a major change in the size of the multiplier. We returns to this issue in section 4.5 below.

3.2.3. Finite Horizons

Up to this point attention has been focused on the Barro-Ramsey case of infinitely lived consumers. It is straightforward to analyze the complications that occur when finite horizons are assumed. The main changes that occur can be illustrated with the aid of Figure 2. The MKR curve is given in (3.5). First consider the case of inelastic labour supply ($\phi=1$). In that case the MKR curve is upward sloping because of the positive tilt to the consumption profiles of individual agents ($r>\alpha$), an effect that is absent in the Barro-Ramsey case. Hence, the configuration is as in Figure 2-a. As labour supply becomes more elastic, the labour supply effect starts to dominate the horizon effect and the MKR curve rotates counter-clockwise. These cases are illustrated in Figures

2-b to 2-d respectively. Saddle point stability again implies that the MKR curve cannot become steeper than the IS curve as ϕ increases. Since the IS curve is unchanged, it is clear that the existence of finite horizons unambiguously *reduces* the size of the output multiplier in the benchmark model. Under finite horizons composite consumption and leisure are cut back after the fiscal impulse has occurred, but by less than under infinite horizons. Agents do not live forever and therefore do not feel the full burden of the tax implications of the fiscal impulse. Because labour supply is increased by less, the increase in the steady-state capital stock will also be smaller. In the case of inelastic labour supply ($\phi=1$), capital will be crowded out, and output will fall. This was analyzed by Blanchard (1985) for the case of perfect competition.

3.3. Labour Tax Financing

The second fiscal policy experiment that can be analyzed with the aid of the benchmark model is an *unanticipated permanent* increase in government spending financed by the labour tax. The dynamic system can be written as follows.

$$\begin{bmatrix} \dot{\tilde{K}} \\ \dot{\tilde{X}} \end{bmatrix} = \begin{bmatrix} (\delta/\omega_Q)(\mu\phi(1-\epsilon_L)-\omega_Q) & (\delta/\omega_Q)(1-\phi-\omega_C) \\ -(r-\alpha)+(r+\delta)[\mu\phi(1-\epsilon_L)-1] & (r-\alpha)+(r+\delta)(1-\phi) \end{bmatrix} \begin{bmatrix} \tilde{K} \\ \tilde{X} \end{bmatrix} + \begin{bmatrix} \delta/\omega_Q[(1-\phi)/\epsilon_L-1] \\ (r+\delta)(1-\phi)/\epsilon_L \end{bmatrix} \omega_G \tilde{G}_0 \tag{3.14}$$

Note that the coefficient matrix on the RHS(3.14) is identical to the one that is relevant under lump-sum tax financing (Δ_1 in equation (3.2)) so that the stability issues are the same for this case. The labour tax has a direct effect on labour supply, however, so that the increased government spending now influences both the MKR curve and the IS curve.[19] As in the previous section, the case of infinite horizons is discussed first.

The IS and MKR curves are defined as follows.

$$\tilde{X} = \left[\frac{\omega_A+(\mu\phi-1)(1-\epsilon_L)}{\omega_C+\phi-1}\right]\tilde{K} - \left[\frac{\omega_G(1-\phi-\epsilon_L)}{\epsilon_L(\omega_C+\phi-1)}\right]\tilde{G}_0 \tag{3.15}$$

$$\tilde{X} = \left[\frac{1-\mu\phi(1-\epsilon_L)}{1-\phi}\right]\tilde{K} - \left[\frac{\omega_G}{\epsilon_L}\right]\tilde{G}_0 \tag{3.16}$$

If government spending is increased, the IS curve shifts to the right as before, and the MKR curve shifts down. The two main cases are illustrated in Figure 3. Following the increase in government spending, both IS and MKR shift down. Depending on the relative strength of the effects, the capital stock may fall, stay the same or rise. It is straightforward to derive that the fall in MKR

[19] The government budget restriction is: $\epsilon_L \tilde{t}_L = \omega_G \tilde{G}_0$. This can be substituted into the labour supply equation to yield the expression (3.14). Obviously the case of inelastic labour supply ($\phi=1$) is not interesting since the labour tax is non-distorting in that case.

dominates the fall in IS if the share of consumption exceeds the share of labour in national income, or, $\omega_C > \epsilon_L$. This has been assumed in Figures 3-a and 3-b, with the opposite cases holding in Figures 3-c and 3-d.

The long-run comparative static effects are given by:

$$\tilde{K}(\infty)\big|_{\epsilon_L} = \frac{(\epsilon_L - \omega_C)}{\epsilon_L\left[1-\omega_Q-\omega_C\left[\dfrac{1-\mu\phi(1-\epsilon_L)}{1-\phi}\right]\right]}\,\omega_G\tilde{G}_0 \tag{3.17}$$

$$\tilde{X}(\infty)\big|_{\epsilon_L} = -\frac{\left[1-\omega_Q-\epsilon_L\left[\dfrac{1-\mu\phi(1-\epsilon_L)}{1-\phi}\right]\right]}{\epsilon_L\left[1-\omega_Q-\omega_C\left[\dfrac{1-\mu\phi(1-\epsilon_L)}{1-\phi}\right]\right]}\,\omega_G\tilde{G}_0 \tag{3.18}$$

Consider the case of a moderately elastic labour supply first. In Figure 3-a the long-run effects on capital and total consumption are unambiguously negative. In the impact period the economy moves from E_0 to A, after which smooth transition occurs until point E_1 is reached. If labour supply is very elastic (but still assuming $\omega_C > \epsilon_L$) a very similar adjustment pattern is observed.

If the share of labour is smaller than the share of wage income ($\omega_C < \epsilon_L$), then the vertical shift in the MKR curve is smaller than that of the IS curve, so that the capital stock rises in the long run. This has been drawn in Figures 3-c and 3-d. In the short run, total consumption ($\tilde{X}$) overshoots its long-run value.

3.4. Debt Financing

The third and final fiscal policy experiment to be analyzed is a permanent increase in government spending financed by means of government debt. The dynamic system can be written as follows.

$$\begin{bmatrix} \dot{\tilde{K}} \\ \dot{\tilde{X}} \\ \dot{\tilde{B}} \end{bmatrix} = \begin{bmatrix} (\delta/\omega_Q)(\mu\phi(1-\epsilon_L)-\omega_Q) & (\delta/\omega_Q)(1-\phi-\omega_C) & 0 \\ -(r-\alpha)+(r+\delta)[\mu\phi(1-\epsilon_L)-1] & (r-\alpha)+(r+\delta)(1-\phi) & -(r-\alpha)/\omega_A \\ 0 & 0 & r/(1-\zeta) \end{bmatrix} \begin{bmatrix} \tilde{K} \\ \tilde{X} \\ \tilde{B} \end{bmatrix} + \begin{bmatrix} -\delta/\omega_Q \\ 0 \\ r/(1-\zeta) \end{bmatrix} \omega_G\tilde{G}_0$$

(3.19)

The determinant of the coefficient matrix on the RHS(3.19) is equal to the product of the characteristic roots, *i.e.*, $|\Delta_2| = r^*h^*z^* = -z^*|\Delta_1|$, where $-z^* \equiv r/(1-\zeta)$ is the characteristic root associated with the bond financing process. Obviously, any $\zeta > 1$ leads to a stable debt process ($z^* > 0$). Note furthermore that the case of infinite horizons is not very interesting here, because in that case Ricardian equivalence holds so that the path of debt does not matter. Formally, $r = \alpha$ in that case and the system is recursive in ($\tilde{K}$,$\tilde{X}$) and $\tilde{B}$.

The long-run comparative static results are:

$$\frac{\tilde{K}(\infty)}{\tilde{G}_0}\Big|_B = \frac{\tilde{K}(\infty)}{\tilde{G}_0}\Big|_Z + \left[\frac{(r-\alpha)\delta\omega_G(1-\phi-\omega_C)}{\omega_Q\omega_A|\Delta_1|}\right] \tag{3.20}$$

$$\frac{\tilde{X}(\infty)}{\tilde{G}_0}\Big|_B = \frac{\tilde{X}(\infty)}{\tilde{G}_0}\Big|_Z + \left[\frac{(r-\alpha)\delta\omega_G(\omega_Q-\mu\phi(1-\epsilon_L))}{\omega_Q\omega_A|\Delta_1|}\right] \tag{3.21}$$

$$\frac{\tilde{B}(\infty)}{\tilde{G}_0}\Big|_B = -\omega_G \tag{3.22}$$

The bond-financed multipliers are related in a simple fashion to the tax financed multipliers derived above. The terms in round brackets appearing in (3.20) and (3.21) are unambiguously positive. This implies that the long-run effect on the capital stock and total consumption are larger under bond-financing than under lump-sum tax financing. The intuition behind this result is straightforward. The increased government spending leads to a lower level of the government debt. Since Ricardian equivalence does not hold, the reduction in government debt is seen by agents as a reduction of their wealth. Consequently, they cut back total consumption and increase labour supply by more than under lump-sum tax financing, so that the long-run capital stock is larger. The long-run effect on real output is equal to:

$$\frac{\tilde{Y}(\infty)}{\tilde{G}_0}\Big|_B = \frac{\tilde{Y}(\infty)}{\tilde{G}_0}\Big|_Z - \left[\frac{(r-\alpha)\delta\omega_G}{\omega_Q\omega_A|\Delta_1|}\right]\left[\mu\phi(1-\epsilon_L)\omega_C+(\phi-1)\omega_Q\right] \tag{3.23}$$

Hence, the long-run output multiplier is larger under bond-financing than under lump-sum tax financing.

The transition path of the economy following a permanent unanticipated change in government spending is equal to the following.

$$\tilde{X}(t) = \tilde{X}(0)e^{-h^*t} + \tilde{X}(\infty)\left(1-e^{-h^*t}\right) + \Omega_X\left[\frac{e^{-z^*t}-e^{-h^*t}}{h^*-z^*}\right]\tilde{G}_0 \tag{3.24a}$$

$$\tilde{K}(t) = \left(1-e^{-h^*t}\right)\tilde{K}(\infty) + \Omega_K\left[\frac{e^{-z^*t}-e^{-h^*t}}{h^*-z^*}\right]\tilde{G}_0 \tag{3.24b}$$

$$\tilde{B}(t) = -\omega_G\left(1-e^{-z^*t}\right)\tilde{G}_0 \tag{3.24c}$$

$$\tilde{X}(0)\,|_B = \tilde{X}(0)\,|_Z - \left[\frac{(r-\alpha)\,\omega_G z^*}{\omega_A r^*(r^*+z^*)}\right]\tilde{G}_0 \tag{3.24d}$$

where $\Omega_K<0$ and $\Omega_X<0$ are constants.[20] Note that the term in square brackets appearing in (3.24a) and (3.24b) is positive regardless of the sign of h^*-z^*. By using bond-financing, the change in total consumption that occurs at impact is smaller under bond financing than under lump-sum tax financing.

4. Extensions

Up to this point the attention has been focused on a theoretical investigation of the benchmark model. This section is aimed at investigating the effects on the output multiplier of relaxing some of the specific assumptions underlying the benchmark model. The focus is on the long-run effects of fiscal policy financed by lump-sum taxes under the assumption of infinite lives (*i.e.*, $\beta=0$ in sections 4.1 to 4.4). The section concludes by presenting some evidence regarding the quantitative significance of the monopolistic competition assumption.

4.1. Ethier Effects

As was pointed out in sections 2.3 and 2.4, the production functions for new investment goods and the public good (equations (2.11) and (2.17)) both exhibit external increasing returns due to the so-called Ethier effect: an increase in the number of varieties N, enables a more roundabout production process and consequently a lower unit price. The strength of this price effect is regulated by the parameters α_Q and α_G respectively. In the benchmark case discussed in section 3, the strength of the Ethier effect was set in such a way as to keep the relative prices of investment goods (P_Q/P) and the public good (P_G/P) constant. This is the case if $\alpha_Q=\alpha_G=\mu$. In this section the consequences of this assumption for the size of the multiplier are discussed.

First, consider the case of $\alpha_Q\neq\mu=\alpha_G$. In that case a fiscal impulse changes the relative price of investment goods, and the resulting long-run output multiplier is equal to the following.

$$\frac{dY(\infty)}{dG_0}\,|_Z = \frac{1}{1-\omega_Q-\omega_C\left[\dfrac{1-\alpha_Q\phi(1-\epsilon_L)}{1-\phi}\right]} \tag{4.1}$$

The multiplier is increasing in α_Q, and coincides with the benchmark expression (3.8) for $\alpha_Q=\mu$. Hence, if $\alpha_Q>\mu$, fiscal policy is more effective than in the benchmark case. The intuition behind this result is that the additional government spending increases output and the number of varieties, and hence lowers the relative price of investment goods and hence the replacement value of the

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[20] Ω_K and Ω_X are defined as follows:

$$\Omega_K \equiv \frac{\delta\omega_G(r-\alpha)(1-\phi-\omega_C)}{\omega_A\omega_Q(r^*+z^*)}<0 \qquad \Omega_X \equiv -\frac{(r-\alpha)\,\omega_G[(\delta/\omega_Q)(\mu\phi(1-\epsilon_L)-\omega_Q)+z^*]}{r^*+z^*}<0$$

where we have made use of the fact that $\omega_V\equiv 1$-ω_L-$\omega_Q>0$.

capital stock. This additional wealth reduction prompts an increase in labour supply that is larger than in the benchmark case.

Second, consider the case of $\alpha_G \neq \mu = \alpha_Q$. In that case a fiscal impulse changes the relative price of the public good, and the resulting long-run output multiplier is equal to the following.

$$\frac{dY(\infty)}{dG_0}\Big|_Z = \frac{1}{1 - \omega_Q - \omega_G\left[1 - \frac{\alpha_G}{\mu}\right] - \omega_C\left[\frac{1 - \mu\phi(1 - \epsilon_L)}{1 - \phi}\right]} \tag{4.2}$$

The multiplier is decreasing in α_G, and coincides with the benchmark expression (3.8) for $\alpha_G = \mu$. Hence, if $\alpha_G > \mu$, fiscal policy is less effective than in the benchmark case. The additional government spending increases output and hence lowers the relative price of the public goods. This in turn implies that the increase in the lump-sum tax is less than in the benchmark case, so that the increase in labour supply is also less than in the benchmark case. As a result output is increased by less than in the benchmark case.

4.2. Mark-Up Effects

In section 2.5 it was shown that the mark-up facing producers in the differentiated sector typically depends on share-weighted elasticities of substitution (see equation (2.23)). If these elasticities are equal ($\sigma_I = \sigma_C = \sigma_G$), as was assumed in the benchmark case, then the composition of demand does not matter and the mark-up is constant ($\mu = \sigma_C/(\sigma_C - 1)$). In this subsection the consequences of allowing for different substitution elasticities are discussed.

First, consider the case of $\sigma_I \neq \sigma_C = \sigma_G$. In that case the *level* of the *actual* mark-up (μ_Q) differs from μ:

$$\mu_Q \equiv \frac{\sigma_C(1 - \omega_Q) + \sigma_I\omega_Q}{\sigma_C(1 - \omega_Q) + \sigma_I\omega_Q - 1} \tag{4.3}$$

so that $\mu - \mu_Q = \text{sgn}(\sigma_I - \sigma_C)$. If $\sigma_I < \sigma_C$ then the mark-up is higher than in the benchmark case because the relatively low price elasticity of demand originating from the investment goods sector makes the differentiated sector less competitive. It turns out, however, that fiscal policy does not change the mark-up in the long run, *i.e.*, $\tilde{\mu}_Q(\infty) = 0$. The equation determining the change in the mark-up is equal to the following.

$$\left[\frac{\mu_Q}{(\mu_Q - 1)^2}\right]\tilde{\mu}_Q = \omega_Q(\sigma_C - \sigma_I)(\tilde{Q} - \tilde{Y}) \tag{4.4}$$

Even though $\sigma_I \neq \sigma_C$, steady-state equilibrium implies $\tilde{K} = \tilde{Q}$ and $\tilde{K} = \tilde{Y}$, so that $\tilde{\mu}_Q = 0$. Hence, the long-run multiplier in this case is equal to the expression derived for the benchmark case (equation

(3.8)).[21]

Second, consider the case of $\sigma_G \neq \sigma_C = \sigma_I$. In that case the *level* of the actual mark-up (μ_G) differs from μ:

$$\mu_G \equiv \frac{\sigma_C(1-\omega_G)+\sigma_G\omega_G}{\sigma_C(1-\omega_G)+\sigma_G\omega_G-1} \tag{4.5}$$

so that μ-μ_G=sgn(σ_G-σ_C). In contrast to the previous case, fiscal policy in general changes the mark-up even in the long run. The model can be condensed to two equations. The equation determining the change in the mark-up is equal to:

$$\left[\frac{\mu_G}{(\mu_G-1)^2}\right]\tilde{\mu}_G = \omega_G(\sigma_C-\sigma_G)(\tilde{G}_0-\tilde{Y}) \tag{4.6a}$$

and the multiplier equation is:

$$\tilde{Y}(\infty)|_Z = \frac{\Lambda\tilde{\mu}_G+\omega_G\tilde{G}_0}{1-\omega_Q-\omega_C\left[\frac{1-\mu\phi(1-\epsilon_L)}{1-\phi}\right]} \qquad \Lambda \equiv \left[\frac{\omega_C\phi}{\phi-1}\right]\left[\frac{\mu-\mu_G}{\mu_G-\lambda}\right] \tag{4.6b}$$

where sgn(Λ)=sgn(σ_G-σ_C). In the Appendix it is shown that the multiplier that is defined implicitly by equations (4.6a) and (4.6b) falls short of the benchmark expression if $\sigma_G \neq \sigma_C$. There are two separate effects that produce this result. First, an increases in government spending increases (lowers) the degree of competition in the economy if $\sigma_G > \sigma_C$ ($\sigma_G < \sigma_C$) and hence lowers (increases) the mark-up (see equation (4.6a)). As a result, the output per firm $\bar{Y}$ increases (decreases). The second effect operates via the number of product varieties N. Together these two effects explain the total effect on Y (recall that $Y \equiv N^{\mu}\bar{Y}$).

4.3. Intertemporal and Intratemporal Substitution Effects

In the benchmark case the intertemporal substitution elasticity is equal to unity ($\xi=1$). This implies that the propensity to consume out of total wealth is constant and equal to ($\alpha+\beta$). As is clear from Table 2, the value of ξ does not affect any of the steady-state equations in the case of infinite horizons. In that case, $\beta=0$, $r(\infty)=\alpha$, $\tilde{r}(\infty)=0$, and the real output multiplier is unaffected. Of course, the transition path is affected by the magnitude of the intertemporal substitution elasticity.

The intratemporal elasticities have a non-trivial effect on the long-run multiplier. First, consider the effect of σ_{CM}, the parameter characterizing the intratemporal substitution elasticity of labour supply. In the benchmark model this parameter is set equal to unity, implying Cobb-Douglas preferences (see equation (2.6)). For the more general case, the following multiplier can

[21]This conclusion hinges on the assumption of infinite horizons, since in that case the steady-state rate of interest is constant.

be derived.

$$\frac{dY(\infty)}{dG_0}\Big|_Z = \frac{1}{1-\omega_Q-\omega_C\Gamma_1} \qquad \Gamma_1 \equiv \frac{\mu\epsilon_L\omega_{LL}\sigma_{CM}-[1-\mu(1-\epsilon_L)](1+\sigma_{CM}\omega_{LL})}{\mu\epsilon_L\omega_{LL}} \tag{4.7}$$

In the Appendix it has been shown that, under monopolistic competition, this real output multiplier exceeds (falls short of) the benchmark expression if $\sigma_{CM}>1$ (<1). The reason is that a higher value of σ_{CM} increases the labour supply response for a given change in the real wage rate. Hence, it also increases the output response.

Under perfect competition, however, the long-run output multiplier does not depend on σ_{CM} (see Appendix). The reason for this is that the long-run results are in that case determined entirely by the long-run capital-labour ratio. Since (in this Barro-Ramsey case) both the real rate of interest and (by the factor price frontier) the real wage rate are constant, the capital-labour ratio is also constant. Hence, the output effect is fully explained by the negative effect on total consumption $\tilde{X}$ of the increased government spending.

The second intratemporal substitution elasticity to be considered is the substitution elasticity between capital and labour in the gross production function (equation (2.10)), σ_{KL}. The real output multiplier for the more general case (*i.e.*, $\sigma_{KL}\neq 1$) is equal to:

$$\frac{dY(\infty)}{dG_0}\Big|_Z = \frac{1}{1-\omega_Q\left[\frac{\Omega}{\mu}\right]-\omega_C\Gamma_2} \qquad \Gamma_2 \equiv \frac{\mu\epsilon_L\left[\frac{\omega_{LL}}{\sigma_{KL}}\right]\left[\frac{\Omega}{\mu}\right]-\left[1-\mu(1-\epsilon_L)\left[\frac{\Omega}{\mu}\right]\right]\left[1+\frac{\omega_{LL}}{\sigma_{KL}}\right]}{\mu\epsilon_L\omega_{LL}} \tag{4.8}$$

where $\Omega\equiv\mu\sigma_{KL}+1-\sigma_{KL}$. In the Appendix it has been shown that this real output multiplier exceeds (falls short of) the benchmark expression if $\sigma_{KL}<1$ (>1), under monopolistic competition. The reason is that a higher value of σ_{KL} lowers the employment response for a given real wage rate change. Hence, it also decreases the output response. As before, under perfect competition, the long-run output multiplier does not depend on σ_{KL} (see Appendix).

4.4. Restricted Entry

As a final extension consider the effects of restricted entry. The number of firms is fixed (at N), and the rate of excess profit for each existing firm can be defined as:[22]

$$\pi \equiv \frac{P_iY_i}{TC_i} - 1 = \frac{\mu}{\lambda}(1-\omega_f)-1 \tag{4.9}$$

where $\omega_f\equiv f/(f+\tilde{Y})$ is the share of fixed cost in gross output. Using the approximation $1/(1+\pi)\approx 1-\pi$, the real output multiplier can be written as:

[22]See footnote 13 for the information used to calculate π.

$$\frac{dY(\infty)}{dG_0}\Big|_Z = \frac{1}{1 - \omega_Q - \omega_C \left[\frac{1 - \mu\phi(1 - \pi - \epsilon_L)}{1 - \phi} \right]} \tag{4.10}$$

From this it is clear that the multiplier is decreasing in the excess profit rate.

4.5. The Quantitative Significance of Imperfect Competition

In order to illustrate the quantitative significance of the introduction of even relatively mild returns to scale, this section presents a calibrated example of the model. The following parameter values were chosen:

Fundamental parameters and shares:

α	0.04	Pure rate of time preference
β	0.04	Instantaneous probability of death
δ	0.10	Rate of depreciation of the capital stock
λ	1.20	Scale parameter in the gross production function
μ	1.30	Initial value of the gross mark-up
σ_{KL}	1.00	Substitution elasticity of the gross production function
$\sigma_C=\sigma_I=\sigma_G$	4.33	Substitution elasticity between varieties of the differentiated product $(=\mu/(\mu\text{-}1))$
r	0.06	Initial real rate of interest
ω_C	0.80	Share of private consumption in real output
ϵ_L	0.70	Share of before tax wage income in real output.
ω_{LL}	2.00	Ratio between leisure and labour.
ω_B	0.00	Share of government interest payments in output.
t_L	0.00	Initial level of the tax on labour

The information at the bottom of Table 3 can be used to derive all other shares from this information. For example, the share of investment in real output is ω_Q=0.130, the share of government consumption is ω_G=0.130, and the share of fixed cost ω_f=0.077. In contrast to the the previous section, in this section the agents' instantaneous probability of death is set at 4 percent per annum, implying a planning horizon of 25 years.

In the benchmark model both the *inter*temporal and the *intra*temporal substitution elasticities are equal to unity. In this section the long-run comparative static effects of a permanent fiscal impulse are calculated for the more general case where both σ_{CM} and ξ can differ from unity. The results of this exercise have been collected in Table 4. This table reports the various comparative static effects for different values of the intertemporal substitution elasticity ξ along the column, and different values of the intratemporal substitution elasticity σ_{CM} along the row. The final row in each case refers to the Barro-Ramsey case of infinite horizons (β=0). In that case the long-run effects are independent from the intertemporal substitution elasticity. The steady-state rate of interest equals α in that case, so that ξ has no long-run effects.[23]

Glancing at the first set of results, it is clear that the output multiplier is large for the chosen parameterisation, and increasing in ξ. A very low degree of intertemporal substitution implies that agents find it very hard to accept an uneven total consumption profile. As government spending and lump-sum taxes are increased the labour supply response is smaller the smaller the value of ξ. The output multiplier is strongly increasing in σ_{CM} (as was to be expected from section 4.3 above). A high value of σ_{CM} implies a strong labour supply response, and a large effect on the capital stock. The two combine to yield a large long-run multiplier. As was shown for the theoretical model, the comparative static effects are strongest for the case of infinite horizons, since in that case the long-run rate of interest is constant so that no crowding out of capital takes place.

In order to ascertain the quantitative significance of the *monopolistic competition* assumption, Table 5 reports the results of the fiscal experiment under the assumption of perfect competition. The calibration values are kept the same as before, excepting the fixed cost parameter and the variety substitution parameter. Both are set at the respective values consistent with perfect competition, *i.e.*, $\sigma_C \rightarrow +\infty$ (so that $\mu=1$), $\omega_f=0$, and $\lambda=1$. Of course, the number of firms is indeterminate under imperfect competition.

The multipliers in Table 5 are still positive but much smaller than the ones reported in Table 4. Furthermore, the comparative static effects now decrease with the intratemporal degree of substitution σ_{CM}. In the Barro-Ramsey case of infinite horizons ($\beta=0$) discussed at length by Baxter and King (1993), the results are independent of both ξ and σ_{CM} (see section 4.3).

5. Conclusions

In this paper an attempt has been made to provide the macroeconomics of imperfect competition with some rigorous microeconomic foundations. The model developed here incorporates the static imperfect competition models by Mankiw (1988) and Startz (1989), and the dynamic perfect competition models by Blanchard (1985) and Baxter and King (1993) as special cases.

A number of conclusions can be drawn on the basis of the model. First, adding imperfect competition has a major influence on the size of the fiscal policy multipliers. The scale economies add a magnification effect to the accelerator effect that also operates under perfect competition. Second, since the multiplier effect operates via labour supply, the method of taxation is of vital importance to the results. If a distortionary labour tax is used to finance extra government spending, then the long-run multiplier may well be negative. If that is the case, then crowding out is worse under imperfect competition than under perfect competition. Third, in many cases considered in the paper, the long-run multiplier exceeds the short-run multiplier. This effect works via capital accumulation, and is of course absent in the static macro models with monopolistic competition.

[23]In order to render the results for the finite and infinite horizon cases compatible, the pure rate of time preference is set at $\alpha=r=0.06$ in the latter case.

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Appendix

A.1. The Three-Stage Solution Method

The optimisation problem faced by the representative consumer can be solved in three stages. The method extends the two-step procedure of Marini and van der Ploeg (1988) by allowing the intertemporal substitution elasticity to differ from unity, and by distinguishing differentiated products and endogenous labour supply. In step 1 the path of total consumption $X(s,t)$ is solved. In step 2 total consumption is allocated between its components $C(s,t)$ and $1\text{-}L(s,t)$. Finally, in step 3 $C(s,t)$ is allocated over the different varieties of the differentiated product, $C_i(s,t)$.

Stage 1.

Define the ideal cost-of-living index as $P_U(v)$.

$$P_U(v)\,U(s,v) = X(s,v) \qquad \text{(A1)}$$

Where $U(s,v) \equiv \Omega[C(s,v), 1\text{-}L(s,v)]$. In the first stage the following optimisation problem is solved for $v \in [t,\infty)$.

$$\underset{\{U(s,v)\}}{Max} \int_t^\infty \frac{1}{1-1/\xi}\left(U(s,v)^{1-1/\xi} - 1\right) e^{(\alpha+\beta)(t-v)} \mathrm{d}v \qquad \text{(A2)}$$

$$\text{s.t.} \quad \dot{A}(s,v) = [r(v)+\beta]A(s,v) + [1 - t_L(v)]\,W(v) - Z(s,v) - P_U(v)\,U(s,v)$$

This leads to the following first-order conditions.

$$U(s,v)^{-1/\xi} = \lambda(s,v)P_U(v) \qquad v \in [t,\infty) \qquad \text{(A3)}$$

$$\frac{\mathrm{d}\lambda(s,v)}{\mathrm{d}v} = [\alpha - r(v)]\,\lambda(s,v) \qquad v \in [t,\infty) \qquad \text{(A4)}$$

Where $\lambda(s,v)$ is the co-state variable of the flow budget restriction. The integrated (life-time) budget restriction (with a NPG condition imposed) is:

$$A(s,t) + H(s,t) = \int_t^\infty P_U(v)\,U(s,v)\,e^{-\int_t^v [r(\mu)+\beta]\mathrm{d}\mu} \mathrm{d}v = \int_t^\infty \lambda(s,v)^{-\xi}\,P_U(v)^{1-\xi}\,e^{-\int_t^v [r(\mu)+\beta]\mathrm{d}\mu} \mathrm{d}v \qquad \text{(A5)}$$

The path of $\lambda(s,v)$ is described by (A4) which can be solved to yield the following.

$$\lambda(s,v) = e^{-\int_t^v [r(\mu)-\alpha]\mathrm{d}\mu}\,\lambda(s,t) \qquad v \geq t \qquad \text{(A6)}$$

Using this in (A5) yields the following.

$$A(s,t) + H(s,t) = \int_t^\infty \left[e^{\int_t^v [\alpha - r(\mu)]d\mu} \lambda(s,t) \right]^{-\xi} P_U(v)^{1-\xi} e^{-\int_t^v [r(\mu)+\beta]d\mu} dv \qquad (A7)$$

But $P_U(t)U(s,t) \equiv X(s,t) = \lambda(s,t)^{-\xi} P_U(t)^{1-\xi}$, so that (A7) can be rewritten as follows.

$$X(s,t) = \left[\Delta(t)\right]^{-1} [A(s,t) + H(s,t)] \qquad (A8)$$

Where $\Delta(t)$ is defined as follows.

$$\Delta(t) \equiv P_U(t)^{\xi-1} \int_t^\infty e^{-(1-\xi)\int_t^v [r(\mu)+\beta]d\mu} P_U(v)^{1-\xi} e^{\xi(\alpha+\beta)(t-v)} dv \qquad (A9)$$

The differential equation for $\Delta(t)$ is obtained by differentiating (A9) with respect to t.

$$\dot{\Delta}(t) = -1 + \left[(1-\xi)[r(t) - \dot{P}_U(t)/P_U(t) + \beta] + \xi(\alpha+\beta)\right]\Delta(t) \qquad (A10)$$

where $\dot{P}_U(t)/P_U(t)$ is the rate of change in the true price index $P_U(t)$ (defined in (A16) below). Note that equations (A8)-(A10) generalize the expressions found in Blanchard (1985, pp. 233-4) to the case of endogenous labour supply.

Stage 2

Total consumption $X(s,t)$ is now allocated over consumption of the composite differentiated good ($C(s,t)$) and leisure ($1-L(s,t)$).

$$\underset{\{C(s,t),1-L(s,t)\}}{Max} U(s,t) = \Omega[C(s,t), 1-L(s,t)]$$
$$s.t. \quad C(s,t) + [1-t_L(t)]\, W(t)(1-L(s,t)) = X(s,t) \qquad (A11)$$

This implies the following for the modified CES utility function used in the text [equation (2.6)].

$$C(s,t) = \frac{\gamma_C}{1-\gamma_C}[(1-t_L(t))\, W(t)]^{\sigma_{CM}} (1-L(s,t)) \qquad (A12)$$

Substituting (A12) into (2.4) yields the expression for $L(s,t)$ and $C(s,t)$ in terms of total consumption $X(s,t)$.

$$L(s,t) = 1 - \frac{(1-\gamma_C)[(1-t_L(t))\, W(t)]^{-\sigma_{CM}}}{\left[\gamma_C + (1-\gamma_C)[(1-t_L(t))\, W(t)]^{1-\sigma_{CM}}\right]} X(s,t) \qquad (A13)$$

$$C(s,t) = \frac{\gamma_C}{\left[\gamma_C + (1-\gamma_C)[(1-t_L(t))W(t)]^{1-\sigma_{CM}}\right]} X(s,t) \qquad (A14)$$

The expression for the true price index is obtained by substituting (A13) and (A14) into the utility function (2.6) and noting (A1). The result is as follows.

$$P_U(t) = \left[\gamma_C + (1-\gamma_C)\left[W(t)(1-t_L(t))\right]^{1-\sigma_{CM}}\right]^{\frac{1}{1-\sigma_{CM}}} \tag{A15}$$

Stage 3

The agent now chooses $C_i(s,t)$ such that the following static maximisation program is solved.

$$\underset{\{C_i(s,t)\}}{Max} \left[\sum_{i=1}^{N(t)} C_i(s,t)^{\frac{\sigma_C-1}{\sigma_C}}\right]^{\frac{\sigma_C}{\sigma_C-1}} \text{ s.t. } \sum_{i=1}^{N(t)} P_i(t)C_i(s,t) = P(t)C(s,t) \tag{A16}$$

Straightforward manipulation yields the demand functions for the differentiated commodities by the agent of vintage s.

$$C_i(s,t) = \left[\frac{P_i(t)}{P(t)}\right]^{-\sigma_C} C(s,t) \tag{A17}$$

(See, *e.g.*, Heijdra and Yang (1993, Appendix)).

A.2. The Optimisation Problem for a Representative Firm

The representative firm i aims to maximise (2.14) subject to (2.15a-d). The current-value Lagrangian is defined as follows.

$$\begin{aligned}\mathcal{L} &= P_i(C_i + I_i + G_{0i}) - W^n L_i - P_Q Q_i \\ &+ \lambda_K(Q_i - \delta^* K_i) + \lambda_Y(\mathrm{F}(L_i, K_i) - f - C_i - I_i - G_{0i})\end{aligned} \tag{A18}$$

The control variables P_i, L_i, and Q_i, the state variable is K_i, the co-state variable is λ_K, and λ_Y is the Lagrange multiplier for the demand restriction. The first-order necessary conditions are:

$$\frac{\partial \mathcal{L}}{\partial Q_i} = 0: \quad -P_Q + \lambda_K = 0 \tag{A19}$$

$$\frac{\partial \mathcal{L}}{\partial L_i} = 0: \quad -W^n + \lambda_Y \frac{\partial \mathrm{F}}{\partial L_i} = 0 \tag{A20}$$

$$\frac{\partial \mathcal{L}}{\partial P_i} = 0: \quad C_i + I_i + G_{0i} + (P_i - \lambda_Y)\left[\frac{\partial C_i}{\partial P_i} + \frac{\partial I_i}{\partial P_i} + \frac{\partial G_{0i}}{\partial P_i}\right] = 0 \tag{A21}$$

$$-\frac{\partial \mathcal{L}}{\partial K_i} = \dot{\lambda}_K - R^* \lambda_K: \quad \dot{\lambda}_K - (R^* + \delta^*)\lambda_K = -\lambda_Y \frac{\partial \mathrm{F}}{\partial K_i} \tag{A22}$$

$$\dot{K}_i = \frac{\partial \mathcal{L}}{\partial \lambda_K}: \quad \dot{K}_i = Q_i - \delta^* K_i \tag{A23}$$

Equation (A19) implies that $\lambda_K = P_Q$. Equation (A21) can be used to solve for λ_Y in terms of the mark-up (μ_i) and the price chosen by the firm: $\lambda_Y = P_i/\mu_i$. Hence, λ_Y has the interpretation of marginal cost. Substituting these expressions for λ_Y and λ_K into (A20) and (A22) yields equations (2.16a-b) in the text.

A.3. Proofs

Dependence of the multiplier on σ_G-σ_C

The more general model is summarized by equations (4.6a)-(4.6b) and can be written as:

$$\tilde{Y}(\infty) = \Omega_Y^G \tilde{G}_0 + \Omega_Y^\mu \tilde{\mu}_G(\infty)$$

$$\tilde{\mu}_G(\infty) = \Omega_\mu [\tilde{G}_0 - \tilde{Y}(\infty)]$$

where the coefficients Ω_Y^μ, Ω_Y^G, and Ω_μ can be obtained from (4.6a) and (4.6b) in a transparent fashion. It is straightforward to prove that $0 < \Omega_Y^G < 1$, $\text{sgn}(\Omega_\mu) = -\text{sgn}(\Omega_Y^\mu) = \text{sgn}(\sigma_G - \sigma_C)$. This implies that the product $\Omega_Y^\mu \Omega_\mu < 0$ for $\sigma_G \neq \sigma_C$. The output effect can be solved as:

$$\tilde{Y}(\infty) = \left[\frac{\Omega_Y^G + \Omega_Y^\mu \Omega_\mu}{1 + \Omega_Y^\mu \Omega_\mu} \right] \tilde{G}_0$$

The benchmark expression is obtained by setting $\Omega_Y^\mu = \Omega_\mu = 0$. Comparison of the two expressions shows that the benchmark output effect exceeds the more general output effect provided $\Omega_Y^\mu \Omega_\mu > -1$. This is certainly the case if the initial level of government spending is not too large. This establishes the results mentioned in the text. □

Dependence of the multiplier on σ_{CM}

The comparison is between the benchmark multiplier given in equation (3.8) and the more general expression derived for an unrestricted value of the substitution elasticity σ_{CM} given in equation (4.7). By using the definition of ϕ given in equation (3.3), the benchmark expression can be written as:

$$\frac{dY(\infty)}{dG_0}\Big|_Z = \frac{1}{1 - \omega_Q - \omega_C \Gamma^*} \qquad \Gamma^* \equiv \frac{\mu\epsilon_L\omega_{LL} - [1 - \mu(1 - \epsilon_L)](1 + \omega_{LL})}{\mu\epsilon_L\omega_{LL}} \tag{A24}$$

Hence, the difference between the two expressions is concentrated in Γ_1 and Γ^*. After some manipulation it can be shown that Γ_1-Γ^* is equal to:

$$\Gamma_1 - \Gamma^* = \frac{(\sigma_{CM} - 1)(\mu - 1)}{\mu\epsilon_L}$$

Hence, $\text{sgn}(\Gamma_1 - \Gamma^*) = \text{sgn}(\sigma_{CM} - 1)(\mu - 1)$. This establishes the results mentioned in the text. □

Dependence of the multiplier on σ_{KL}

The comparison is between the benchmark multiplier given in equation (3.8) and the more general expression derived for an unrestricted value of the substitution elasticity σ_{KL} given in equation (4.8). By comparing this expression to the benchmark expression (A24), it is possible to derive the following:

$$\left[\frac{dY}{dG_0}\bigg|_{\sigma_{KL}\neq 1}\right]^{-1} - \left[\frac{dY}{dG_0}\bigg|_{\sigma_{KL}=1}\right]^{-1} = \omega_Q\left[\frac{\Omega}{\mu} - 1\right] + \omega_C\left[\Gamma_2 - \Gamma^*\right]$$

It is straightforward to derive that $\mu\epsilon_L\omega_{LL}(\Gamma_2\text{-}\Gamma^*)=(1\text{-}\epsilon_L)(\mu\text{-}1)(\sigma_{KL}\text{-}1)$ and $\Omega\text{-}\mu=(\mu\text{-}1)(\sigma_{KL}\text{-}1)$. This establishes the results mentioned in the text. □

Table 1: Short-Run Version of the Complete Model[a]

Dynamic:

$$\dot{K} = Q - \delta K \tag{A}$$

$$\dot{X} = \left[\xi(r-\alpha) + (1-\xi)\dot{P}_U/P_U\right]X - \beta\Delta^{-1}[V + B] \tag{B}$$

$$\dot{\Delta} = -1 + [(1-\xi)[r - \dot{P}_U/P_U + \beta] + \xi(\alpha+\beta)]\Delta \tag{C}$$

$$\dot{B} = (P_G/P)G_0 + rB - t_L WL - Z \tag{D}$$

$$Z = Z_0 - \zeta\dot{B} \qquad \zeta > 1 \tag{E}$$

Static:

$$V = (P_Q/P)K \tag{F}$$

$$\lambda^{\sigma_{KL}-1}\epsilon_L^{\lambda\sigma_{KL}} N^{(1-\sigma_{KL})\left(\lambda - \frac{\sigma_C}{\sigma_C - 1}\right)} Y^{1+(\lambda-1)\sigma_{KL}} = \mu^{\sigma_{KL}-1} W^{\lambda\sigma_{KL}} L^{\lambda} \tag{G}$$

$$K = \left[\frac{1-\epsilon_L}{\epsilon_L}\right]^{\sigma_{KL}} \left[\frac{WP}{\psi P_Q}\right]^{\sigma_{KL}} L \tag{H}$$

$$PY = PC + P_Q Q + P_G G_0 \tag{I}$$

$$C = \left[\frac{\gamma_C}{P_U^{1-\sigma_{CM}}}\right] X \tag{J}$$

$$L = 1 - \left[\frac{(1-\gamma_C)[(1-t_L)W]^{-\sigma_{CM}}}{P_U^{1-\sigma_{CM}}}\right] X \tag{K}$$

$$Y + fN^{\frac{\sigma_C}{\sigma_C - 1}} = N^{\frac{\sigma_C}{\sigma_C - 1} - \lambda} \left[\epsilon_L L^{\frac{\sigma_{KL}-1}{\sigma_{KL}}} + (1-\epsilon_L) K^{\frac{\sigma_{KL}-1}{\sigma_{KL}}} \right]^{\frac{\lambda \sigma_{KL}}{\sigma_{KL}-1}} \tag{L}$$

$$(\epsilon_i \equiv)\ \omega_C \sigma_C + \omega_Q \sigma_I + \omega_G \sigma_G = \frac{\mu}{\mu - 1} \tag{M}$$

$$(\mu - \lambda) Y = \lambda f N^{\frac{\sigma_C}{\sigma_C - 1}} \tag{N}$$

Definitions:

$$P_Q = N^{1-\alpha_Q} \bar{P} \quad P_G \equiv N^{1-\alpha_G} \bar{P} \quad P \equiv N^{\frac{1}{1-\sigma_C}} \bar{P} \tag{O}$$

$$P_U \equiv \left[\gamma_C + (1-\gamma_C)[(1-t_L) W]^{1-\sigma_{CM}} \right]^{\frac{1}{1-\sigma_{CM}}} \tag{P}$$

$$\psi \equiv r + \delta + \dot{P}/P - \dot{P}_Q/P_Q \tag{Q}$$

a. Equation (E) is relevant only under bond-financing. A value of ζ in excess of unity is needed to ensure that the government's solvency condition (2.21) is satisfied. See section 3.4 in the text.

b. Equation (F) becomes dynamic if entry is restricted ($\dot{N}=0$) in which case the zero pure-profit condition (O) is irrelevant. The expression for $\dot{V}$ is then:

$$\dot{V} = rV - [Y - WL - (P_Q/P) Q] \tag{F*}$$

Table 2: Log-Linearized Version of the Complete Model[a]

$$\dot{\tilde{K}} = \delta[\tilde{Q} - \tilde{K}] \tag{A'}$$

$$\dot{\tilde{X}} = \xi(r-\alpha)[\tilde{X}+\tilde{\Delta}] + \xi r\tilde{r} + (1-\xi)\dot{\tilde{P}}_U - \left[\frac{\xi(r-\alpha)}{\omega_A}\right][\tilde{V} + \tilde{B}] \tag{B'}$$

$$\dot{\tilde{\Delta}} = (1-\xi)[r\tilde{r} - \dot{\tilde{P}}_U] + \Delta^{-1}\tilde{\Delta} \tag{C'}$$

$$\dot{\tilde{B}} = r\omega_G[\tilde{G}_0 + \tilde{P}_G - \tilde{P}] + r\tilde{B} + r\omega_B\tilde{r} - rt_L\omega_L[\tilde{L} + \tilde{W}] - r\omega_Z\tilde{Z} - r(1-t_L)\omega_L\tilde{t}_L \tag{D'}$$

$$\zeta\dot{\tilde{B}} = r\omega_Z[\tilde{Z}_0 - \tilde{Z}] \tag{E'}$$

$$\tilde{V} = \omega_V(\tilde{K} + \tilde{P}_Q - \tilde{P}) \tag{F'}$$

$$\lambda\tilde{L} = \left(1+(\lambda-1)\sigma_{KL}\right)\tilde{Y} + \left(1-\sigma_{KL}\right)\tilde{\mu} + \left[\frac{(1-\sigma_{KL})(\lambda(1-\sigma_C)+\sigma_C)}{1-\sigma_C}\right]\tilde{N} - \lambda\sigma_{KL}\tilde{W} \tag{G'}$$

$$\tilde{K} - \tilde{L} = \sigma_{KL}\left(\tilde{W} + \tilde{P} - \tilde{\psi} - \tilde{P}_Q\right) \tag{H'}$$

$$\tilde{Y} = \omega_C\tilde{C} + \omega_Q(\tilde{Q} + \tilde{P}_Q - \tilde{P}) + \omega_G(\tilde{G}_0 + \tilde{P}_G - \tilde{P}) \tag{I'}$$

$$\tilde{C} = (\sigma_{CM}-1)\tilde{P}_U + \tilde{X} \tag{J'}$$

$$\tilde{L} = \omega_{LL}\left[\sigma_{CM}[\tilde{W} - \tilde{t}_L] + (1-\sigma_{CM})\tilde{P}_U - \tilde{X}\right] \tag{K'}$$

$$(1-\omega_f)\tilde{Y} + \omega_f\left[\frac{\sigma_C}{\sigma_C-1}\right]\tilde{N} = \left[\frac{\sigma_C}{\sigma_C-1} - \lambda\right]\tilde{N} + \lambda[\omega_L^*\tilde{L} + (1-\omega_L^*)\tilde{K}] \tag{L'}$$

$$-\frac{\mu}{(\mu-1)^2}\tilde{\mu} = \omega_C(\sigma_C-\sigma_G)(\tilde{C}-\tilde{Y}) + \omega_Q(\sigma_I-\sigma_G)(\tilde{Q}+\tilde{P}_Q-\tilde{P}-\tilde{Y}) \tag{M'}$$

$$\tilde{\mu} = \omega_f \left[\frac{\sigma_C}{\sigma_C - 1} \tilde{N} - \tilde{Y} \right] \qquad \text{(N')}$$

Definitions:

$$\tilde{P}_G = (1-\alpha_G)\tilde{N} + \tilde{\tilde{P}} \qquad \tilde{P} = \frac{\tilde{N}}{1-\sigma_C} + \tilde{\tilde{P}} \qquad \tilde{P}_Q = (1-\alpha_Q)\tilde{N} + \tilde{\tilde{P}} \qquad \text{(O')}$$

$$\tilde{P}_U = \left[\frac{(1-t_L)\omega_L \omega_{LL}}{\omega_C + (1-t_L)\omega_L \omega_{LL}} \right] [\tilde{W} - \tilde{t}_L] \qquad \text{(P')}$$

$$(r+\delta)\dot{\tilde{\psi}} = r\tilde{r} - \left[1 - \alpha_Q - \frac{1}{1-\sigma_C} \right] \dot{\tilde{N}} \qquad \text{(Q')}$$

Shares and parameters:

ω_W	*W/Y.*	Share of potential wage in real output, $\omega_W=(1+\omega_{LL})\omega_L$.
ω_Z	*Z/Y.*	Share of lump-sum taxes in real output.
ω_L	*WL/Y.*	Share of before tax wage income in real output.
ω_H	*rH/Y.*	Share of "risk-free" income from human wealth in real output, $(r+\beta)\omega_H = r((1-t_L)\omega_W-\omega_Z)$.
ω_f	$f/(f+\tilde{Y})$	Proportion of fixed cost in gross output. If entry/exit is free then $\mu(1-\omega_f)/\lambda=1$.
ω_L^*		Adjusted share of before tax wage income in real output, $\omega_L^*=\mu(1-\omega_f)\omega_L/\lambda$.
ω_A	*rA/Y.*	Share of income from financial assets in real output, $\omega_A=\omega_C+\omega_Z-(1-t_L)\omega_L$.
ω_G	G_0P_G/PY.	Share of government spending on differentiated goods in output.
ω_B	*rB/Y.*	Share of government interest payments in output, $\omega_Z=\omega_G+\omega_B-t_L\omega_L$.
ω_C	*C/Y.*	Share of private consumption in real output
ω_Q	QP_Q/PY.	Share of investment spending on differentiated goods in output, $\omega_C+\omega_Q+\omega_G=1$).
ω_{LL}	$(1-L)/L$	Ratio between leisure and labour.
ω_V	*rV/Y.*	Share of income from company shares in real output, $\omega_V=\omega_A-\omega_B=1-\omega_L-\omega_Q$.
t_L		Proportional tax rate on labour levied on households.

a. Equation (E') is relevant only under bond-financing. A value of ζ in excess of unity is needed to ensure that the government's solvency condition (2.21) is satisfied. See section 3.4 in the text.

b. Equation (F*') must be used under restricted entry ($\tilde{N}=0$) in which case the zero pure-profit condition (N') is irrelevant.

$$\tilde{V} = r\tilde{V} + r\omega_V\tilde{r} - r[\tilde{Y} - \omega_L(\tilde{W} + \tilde{L}) - \omega_Q(\tilde{Q} + \tilde{P}_Q - \tilde{P})] \qquad \text{(F*')}$$

Table 3: Log-Linearized Version of the Benchmark Model[a]

$$\dot{\tilde{K}} = \delta[\tilde{Q} - \tilde{K}] \quad \text{(A}'\text{)}$$

$$\dot{\tilde{X}} = (r-\alpha)\tilde{X} + r\tilde{r} - \left[\frac{r-\alpha}{\omega_A}\right][\omega_V\tilde{K} + \tilde{B}] \quad \text{(B}'\text{)}$$

$$\dot{\tilde{B}} = r\left[\omega_G\tilde{G}_0 + \tilde{B} + t_L\epsilon_L[\tilde{L} + \tilde{W}] - \omega_Z\tilde{Z} - (1-t_L)\epsilon_L\tilde{t}_L\right] \quad \text{(C}'\text{)}$$

$$\zeta\dot{\tilde{B}} = r\omega_Z[\tilde{Z}_0 - \tilde{Z}] \quad \text{(D}'\text{)}$$

$$\tilde{L} = \tilde{Y} - \tilde{W} \qquad \tilde{K} - \tilde{L} = \tilde{W} - \left[\frac{r}{r+\delta}\right]\tilde{r} \quad \text{(E}'\text{)}$$

$$\tilde{Y} = \omega_C\tilde{C} + \omega_Q\tilde{Q} + \omega_G\tilde{G}_0 \quad \text{(F}'\text{)}$$

$$\tilde{C} = \tilde{X} \qquad \tilde{L} = \omega_{LL}\left[\tilde{W} - \tilde{t}_L - \tilde{X}\right] \quad \text{(G}'\text{)}$$

$$\tilde{Y} = \mu[\epsilon_L\tilde{L} + (1-\epsilon_L)\tilde{K}] \qquad \mu \equiv \frac{\sigma_C}{\sigma_C - 1} \quad \text{(H}'\text{)}$$

Definitions:

ϵ_L	WL/Y.	Share of before tax wage income in real output.
ω_A	rA/Y.	Share of income from financial assets in real output, $\omega_A = \omega_C + \omega_Z - (1-t_L)\epsilon_L$.
ω_G	G_0/Y.	Share of government spending on differentiated goods in output.
ω_C	C/Y.	Share of private consumption in real output
ω_Q	Q/Y.	Share of investment spending on differentiated goods in output, $\omega_C + \omega_Q + \omega_G = 1$).
ω_{LL}	$(1-L)/L$	Ratio between leisure and labour.
ω_Z	Z/Y.	Share of lump-sum taxes in real output, $\omega_Z = \omega_G + t_L\epsilon_L$.
ω_V	rV/Y.	Share of income from company shares in real output, $\omega_V = \omega_A = 1 - \epsilon_L - \omega_Q$.
t_L		Proportional tax rate on labour levied on households.

a. Equation (D′) is relevant only under bond-financing. A value of ζ in excess of unity is needed to ensure that the government's solvency condition (2.21) is satisfied.

Table 4. Long-Run Effects of Fiscal Policy

(lump-sum financed, free entry/exit, finite horizons)
Shock: $\tilde{G}_0 = 1$

Intratemporal substitution elasticity between leisure and consumption: σ_{CM}

Intertemporal substitution parameter:		0	0.5	1	2	3
			Output multiplier: dY/dG_0			
ξ	2	0.86	0.95	1.07	1.45	2.26
	1	0.85	0.94	1.05	1.39	2.11
	0.8	0.85	0.94	1.05	1.38	2.08
	0.5	0.85	0.93	1.04	1.37	2.04
	0.2	0.84	0.93	1.03	1.36	2.01
$\beta=0$		0.88	0.99	1.14	1.64	2.88
			Consumption multiplier: dC/dG_0			
ξ	2	-0.24	-0.16	-0.06	0.27	0.99
	1	-0.25	-0.17	-0.07	0.23	0.86
	0.8	-0.25	-0.17	-0.07	0.22	0.84
	0.5	-0.25	-0.17	-0.08	0.21	0.81
	0.2	-0.25	-0.17	-0.09	0.20	0.77
$\beta=0$		-0.24	-0.14	-0.01	0.42	1.51
			Capital stock effect: $\tilde{K}$ (in %)			
ξ	2	5.39	6.02	6.82	9.30	14.71
	1	5.16	5.74	6.47	8.71	13.38
	0.8	5.11	5.68	6.40	8.59	13.13
	0.5	5.05	5.60	6.30	8.43	12.78
	0.2	4.98	5.52	6.20	8.26	12.44
$\beta=0$		6.15	6.96	8.01	11.46	20.18
			Employment effect: $\tilde{L}$ (in %)			
ξ	2	4.27	4.74	5.32	7.14	11.10
	1	4.32	4.76	5.31	6.99	10.51
	0.8	4.33	4.76	5.31	6.97	10.40
	0.5	4.34	4.77	5.30	6.92	10.25
	0.2	4.36	4.77	5.29	6.88	10.10
$\beta=0$		4.12	4.66	5.37	7.68	13.53
			Real wage effect: $\tilde{W}$ (in %)			
ξ	2	1.72	1.92	2.18	2.99	4.74
	1	1.62	1.81	2.05	2.77	4.27
	0.8	1.60	1.79	2.02	2.72	4.19
	0.5	1.58	1.76	1.98	2.66	4.06
	0.2	1.55	1.72	1.94	2.60	3.94
$\beta=0$		2.03	2.29	2.64	3.78	6.65

Table 5. Long-Run Effects of Fiscal Policy under Perfect Competition

(lump-sum financed, finite horizons)
Shock: $\tilde{G}_0 = 1$

Intratemporal substitution elasticity between leisure and consumption: σ_{CM}

Intertemporal substitution parameter:		0	0.5	1	2	3
		Output multiplier: dY/dG_0				
ξ	2	0.76	0.75	0.74	0.71	0.69
	1	0.76	0.74	0.72	0.69	0.66
	0.8	0.75	0.74	0.72	0.69	0.65
	0.5	0.75	0.73	0.71	0.68	0.65
	0.2	0.75	0.73	0.71	0.67	0.64
$\beta=0$		0.79	0.79	0.79	0.79	0.79
		Consumption multiplier: dC/dG_0				
ξ	2	-0.32	-0.33	-0.35	-0.37	-0.39
	1	-0.33	-0.34	-0.36	-0.38	-0.41
	0.8	-0.33	-0.34	-0.36	-0.39	-0.42
	0.5	-0.33	-0.34	-0.36	-0.39	-0.42
	0.2	-0.33	-0.34	-0.36	-0.40	-0.43
$\beta=0$		-0.32	-0.32	-0.32	-0.32	-0.32
		Capital stock effect: $\tilde{K}$ (in %)				
ξ	2	4.62	4.54	4.45	4.29	4.13
	1	4.36	4.25	4.15	3.94	3.75
	0.8	4.31	4.19	4.09	3.88	3.68
	0.5	4.23	4.11	4.00	3.78	3.57
	0.2	4.15	4.03	3.91	3.68	3.47
$\beta=0$		5.51	5.51	5.51	5.51	5.51
		Employment effect: $\tilde{L}$ (in %)				
ξ	2	5.65	5.55	5.46	5.27	5.10
	1	5.69	5.56	5.44	5.20	4.98
	0.8	5.70	5.57	5.44	5.19	4.96
	0.5	5.71	5.57	5.43	5.17	4.93
	0.2	5.72	5.57	5.43	5.15	4.89
$\beta=0$		5.51	5.51	5.51	5.51	5.51
		Real wage effect: $\tilde{W}$ (in %)				
ξ	2	-0.31	-0.30	-0.30	-0.29	-0.29
	1	-0.40	-0.39	-0.39	-0.38	-0.37
	0.8	-0.42	-0.41	-0.41	-0.39	-0.38
	0.5	-0.44	-0.44	-0.43	-0.42	-0.41
	0.2	-0.47	-0.46	-0.46	-0.44	-0.43
$\beta=0$		0.00	0.00	0.00	0.00	0.00

Figure 1. Fiscal Policy (Lump-Sum Taxes, Barro-Ramsey)

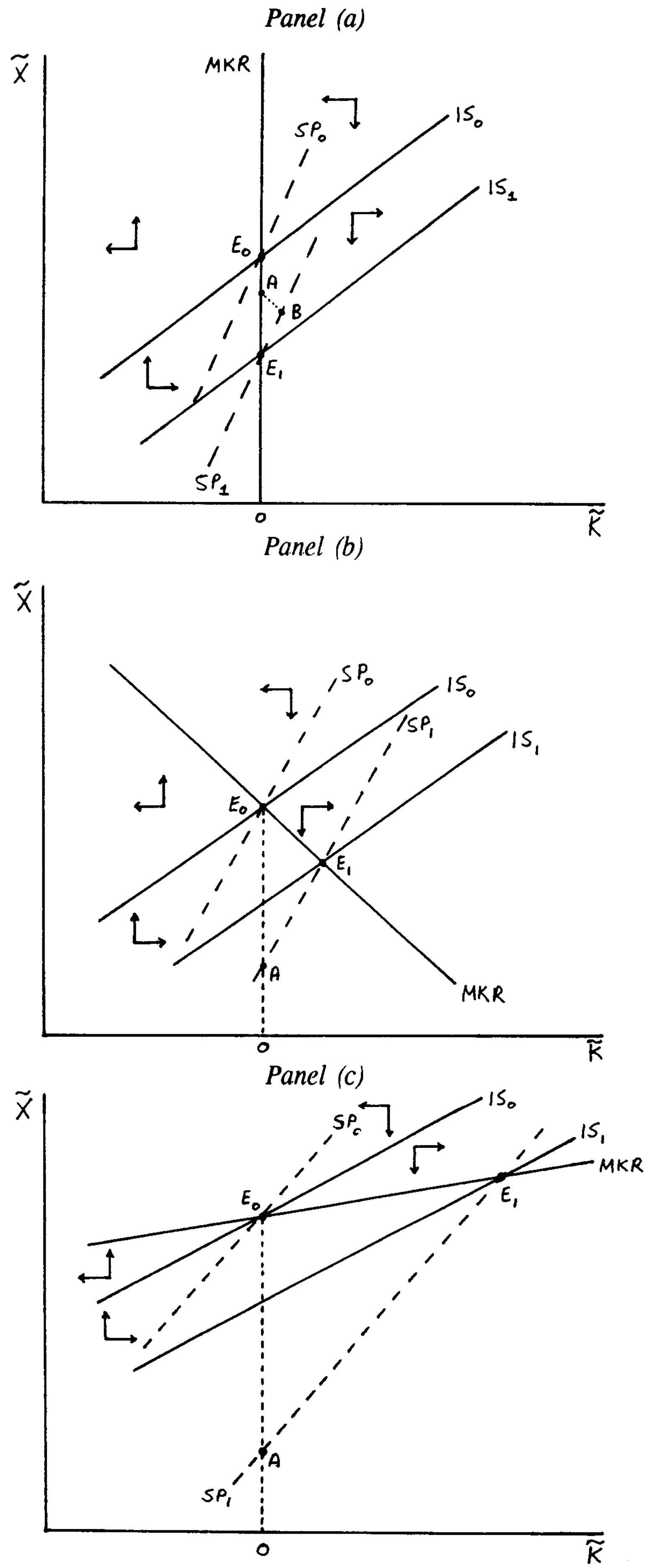

Figure 2. Fiscal Policy (Lump-Sum Taxes, Blanchard)

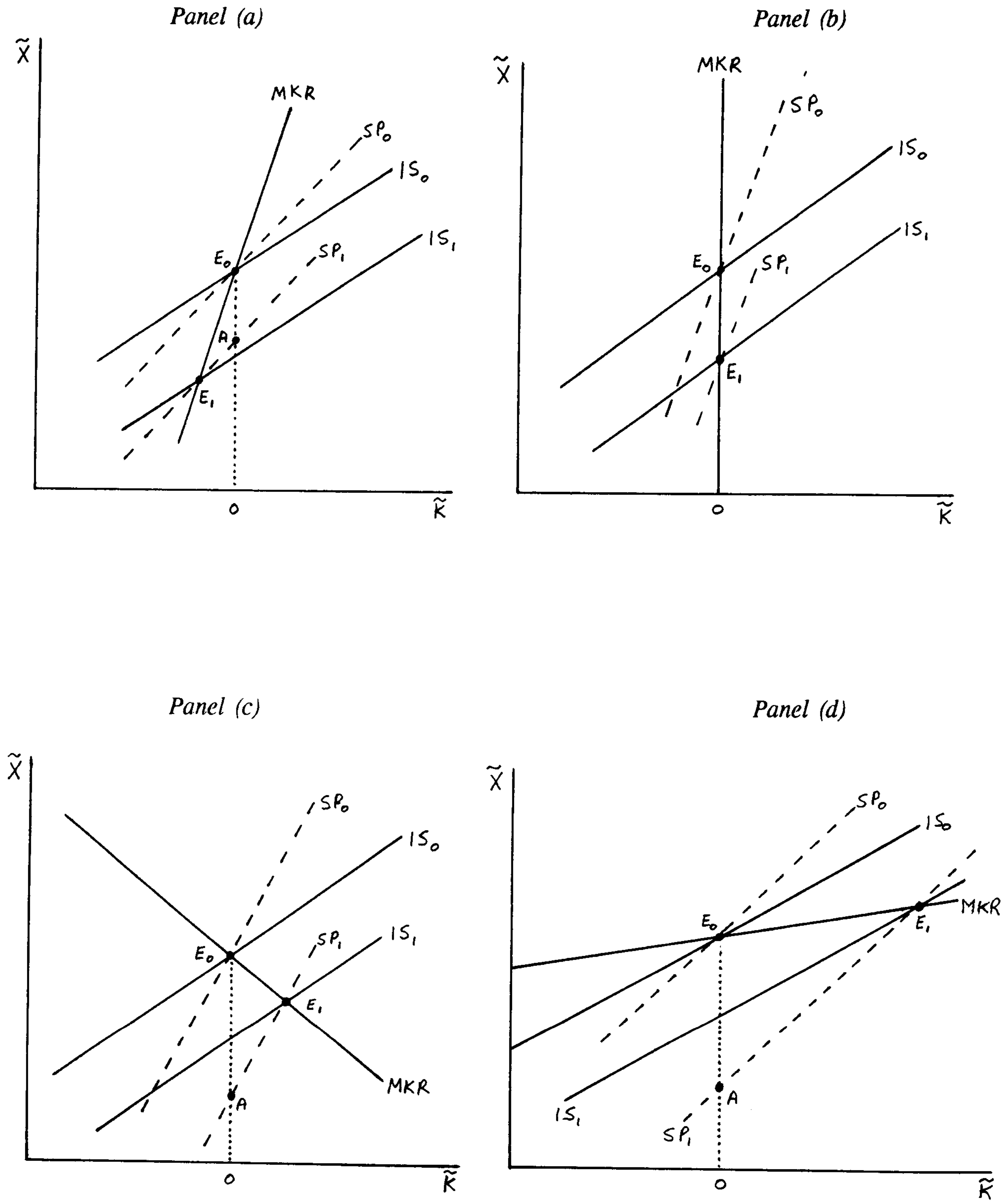

Figure 3. Fiscal Policy (Labour Taxes, Barro-Ramsey)