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The Macroeconomic Effects of Longevity Risk under Private and Public Insurance and Asymmetric Information

Abstract

We study the impact of a fully-funded social security system in an economy with heterogeneous consumers. The unobservability of individual health conditions leads to adverse selection in the private annuity market. Introducing social security—which is immune to adverse selection—affects capital accumulation and individual welfare depending on its size and on the pension benefit rule that is adopted. If this rule incorporates some implicit or explicit redistribution from healthy to unhealthy individuals then the latter types are better off as a result of the pension system. In the absence of redistribution the public pension system makes everybody worse off in the long run. Though attractive to distant generations, privatization of social security is not generally Pareto improving to all generations.

JEL-Codes: D910, E100, H550, J100.

Keywords: social security, annuity market, adverse selection, overlapping generations, redistribution.

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1 Introduction

More than half a century ago Yaari (1965) proved convincingly that private annuities are very attractive insurance instruments when non-altruistic individuals face longevity risk. Simply put, annuities are desirable because they insure such agents against the risk of outliving their assets. Yaari also proved a much stronger result: in the absence of an intentional bequest motive, rational utility-maximizing individuals should *fully* annuitize all of their savings. Yaari derives this result under the strong assumption that actuarially fair annuities are available. In a more recent paper, however, Davidoff *et al.* (2005) have demonstrated that the full annuitization result holds in a much more general setting than the one adopted by Yaari, for example when annuities are less than actuarially fair.

Despite the theoretical attractiveness of annuities, there is a vast body of empirical evidence showing that in reality people do not invest heavily in private annuity markets. The discrepancy between the theoretical predictions and the observable facts regarding annuity markets is know as the annuity puzzle. Of course there are many reasons why individuals may not choose to fully annuitize their wealth. Friedman and Warshawsky (1990, pp. 136-7), for example, argue that purchases of private annuities are low because (a) individuals may want to leave bequests to their offspring, (b) agents may already implicitly hold social annuities because they are participating in a system of mandatory public pensions, and (c) private annuities may be priced unattractively, for example because of transaction costs and taxes, excessive profits extracted by imperfectly competitive annuity firms, and adverse selection. Intuitively, under asymmetric information annuity companies cannot observe an individual's health status. Adverse selection arises in such a setting because agents with above-average health are more likely to buy annuities. This implies that such "high-risk types" are over-represented in the group of clients of annuity firms and that pricing of annuities cannot be based on the average health status of the population at large.

While recognizing their potential role in accounting for parts of the annuity puzzle, we ignore intentional bequest motives, administrative costs, and imperfect competition in this paper. Instead, we follow *inter alia* Abel (1986), Walliser (2000), Palmon and Spivak (2007), Sheshinski (2008), and Heijdra and Reijnders (2012) by focusing on the adverse selection channel. We approach the material sequentially by first demonstrating the adverse selection effect in an economy without public pensions. In the next step we introduce social annuities and study the general equilibrium interactions between private and public annuity markets under different pension benefit rules.

Our paper is most closely related to earlier work by Heijdra and Reijnders (2012). They study a discrete-time overlapping generations model in which non-altruistic agents differ in their innate health status, which is assumed to be private information. The private annuity market settles in a risk-pooling equilibrium in which the unhealthiest segment of the population experiences binding borrowing constraints (because they are unable to go short on annuities) and the other agents receive a common yield on their annuity purchases. They also show that the introduction of a mandatory public pension system—though immune to adverse selection by design—leads to a reduction in steady-state welfare, an aggravation of adverse selection in the private annuity market, and a reduction in the economy-wide capital intensity.

We extend the work by Heijdra and Reijnders (2012) by assuming that the individuals populating the economy differ by *two dimensions* of heterogeneity (health and ability) rather than just a single one (health). The introduction of heterogeneous abilities serves two purposes. First, as was shown by Walliser (2000, pp. 374-375) in a partial equilibrium setting, "(the simulations reveal that) between 40 and 60 percent of the measured adverse selection is due to the positive correlation between income and mortality..." By incorporating health-ability heterogeneity, and by assuming that there is a positive correlation between the two characteristics, we are able to capture this reputedly important source of adverse selection in the private annuity market. There is a second reason why heterogeneity matters which is related to the type of funded public pension system that is in place. Indeed, depending on the details regarding pension contributions and receipts, social security systems can have vastly different welfare implications for consumers with different health status and/or ability. In this paper we consider three different public pension schemes which differ in the degree to which they lead to (implicit or explicit) redistribution from healthy to unhealthy individuals.

Our main findings are as follows. Firstly, a plausibly calibrated version of the model reveals that, compared to the case with full information, asymmetric information on the part of annuity companies is important quantitatively in that it causes substantial reductions in steady-state output per efficiency unit of labour and the capital intensity. The general equilibrium effects are thus shown to matter a lot. Second, the introduction of a funded social security system reduces the capital intensity and output per efficiency unit even further, more so the larger is the system, i.e. the higher is the replacement rate it incorporates. These results are consistent with Palmon and Spivak (2007) and Heijdra and Reijnders (2012). Third, privatizing social security (by abolishing the public pension system) is not generally Pareto improving to *all* generations. Indeed, in our simulations we find that healthy agents born at the time of the shock would have been better off if the social security system had not been privatized. Just as for unfunded pensions, getting rid of a pre-existing funded system is not an easy task to accomplish.

The remainder of the paper is organized as follows. In Section 2 we set up the model and characterize the microeconomic choices and the resulting macroeconomic equilibrium under full information, i.e. the hypothetical case in which insurance companies can perfectly observe an individual's characteristics. In Section 3 we introduce asymmetric information inhibiting insurance companies and show that it leads to a pooling equilibrium in the annuity market. In Section 4 we introduce a fully-funded social security system in which pension contributions are proportional to labour income during youth. We analyze three specific versions of this system which differ with respect to the pension receipts during old age. Section 5 considers the consequences of privatizing social security. The final section concludes. Some technical issues are dealt with in three brief appendices.

2 Model

2.1 Consumers

In each period the population in the closed economy under consideration features two overlapping generations of heterogeneous agents. Each person can live at most for two periods, namely 'youth' (superscript y) and 'old age' (superscript o). Individuals are heterogenous along two exogenously given dimensions. First, they differ by health status which we capture by the probability of surviving into old-age. Everyone faces lifetime uncertainty at the end of the first period, and the survival probability is denoted by μ. This means that unhealthy people have a higher risk of dying and a shorter expected life span (which equals $1 + \mu$ periods). Second, individuals differ in their working ability as proxied by innate labour productivity η.

We assume that consumer types are continuous and uniformly distributed on these two dimensions, i.e. $\mu \in [\mu_L, \mu_H]$ (such that $0 < \mu_L < \mu_H < 1$) and $\eta \in [\eta_L, \eta_H]$ (such that $0 < \eta_L < \eta_H$). Furthermore, we postulate that μ and η are positively correlated. Hence, a person in better health is more likely to possess higher working abilities, and vice versa. The bivariate uniform distribution used in this paper is characterized by the following probability density function:

$$h(\mu, \eta) = \frac{1 + \xi\,(\mu - \bar{\mu})(\eta - \bar{\eta})}{(\mu_H - \mu_L)(\eta_H - \eta_L)}, \tag{1}$$

where ξ is a parameter regulating the correlation between μ and η (such that $\xi > 0$), and $\bar{\mu}$ and $\bar{\eta}$ denote the unconditional means of μ and η, respectively. In Figure 1 the distribution function is depicted in panel (a) whilst the probability density function is shown in panel (b). From the graph of the density function it is clear that there is a higher probability for healthier consumers to possess higher working abilities, and vice versa. For future reference we postulate Lemma 1 which summarizes some useful properties of the bivariate distribution that we employ.

Lemma 1 *The distribution function for the survival probability μ and labour productivity η is given by:*

$$H(\mu, \eta) = \frac{(\mu - \mu_L)(\eta - \eta_L)}{(\mu_H - \mu_L)(\eta_H - \eta_L)} \left[1 + \frac{\xi}{4}\,(\mu_H - \mu)(\eta_H - \eta)\right],$$

where $\mu_L \leq \mu \leq \mu_H$ and $\eta_L \leq \eta \leq \eta_H$. The density function is given in (1). Further *properties of the distribution are: (i) the marginal density functions are* $h_\mu(\mu) = 1/(\mu_H - \mu_L)$ *and* $h_\eta(\mu) = 1/(\eta_H - \eta_L)$*; (ii) the unconditional means are* $\bar{\mu} = (\mu_L + \mu_H)/2$ *and* $\bar{\eta} = (\eta_L + \eta_H)/2$*; (iii) the unconditional variances are* $\sigma_\mu^2 = (\mu_H - \mu_L)^2/12$ *and* $\sigma_\eta^2 = (\eta_H - \eta_L)^2/12$*; (iv) the covariance is* $\text{cov}(\eta, \mu) = \xi \sigma_\eta^2 \sigma_\mu^2$ *and the correlation is* $\text{cor}(\eta, \mu) = \xi \sigma_\eta \sigma_\mu$*; (v) the conditional probability density functions are:*

$$h_{\mu|\eta}(\mu) \equiv \frac{h(\eta, \mu)}{h_\eta(\eta)} = \frac{1 + \xi(\mu - \bar{\mu})(\eta - \bar{\eta})}{\mu_H - \mu_L},$$
$$h_{\eta|\mu}(\eta) \equiv \frac{h(\eta, \mu)}{h_\mu(\mu)} = \frac{1 + \xi(\mu - \bar{\mu})(\eta - \bar{\eta})}{\eta_H - \eta_L},$$

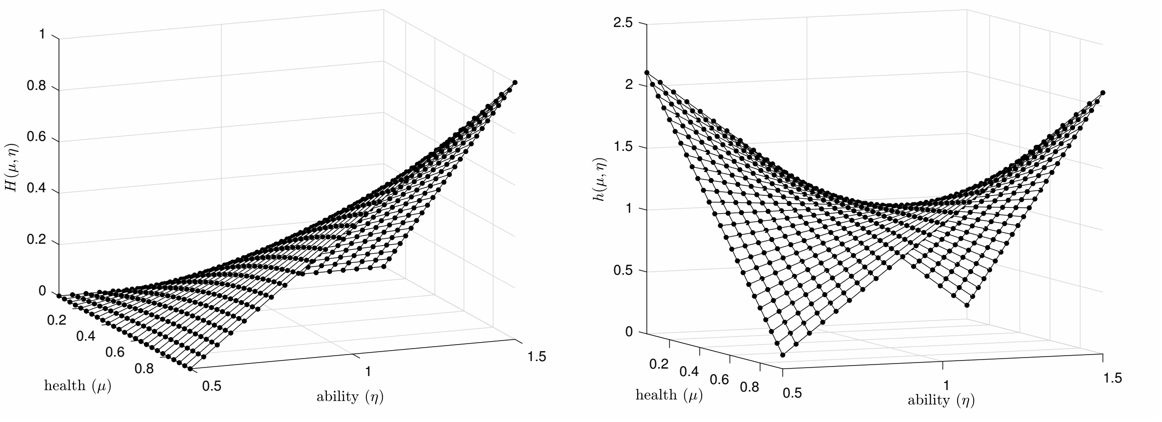
and (vi) the conditional mean of η *for a given* μ *is:*

$$\Gamma_1(\mu) \equiv \frac{\int_{\eta_L}^{\eta_H} \eta h(\eta, \mu)\, d\eta}{\int_{\eta_L}^{\eta_H} h(\eta, \mu)\, d\eta} = \frac{h_\mu(\mu) \int_{\eta_L}^{\eta_H} \eta h_{\eta|\mu}(\eta)\, d\eta}{h_\mu(\mu)} = \bar{\eta} + \xi \sigma_\eta^2 (\mu - \bar{\mu}).$$

Proof: *see Appendix A.* ■

Figure 1: Features of the distribution for μ and η

(a) Distribution $H(\mu, \eta)$ (b) Density $h(\mu, \eta)$



Legend Health and innate ability are proxied by, respectively, the survival probability μ and the labour productvity parameter η. The two characteristics of an individual are positively correlated. The distribution $H(\mu, \eta)$ is bivariate uniform. The marginal distributions of μ and η are both uniform. See Appendix A and Lemma 1 for further features of the distribution.

From the perspective of birth, the expected lifetime utility of a person with health status

μ and working ability η is given by:

$$\mathbb{E}\Lambda_t(\mu,\eta) \equiv U\left(C_t^y(\mu,\eta)\right) + \mu\beta U\left(C_{t+1}^o(\mu,\eta)\right), \tag{2}$$

where $C_t^y(\mu,\eta)$ and $C_{t+1}^o(\mu,\eta)$ are consumption during youth and old age, respectively, β is the parameter capturing pure time preference ($0<\beta<1$), and $U(C)$ is the felicity function:

$$U(C) \equiv \begin{cases} \dfrac{C^{1-1/\sigma}-1}{1-1/\sigma}, & \text{for } \sigma \neq 1, \\ \ln C & \text{for } \sigma = 1, \end{cases} \tag{3}$$

where σ is the intertemporal elasticity of substitution ($\sigma > 0$). Equation (2) incorporates the assumption that individuals do not have a bequest motive, i.e. utility solely depends on own consumption during one's lifetime.

In this section we postulate the existence of perfect private annuities. Specifically, we adopt the following assumptions regarding the market for private annuities:

(A0) Health status is public information.

(A1) The annuity market is perfectly competitive. A large number of risk-neutral firms offer annuities to individuals, and annuity firms can freely enter or exit the market.

(A2) Annuity firms do not use up any real resources.

As is explained by Heijdra and Reijnders (2012, pp. 322–3), in this *Full Information* case (abbreviated as FI) each health type receives its actuarially fair rate of return and achieves perfect insurance against longevity risk. If $A_t^p(\mu,\eta)$ denotes the private annuity holdings of an agent of health type μ then the net rate of return on annuities will be equal to:

$$1 + r_{t+1}^p(\mu) = \frac{1+r_{t+1}}{\mu}, \tag{4}$$

where r_{t+1} is the net rate of return on physical capital (see also below). Since the survival rate is such that $0<\mu<1$, it follows from (4) that $r_{t+1}^p(\mu)$ exceeds r_{t+1} so that all agents will completely annuitize their wealth. This classic result was first derived by Yaari (1965).

We assume that individuals work full time during youth and part time in old age as a result of a system of mandatory retirement. With full annuitization of assets the periodic budget identities are given by:

$$C_t^y(\mu,\eta) + A_t^p(\mu,\eta) = w_t(\eta), \tag{5}$$

$$C_{t+1}^o(\mu,\eta) = \lambda w_{t+1}(\eta) + (1 + r_{t+1}^p(\mu))A_t^p(\mu,\eta), \tag{6}$$

where $w_t(\eta)$ is the wage rate of an η type in period t, λ is the proportion of time that is devoted to work in old age ($0 < \lambda < 1$), and $1 + r^p_{t+1}(\mu)$ is the rate of return on private annuities. The periodic budget identities can be combined to obtain the consolidated budget constraint:

$$C^y_t(\mu,\eta) + \frac{C^o_{t+1}(\mu,\eta)}{1 + r^p_{t+1}(\mu)} = w_t(\eta) + \frac{\lambda w_{t+1}(\eta)}{1 + r^p_{t+1}(\mu)}. \tag{7}$$

The present value of lifetime consumption (left-hand side) equals the present value of lifetime income (right-hand side). That is, people consume their human wealth.

Consumers choose $C^y_t(\mu,\eta)$ and $C^o_{t+1}(\mu,\eta)$ in order to maximize expected lifetime utility (2) subject to the budget constraint (7). The optimal consumption plans and annuity demands are fully characterized by:

$$C^y_t(\mu,\eta) = \Phi\left(\mu, \frac{1 + r_{t+1}}{\mu}\right)\left[w_t(\eta) + \frac{\lambda\mu w_{t+1}(\eta)}{1 + r_{t+1}}\right], \tag{8}$$

$$\frac{\mu C^o_{t+1}(\mu,\eta)}{1 + r_{t+1}} = \left[1 - \Phi\left(\frac{1 + r_{t+1}}{\mu}\right)\right]\left[w_t(\eta) + \frac{\lambda\mu w_{t+1}(\eta)}{1 + r_{t+1}}\right], \tag{9}$$

$$A^p_t(\mu,\eta) = \left[1 - \Phi\left(\mu, \frac{1 + r_{t+1}}{\mu}\right)\right] w_t(\eta) - \Phi\left(\mu, \frac{1 + r_{t+1}}{\mu}\right)\frac{\lambda\mu w_{t+1}(\eta)}{1 + r_{t+1}}, \tag{10}$$

where we have substituted the expression for the actuarially fair annuity rate (4), and where $\Phi(\mu, x)$ is the marginal propensity to consume out of lifetime income during youth:

$$\Phi(\mu, x) \equiv \frac{1}{1 + (\mu\beta)^\sigma x^{\sigma-1}}. \tag{11}$$

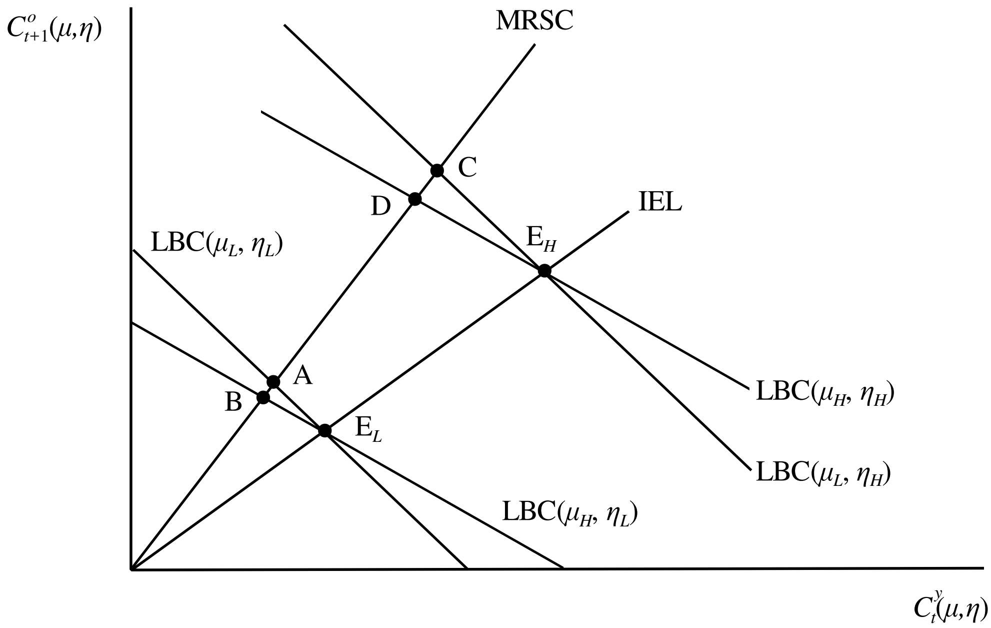
From equations (8) and (9) we find that consumption during youth and old-age are both proportional to human wealth. Furthermore, equation (10) shows that annuity demand depends positively on the wage income during youth and negatively on old-age labour income.

The optimal consumption choices of different types of consumers are illustrated in Figure 2. To avoid cluttering the diagram we illustrate the choices made by the four extreme types, unhealthy and healthy lowest-skilled (μ_L, η_L) and (μ_H, η_L), and unhealthy and healthy highest-skilled (μ_L, η_H) and (μ_H, η_H). For a given working ability type η_i, the line labelled LBC(μ_L, η_i) and LBC(μ_H, η_i) are the lifetime budget constraints as given in (7). For skill type η_L the income endowment point $(w_t(\eta), \lambda w_{t+1}(\eta))$ is located at point E_L. With perfect annuities, LBC(μ_L, η_i) is steeper than LBC(μ_H, η_i) because the unhealthy get a much higher annuity rate than the healthy.

In the presence of perfect annuities and under full annuitization, the consumption Euler equation is given by:

$$\frac{U'(C^y_t(\mu,\eta))}{\beta U'(C^o_{t+1}(\mu,\eta))} = \mu\left(1 + r^p_{t+1}(\mu)\right) = 1 + r_{t+1}, \tag{12}$$

Figure 2: Consumption-saving choices under full information



Legend LBC(μ_i, η_j) is the lifetime budget constraint for an individual with survival probability μ_i and productivity level η_j. IEL is the income endowment line and agents are located on the line segment $E_L E_H$. MRSC is the consumption Euler equation under perfect information with actuarially fair annuities at the individual level. Optimal consumption for individual (μ_i, η_j) is located at the intersection of MRSC and LBC(μ_i, η_j). All individuals purchase annuities.

where we have used (4) to get from the first to the second equality. The crucial thing to note is that all agents equate the marginal rate of substitution between current and future consumption to the gross interest factor on capital. Intuitively, as was first pointed out by Yaari (1965), the mortality rate drops out of the expression characterizing the life-cycle profile of consumption because agents are fully insured against the unpleasant aspects of lifetime uncertainty. For the homothetic felicity function (3) it is easy to show that (12) is a ray from the origin—see the locus labelled MRSC in Figure 2. Optimal choices are located at the intersection of MRSC and the relevant lifetime budget constraint. It follows that types (μ_L, η_L) and (μ_H, η_L) consume at points A and B respectively.

What about the choices made by the highest-ability types? Given the specification of technology adopted below, it follows that $w_t(\eta) = \eta w_t$ and $w_{t+1}(\eta) = \eta w_{t+1}$ so that income endowment points lie along the ray from the origin labelled IEL. Furthermore, it follows from (7) that LBC(μ_L, η_H) is parallel to LBC(μ_L, η_L) whilst LBC(μ_H, η_H) is parallel to LBC(μ_H, η_L). Hence types (μ_L, η_H) and (μ_H, η_H) consume at points C and D respectively.

Several conclusions can be drawn from the microeconomic behaviour discussed in this subsection. First, in this closed economy featuring a positive capital stock (see below) all agents are net savers, i.e. everybody expresses a positive demand for private annuities, $A_t^p(\mu, \eta) > 0$ for all μ and η. This result follows readily from Figure 2 because the MRSC line lies to the left of the IEL line. Second, for a given value of agent productivity η, the demand for annuities is increasing in the survival probability μ, i.e. $\partial A_t^p(\mu, \eta)/\partial \mu > 0$. Intuitively, healthy people buy more annuities than do unhealthy people of the same skill category because they expect to live longer a priori. Again this result follows readily from Figure 2 because LBC(μ_L, η_i) is steeper than LBC(μ_H, η_i). Third, the demand for annuities is increasing in the skill level, i.e. $\partial A_t^p(\mu, \eta)/\partial \eta > 0$. This can be see graphically in Figure 2 and can be proved formally by noting that $A_t^p(\mu, \eta)$ in (10) is linear in η.

2.2 Demography

Let L_t denote the size of the population cohort born at time t. The density of consumers with health type μ and working ability η is thus:

$$L_t(\mu, \eta) \equiv h(\mu, \eta) L_t, \tag{13}$$

where the density function $h(\mu, \eta)$ is stated in (1) above. The density of (young and old) consumers of type μ alive at time t is given by:

$$P_t(\mu) \equiv \mu \int_{\eta_L}^{\eta_H} L_{t-1}(\mu, \eta) d\eta + \int_{\eta_L}^{\eta_H} L_t(\mu, \eta) d\eta = \mu h_\mu(\mu) L_{t-1} + h_\mu(\mu) L_t, \tag{14}$$

where $h_\mu(\mu)$ is the marginal distribution of μ (see Lemma 1(i)). If newborn cohort sizes evolves according to $L_t = (1+n)L_{t-1}$ (with $n > -1$), the total population at time t is given by:

$$P_t \equiv \int_{\mu_L}^{\mu_H} P_t(\mu)d\mu = \frac{1+n+\bar{\mu}}{1+n}L_t, \tag{15}$$

where $\bar{\mu} \equiv \int_{\mu_L}^{\mu_H} \mu h_\mu(\mu)d\mu$ is the average survival rate of a newborn cohort.

2.3 Production

We assume that perfect competition prevails in the goods market. The technology is represented by the following Cobb-Douglas production function:

$$Y_t = \Omega_0 K_t^\varepsilon N_t^{1-\varepsilon}, \tag{16}$$

where Y_t is total production, K_t is the aggregate capital stock, ε is the efficiency parameter of capital ($0 < \varepsilon < 1$), Ω_0 is total factor productivity (assumed to be constant), and N_t is the *effective* labor force, which is defined as:

$$N_t \equiv \int_{\eta_L}^{\eta_H} \int_{\mu_L}^{\mu_H} \eta \left[L_t(\mu,\eta) + \lambda L_{t-1}(\mu,\eta)\right] d\mu d\eta. \tag{17}$$

Note that N_t has the dimension of worker efficiency (denoted by η) times number of working hours. By using (13) in (17) and noting that $L_t = (1+n)L_{t-1}$ we find that N_t/L_t can be written as:

$$\frac{N_t}{L_t} = \bar{\eta} + \frac{\lambda}{1+n}\left[\bar{\eta}\bar{\mu} + \text{cov}\,(\eta,\mu)\right], \tag{18}$$

where $\text{cov}\,(\eta,\mu) \equiv \xi\sigma_\eta^2\sigma_\mu^2$ is the (positive) covariance between μ and η (see Lemma 1(iv)).

By defining $y_t \equiv Y_t/N_t$ and $k_t \equiv K_t/N_t$, the intensive-form production function can be written as:

$$y_t = \Omega_0 k_t^\varepsilon. \tag{19}$$

Firms choose efficiency units of labour and the capital stock such that profits are maximized. This optimization problem gives the following factor demand equations:

$$r_t + \delta = \varepsilon\Omega_0 k_t^{\varepsilon-1}, \tag{20}$$

$$w_t = (1-\varepsilon)\Omega_0 k_t^\varepsilon, \tag{21}$$

$$w_t(\eta) = \eta w_t, \tag{22}$$

where r_t is the net rate of return on physical capital, δ is the depreciation rate of capital ($0 < \delta < 1$), and w_t is the rental rate on efficiency units of labour. With perfect substitutability of efficiency units of labour, the wage rate of a η type worker, $w_t(\eta)$, is η times the rental rate w_t (as was asserted above).

2.4 Equilibrium

The model is completed by a description of the macroeconomic equilibrium. Since all annuity purchases are invested in the capital market we find that:

$$K_{t+1} = L_t \int_{\mu_L}^{\mu_H} \int_{\eta_L}^{\eta_H} A_t^p(\mu,\eta) h(\mu,\eta) d\eta d\mu, \tag{23}$$

where $A_t^p(\mu,\eta)$ is given in (10) above. Intuitively, equation (23) says that next period's aggregate capital stock is equal to total savings in the current period (consisting of private annuities). By substituting the demand for annuities (10) and the wage equation (22) into (23) we obtain the fundamental difference equation for the capital intensity:

$$k_{t+1} = \frac{1}{1+n}\frac{L_t}{N_t}\left[\bar{\eta} w_t - \int_{\mu_L}^{\mu_H} \Phi\left(\mu, \frac{1+r_{t+1}}{\mu}\right)\left[w_t + \frac{\lambda \mu w_{t+1}}{1+r_{t+1}}\right] h_\mu(\mu)\Gamma_1(\mu) d\mu\right], \tag{24}$$

where $\Gamma_1(\mu)$ is the conditional mean of η given μ (see Lemma 1(vi)). In view of (20)–(21) w_t and r_{t+1} depend on, respectively, k_t and k_{t+1} so (24) is a non-linear implicit function relating k_{t+1} to k_t and the exogenous variables.

2.5 Parameterization and visualization

In order to visualize the main features of the economy we parameterize the model by selecting plausible values for the structural parameters—see Table 1. We follow Heijdra and Reijnders (2012) in the parameterization procedure. First, we postulate plausible values for the intertemporal elasticity of substitution ($\sigma = 0.7$), the efficiency parameter of capital ($\varepsilon = 0.275$), the annual capital depreciation rate ($\delta_a = 0.06$), the annual growth rate of the population ($n_a = 0.01$) and the target annual steady-state interest rate ($\hat{r}_a = 0.05$). Using these parameters we can determine the steady-state (annual) capital-output ratio ($\hat{K}/\hat{Y} = \varepsilon/(\hat{r}_a + \delta_a) = 2.5$). Second, we set the length of each period to be 40 years and compute the values for n, δ and $\hat{r}$ (noting that $n = (1+n_a)^{40} - 1$, $\delta = 1 - (1-\delta_a)^{40}$ and $\hat{r} = (1+r_a)^{40} - 1$). Third, we assume that the mandatory retirement age is 65 years so that $\lambda = 25/40 = 0.625$. In the fourth step, we choose $\eta_L = 0.5$, $\eta_H = 1.5$, $\mu_L = 0.05$, $\mu_H = 0.95$, so that the average health status is $\bar{\mu} = 0.5$, average working ability is $\bar{\eta} = 1$, and the variances are $\sigma_\eta^2 = 0.0833$ and $\sigma_\eta^2 = 0.0675$. By setting $\xi = 4$ we ensure that there is a strong correlation between health and ability, i.e. $\text{cor}(\mu, \eta) = 0.300$.[1] In the fifth step we choose Ω_0 such that $\hat{y} = 10$ in the initial steady state. This also pins down the steady state values for $\hat{k}$ and $\hat{w}$. In the final step the discount factor β is used as a calibration parameter, i.e. it is set at the value such that the steady-state version of the fundamental difference equation (24) is satisfied. To interpret the value of β in Table 1, note that the annual rate of time preference is $\rho_a = \beta^{-1/40} - 1 = 0.0204$ (a little over two percent per annum).

Table 1: Structural parameters

σ	intertemporal substitution elasticity		0.7000
ε	capital efficiency parameter		0.2750
δ_a	annual capital depreciation rate		0.0600
δ	capital depreciation factor		0.9158
n_a	population growth rate		0.0100
n	population growth factor		0.4889
β	time preference parameter	c	0.4462
λ	mandatory retirement parameter		0.6250
Ω_0	scale factor production function	c	12.9071
μ_L	survival rate of the unhealthiest		0.0500
μ_H	survival rate of the healthiest		0.9500
η_L	lowest working ability		0.5000
η_H	highest working ability		1.5000
ξ	covariance parameter of the distribution function		4.0000

Note The parameters labelled 'c' are calibrated as is explained in the text. The remaining parameters are postulated a priori. The values for δ and n follow from, respectively, δ_a and n_a, by noting that each model period represents 40 years.

The main features of the steady-state FI equilibrium are reported in column (a) of Table 2. Consistent with the calibration procedure, output per efficiency unit of labour is equal to ten ($\hat{y} = 10$) whilst the steady-state interest rate is five percent on an annual basis ($\hat{r}^a = 0.05$). The steady-state capital intensity equals $\hat{k} = 0.395$. Ownership of the capital stock is highly uneven due to the fact that individuals differ in terms of labour productivity. Indeed, as is noted in the table, the first ability quartile of agents (averaged over all survival rates) owns

[1] The positive correlation between health and income (or productivity) is mentioned by many authors in the literature on annuities—see, for example, Walliser (2000), Brunner and Pech (2008), Direr (2010), and Cremer *et al.* (2010). Firm empirical evidence on this correlation is, however, hard to come by. In a recent paper Chetty *et al.* (2016) employ US data for the period 2001-2014 and find that the gap in life expectancy between the richest and poorest 1% of individuals was 14.6 years for men and 10.1 years for women. In our calibration the expected lifetime at birth of the bottom and top 1% individuals (by productivity) are 54.65 and 65.35.

Table 2: Allocation and welfare

	(a) FI	(b) AI	(c) SA_A $\theta = 0.010$	(d) SA_A $\theta = 0.025$	(e) SA_B $\theta = 0.010$	(f) SA_B $\theta = 0.025$	(g) SA_C $\theta = 0.010$	(g) SA_C $\theta = 0.025$
$\hat{y}$	10.000	9.840	9.776	9.680	9.768	9.668	9.762	9.660
$\hat{k}$	0.395	0.373	0.364	0.351	0.363	0.350	0.362	0.349
%Q1	12.34	11.78	10.15	7.73	9.69	6.69	9.15	5.50
%Q2	19.81	19.46	17.14	13.59	16.90	12.98	16.75	12.60
%Q3	28.73	28.84	25.81	21.05	25.92	21.26	26.12	21.66
%Q4	39.12	39.93	36.18	30.11	36.74	31.46	37.22	32.58
%SAS			10.72	27.51	10.74	27.60	10.76	27.67
$\hat{r}$	6.04	6.34	6.47	6.66	6.48	6.69	6.50	6.70
$\hat{r}^a$	5.00%	5.11%	5.16%	5.22%	5.16%	5.23%	5.17%	5.24%
$\hat{w}$	7.250	7.134	7.087	7.018	7.082	7.010	7.077	7.003
$\widehat{BC}$	0.00%	5.83%	10.03%	17.66%	10.63%	19.33%	10.63%	19.33%
$\hat{\hat{r}}^p$		10.18	10.12	9.99	10.12	9.98	10.12	9.96
$\hat{\hat{\mu}}^p$		0.66	0.67	0.70	0.67	0.70	0.67	0.70
$\widehat{AS}$		1.31	1.34	1.39	1.35	1.40	1.35	1.41
$\hat{c}^y$	5.357	5.296	5.270	5.233	5.268	5.228	5.265	5.225
%Q1	15.99	16.03	16.02	15.98	16.06	16.09	16.12	16.20
%Q2	22.10	22.13	22.12	22.10	22.14	22.16	22.16	22.20
%Q3	28.06	28.05	28.05	28.06	28.04	28.04	28.02	27.99
%Q4	33.85	33.79	33.81	33.86	33.75	33.72	33.70	33.61
$\hat{c}^o$	4.087	4.021	3.994	3.954	3.991	3.949	3.988	3.946
%Q1	12.23	10.70	10.72	10.77	10.77	10.93	10.83	11.14
%Q2	19.74	18.78	18.79	18.82	18.82	18.90	18.83	18.95
%Q3	28.75	29.04	29.03	29.02	29.02	28.98	29.00	28.91
%Q4	39.28	41.48	41.46	41.39	41.39	41.18	41.33	41.00
$\mathbb{E}\hat{\Lambda}(\mu_L, \eta_L)$	1.014	0.996	0.989	0.978	1.022	1.020	1.026	1.029
$\mathbb{E}\hat{\Lambda}(\mu_H, \eta_L)$	1.433	1.471	1.468	1.463	1.260	1.261	1.266	1.276
$\mathbb{E}\hat{\Lambda}(\mu_L, \eta_H)$	1.529	1.517	1.513	1.506	1.532	1.527	1.531	1.525
$\mathbb{E}\hat{\Lambda}(\mu_H, \eta_H)$	2.143	2.167	2.164	2.161	2.031	2.026	2.030	2.024

Note Here %Qj denotes the share accounted for by skill quartile j (averaged over all survival rates) of the variable directly above it. %SAS is the share owned by the social annuity system. $\mathbb{E}\hat{\Lambda}(\mu_i, \eta_j)$ gives expected utility for an agent with health type μ_i and skill type η_i. $\widehat{BC}$ is the proportion of the population facing borrowing constraints. $\widehat{AS}$ is an indicator for the severity of adverse selection in the private annuity market.

12.34% of the capital stock. In contrast, the top ability quartile owns 39.12% of the economy's stock of capital.

Steady-state consumption (per efficiency unit of labour) by the young and surviving old are given by:

$$\hat{c}^y \equiv \frac{L_t}{N_t}\int_{\eta_L}^{\eta_H}\int_{\mu_L}^{\mu_H}\hat{C}^y(\mu,\eta)\,h(\mu,\eta)d\mu d\eta, \tag{25}$$

$$\hat{c}^o \equiv \frac{1}{1+n}\frac{L_t}{N_t}\left[\int_{\eta_L}^{\eta_H}\int_{\mu_l}^{\mu_h}\mu\hat{C}^o(\mu,\eta)\,h(\mu,\eta)d\mu d\eta\right]. \tag{26}$$

Inequality due to heterogeneous productivity also shows up in the consumption levels during youth and old-age. The two lowest-ability quartiles enjoy a modest and declining share of total consumption over the life-cycle due to the positive correlation between health and ability. The opposite holds for the two highest-ability quartiles. Finally, Table 2 also reports some welfare indicators. Not surprisingly we find that expected lifetime utility is lowest for individuals with low ability and poor health (μ_L,η_L) and highest for those lucky ones with high ability and excellent health (μ_H,η_H).[2]

In Figure 3 we depict the steady-state profiles for youth consumption, old-age consumption, annuity demand, and expected utility. These profiles have been averaged over η values and are thus a function of the survival probability only:

$$\hat{C}^y(\mu) \equiv \frac{\int_{\eta_L}^{\eta_H}\hat{C}^y(\mu,\eta)\,h(\mu,\eta)d\eta}{\int_{\eta_L}^{\eta_H}h(\mu,\eta)d\eta} = \Phi\left(\mu,\frac{1+\hat{r}}{\mu}\right)\left[1+\frac{\lambda\mu}{1+\hat{r}}\right]\hat{w}\Gamma_1(\mu), \tag{27}$$

$$\hat{C}^o(\mu) \equiv \frac{\int_{\eta_L}^{\eta_H}\hat{C}^o(\mu,\eta)\,h(\mu,\eta)d\eta}{\int_{\eta_L}^{\eta_H}h(\mu,\eta)d\eta} = \left[1-\Phi\left(\mu,\frac{1+\hat{r}}{\mu}\right)\right]\left[\frac{1+\hat{r}}{\mu}+\lambda\right]\hat{w}\Gamma_1(\mu), \tag{28}$$

$$\hat{A}^p(\mu) \equiv \frac{\int_{\eta_L}^{\eta_H}\hat{A}^p(\mu,\eta)\,h(\mu,\eta)d\eta}{\int_{\eta_L}^{\eta_H}h(\mu,\eta)d\eta} = \left[1-\Phi\left(\mu,\frac{1+\hat{r}}{\mu}\right)\left[1+\frac{\lambda\mu}{1+\hat{r}}\right]\right]\hat{w}\Gamma_1(\mu), \tag{29}$$

$$\mathbb{E}\Lambda(\mu) \equiv \frac{\int_{\eta_L}^{\eta_H}\mathbb{E}\Lambda(\mu,\eta)\,h(\mu,\eta)d\eta}{\int_{\eta_L}^{\eta_H}h(\mu,\eta)d\eta}. \tag{30}$$

In panel (a) we find that $\hat{C}^y(\mu)$ is increasing in μ. This result is the opposite of the findings reported by Heijdra and Reijnders (2012, p. 321) who assume that all individuals have the same labour productivity (i.e., $\sigma_\eta^2 = 0$ in their model). In our model, for a given productivity level η, youth consumption is decreasing in the survival probability (see Figure 2). But as a result of the positive correlation between η and μ, healthy agents also tend to be wealthy agents who consume more in youth as a result. Referring to equation (27), the term $\Phi\left(\mu,\frac{1+\hat{r}}{\mu}\right)\left[1+\frac{\lambda\mu}{1+\hat{r}}\right]$ is decreasing in μ but the $\Gamma_1(\mu)$ term is increasing in μ (see Lemma

[2] By scaling steady-state output such that $\hat{y}=10$ for the FI case we avoid the counterintuitive feature noted by Heijdra and Reijnders (2012, p. 321) that lifetime utility is decreasing in the survival probability.

1(vi))). Due to the strong correlation between μ and η the latter effect dominates the former, thus ensuring that $\hat{C}^y(\mu)$ is increasing in the survival probability.

As panel (b) shows, the profile for old-age consumption $\hat{C}^o(\mu)$ is also increasing in μ. Again this result is reversed if all agents feature the same labour productivity, as can be easily verified with the aid of Figure 2. In panel (c) we find that $\hat{A}^p(\mu)$ is increasing in μ. This result even holds if $\sigma_\eta^2 = 0$ (so that $\Gamma_1(\mu)$ is a constant) because $1 - \Phi\left(\mu, \frac{1+\hat{r}}{\mu}\right)\left[1 + \frac{\lambda\mu}{1+\hat{r}}\right]$ is increasing in μ. Finally, as panel (d) illustrates, $\mathbb{E}\hat{\Lambda}(\mu)$ is increasing in the survival probability. Intuitively, for a given productivity level η individual lifetime utility is increasing in μ (people like surviving into old-age). Furthermore, μ and η are positively correlated thus strengthening the positive link between utility and health.

3 Informational asymmetry in the private annuity market

In the previous section we have studied the steady state of an economy populated by heterogeneous individuals facing longevity risk and differing in terms of their innate labour productivity. With full information about the health status of individuals, annuity firms can effectively segment the market for private annuities and offer these insurance products at a price that is actuarially fair for all individuals. In this section we study the less pristine—and arguably much more realistic—scenario under which information regarding a person's health is not perfectly observable by insure firms. Indeed, from here on we drop Assumption (A0) and replace it by the following alternative assumptions:

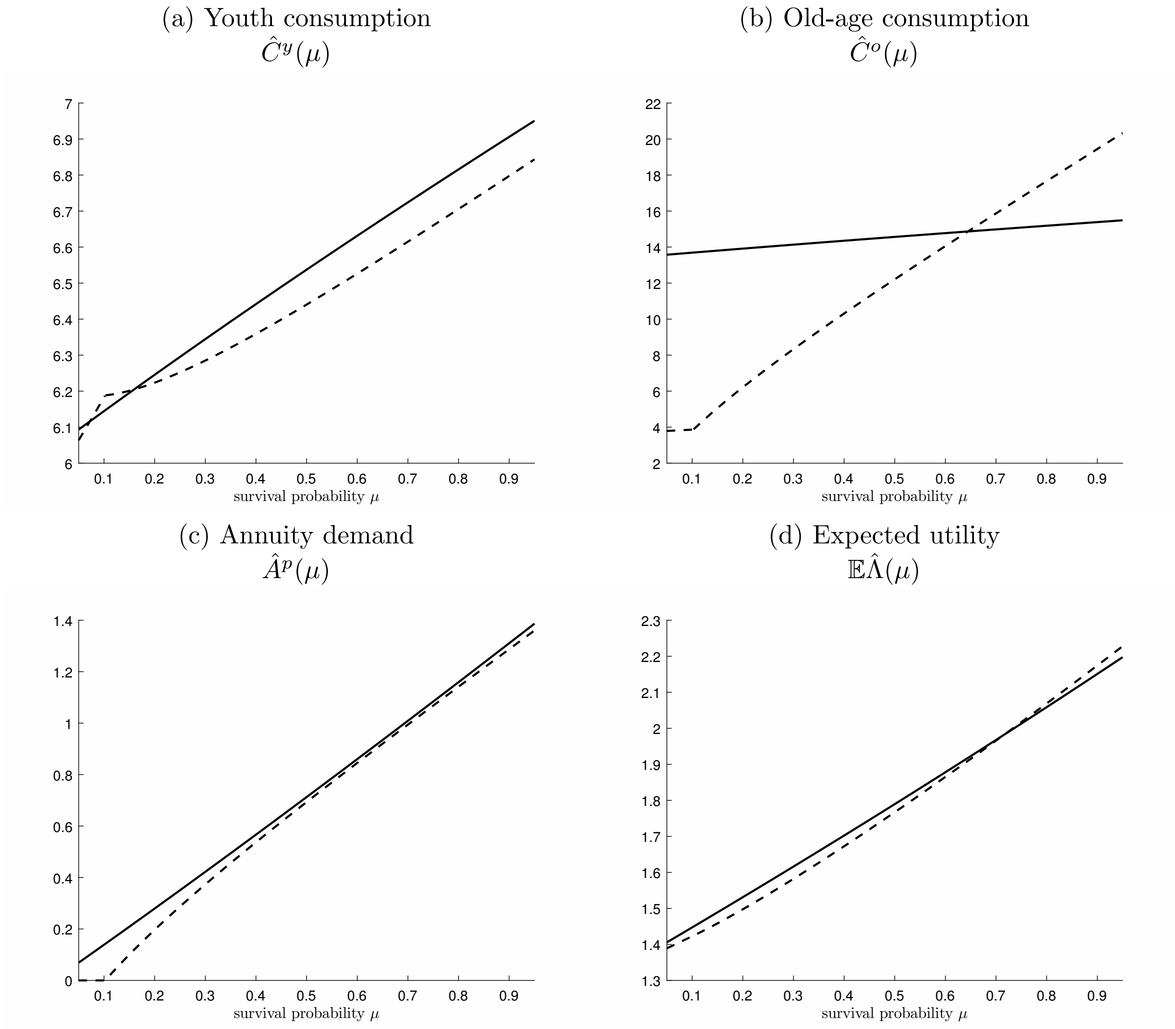
(A3) Health status and productivity are private information of the annuitant. The distribution of health and productivity types in the population, $H(\mu, \eta)$, is common knowledge.

(A4) Annuitants can buy multiple annuities for different amounts and from different annuity firms. Individual annuity firms cannot monitor their clients' wage income or annuity holdings with other firms.

As is explained by Heijdra and Reijnders (2012, pp. 325–6), in this *Asymmetric Information* case (abbreviated as AI) the market for private annuities is characterized by a pooling equilibrium. In this equilibrium there is a single pooled annuity rate, $\bar{r}^p_{t+1}$, which applies to all purchasers of private annuities. Lacking information about an individual's health and productivity, the annuity company cannot obtain full information revelation by setting both price and quantity. As a result, Pauly's (1974) linear pricing concept is the relevant one.[3] A second feature of the pooling equilibrium is that there typically are unhealthy agents who drop out of the annuity market altogether and face binding borrowing constraints. Indeed, since an individual's human wealth is proportional to his/her labour productivity, and individual

[3]See also Abel (1986), Walliser (2000), Palmon and Spivak (2007), and Sheshinski (2008) on linear pricing of annuities.

Figure 3: Steady-state profiles



Legend The solid lines depict the steady-state profiles for the full information (FI) case featuring perfect annuities. The dashed lines visualize the profiles for the asymmetric information (AI) case in which adverse selection results in a single pooling rate of interest on annuities, $\bar{r}^p_{t+1}$. In the AI case agents with poor health face binding borrowing constraints regardless of their productivity in the labour market.

consumption is decreasing in the survival rate, there may exist a cut-off survival probability, μ_t^{bc}, below which individuals would like to go short on annuities. But this is impossible because in doing so they would reveal their poor health status and obtain an offer they cannot possibly accept from annuity firms (more on this below).[4]

The pooled annuity rate, $\bar{r}_{t+1}^p$, is determined as follows. We assume that the cut-off health type is μ_t^{bc} such that consumers with health type $\mu_L \leq \mu < \mu_t^{bc}$ purchase no annuities. Net savers feature a survival probability such that $\mu_t^{bc} \leq \mu \leq \mu_H$ and purchase annuities. The zero-profit condition for the private annuity market is given by:

$$(1+r_{t+1}) \int_{\eta_L}^{\eta_H} \int_{\mu_t^{bc}}^{\mu_H} L_t(\mu,\eta) A_t^p(\mu,\eta) d\mu d\eta = (1+\bar{r}_{t+1}^p) \int_{\eta_L}^{\eta_H} \int_{\mu_t^{bc}}^{\mu_H} \mu L_t(\mu,\eta) A_t^p(\mu,\eta) d\mu d\eta, \quad (31)$$

where $1+r_{t+1}$ is the gross rate of return on physical capital, $1+\bar{r}_{t+1}^p$ is the gross rate of return on private annuities, $L_t(\mu,\eta)$ is the density of type (μ,η) consumers in period t, and $A_t^p(\mu,\eta)$ is the density of private annuities that is purchased by such agents. The gross returns from the annuity savings of all annuitants in period t (left-hand side of (31)) are redistributed to the surviving annuitants in the form of insurance claims in period $t+1$ (right-hand side of (31)). It follows that the pooling rate equals:

$$1+\bar{r}_{t+1}^p = \frac{1+r_{t+1}}{\bar{\mu}_t^p}, \quad (32)$$

where $\bar{\mu}_t^p$ denotes the asset-weighted average survival rate of annuity purchasers:

$$\bar{\mu}_t^p \equiv \int_{\mu_t^{bc}}^{\mu_H} \mu \omega_t(\mu) d\mu, \quad \omega_t(\mu) \equiv \frac{\int_{\eta_L}^{\eta_H} A_t^p(\mu,\eta) h(\mu,\eta) d\eta}{\int_{\eta_L}^{\eta_H} \int_{\mu_t^{bc}}^{\mu_H} A_t^p(\mu,\eta) h(\mu,\eta) d\mu d\eta}. \quad (33)$$

In view of the fact that the asset-weighted survival rate is such that $\mu_t^{bc} < \bar{\mu}_t^p < \mu_H < 1$, it follows from (32) that $\bar{r}_{t+1}^p$ exceeds r_{t+1} so that all net savers will completely annuitize their wealth. Hence, Yaari's (1965) classic result also holds in the pooled annuity market.

The pooling rate (32) is demographically unfair because it is based on the *asset-weighted* survival rate $\bar{\mu}_t^p$ rather than on the *average* survival rate in the population $\bar{\mu}$. The demographically fair pooling rate is given by:

$$1+\bar{r}_{t+1}^{df} = \frac{1+r_{t+1}}{\bar{\mu}}, \quad (34)$$

and, since $\bar{\mu} < \bar{\mu}_t^p$ (see Appendix B), it follows readily from the comparison of (32) and (34)

[4] Villeneuve formulates a partial equilibrium model with heterogeneous survival rates (and identical labour productivity). He argues that only one insurance market can be active at any time, i.e. either the annuity market or the life-insurance market is active but not both. If there is no demand for life insurance in the full information case—as is the case in our model of the closed economy—then adverse selection in the market for private annuities cannot result in the activation of the life insurance market (2003, p. 534).

that $\bar{r}^p_{t+1} < \bar{r}^{df}_{t+1}$. In our numerical exercise we follow Walliser (2000, p. 380) by constructing an adverse selection index AS_t (or 'load factor') which shows by how much the asking price of an annuity insurance company exceeds the demographically fair price:

$$AS_t \equiv \frac{1/(1+\bar{r}^p_{t+1})}{1/(1+\bar{r}^{df}_{t+1})} = \frac{\bar{\mu}^p_t}{\bar{\mu}}. \tag{35}$$

As a result of adverse selection in the private annuity market, AS_t exceeds unity. Furthermore, the larger is AS_t, the more severe is the adverse selection problem.

Under the maintained assumption that $\mu_L < \mu^{bc}_t < \mu_H$, there are two types of agents in the economy. Individuals with a relatively low survival probability ($\mu_L \leq \mu < \mu^{bc}_t$) will face a binding borrowing constraint, whilst healthier individuals ($\mu^{bc}_t \leq \mu \leq \mu_H$) will be net savers. It follows that constrained individuals simply consume their endowment incomes in the two periods:

$$C^y_t(\mu,\eta) = w_t(\eta), \tag{36}$$

$$C^o_{t+1}(\mu,\eta) = \lambda w_{t+1}(\eta). \tag{37}$$

For unconstrained individuals the consolidated budget constraint in a pooled annuity market is given by:

$$C^y_t(\mu,\eta) + \frac{C^o_{t+1}(\mu,\eta)}{1+\bar{r}^p_{t+1}} = w_t(\eta) + \frac{\lambda w_{t+1}(\eta)}{1+\bar{r}^p_{t+1}}, \tag{38}$$

where $\bar{r}^p_{t+1}$ is the pooling rate of interest. Such consumers choose $C^y_t(\mu,\eta)$ and $C^o_{t+1}(\mu,\eta)$ in order to maximize expected lifetime utility (2) subject to the budget constraint (38). The optimal consumption plans and annuity demand are fully characterized by:

$$C^y_t(\mu,\eta) = \Phi\left(\mu, \frac{1+r_{t+1}}{\bar{\mu}^p_t}\right)\left[w_t(\eta) + \frac{\lambda\bar{\mu}^p_t w_{t+1}(\eta)}{1+r_{t+1}}\right], \tag{39}$$

$$\frac{\bar{\mu}^p_t C^o_{t+1}(\mu,\eta)}{1+r_{t+1}} = \left[1-\Phi\left(\mu, \frac{1+r_{t+1}}{\bar{\mu}^p_t}\right)\right]\left[w_t(\eta) + \frac{\lambda\bar{\mu}^p_t w_{t+1}(\eta)}{1+r_{t+1}}\right], \tag{40}$$

$$A^p_t(\mu,\eta) = \left[1-\Phi\left(\mu, \frac{1+r_{t+1}}{\bar{\mu}^p_t}\right)\right] w_t(\eta) - \Phi\left(\mu, \frac{1+r_{t+1}}{\bar{\mu}^p_t}\right)\frac{\lambda\bar{\mu}^p_t w_{t+1}(\eta)}{1+r_{t+1}}, \tag{41}$$

where we have used the expression for the pooled annuity rate as given in (32).

The optimal consumption choices of different types of consumers are illustrated in Figure 4. Just as for the FI case we only illustrate the choices made by the four extreme types, unhealthy and healthy lowest-skilled (μ_L,η_L) and (μ_H,η_L), and unhealthy and healthy highest-skilled (μ_L,η_H) and (μ_H,η_H). In view of (38) the location of an individual's lifetime budget constraint only depends on the person's productivity level, so that LBC(η_L) and LBC(η_H) are parallel. As before the income endowment line is given by IEL, so that the two relevant endowment points are given by, respectively, points E_L and E_H. The consumption Euler equation for unconstrained consumers operating in a pooled annuity market is given by:

$$\frac{U'(C_t^y(\mu,\eta))}{\beta U'(C_{t+1}^o(\mu,\eta))} = \mu\left(1+\bar{r}_{t+1}^p\right) = \frac{\mu}{\bar{\mu}_t^p}(1+r_{t+1}), \tag{42}$$

where we have used (32) to get from the first to the second equality. Using the CRRA felicity function stated in (3), we easily find that the Euler equation is a straight line from the origin with a slope that depends positively on μ. In Figure 4 we have drawn the Euler equations as MRSC(μ_H) and MRSC(μ_L). Since MRSC(μ_H) lies to the left of IEL, points B and D denote the optimal (unconstrained) consumption points for, respectively, the lowest-skilled and highest-skilled consumers. In contrast, since MRSC(μ_L) lies to the right of IEL, points A and C are infeasible as they would involve going short on annuities. It follows that all lowest-health individuals face borrowing constraints. Furthermore, the Euler equation (42) that coincides with the IEL, MRSC(μ_t^{bc}), determines the cut-off health type μ_t^{bc}:

$$\mu_t^{bc} = \frac{\bar{\mu}_t^p U'(w_t(\eta))}{(1+r_{t+1})\beta U'(\lambda w_{t+1}(\eta))}. \tag{43}$$

Unconstrained consumers are located in the area E_LBDE$_H$ whilst constrained individuals are bunched on the line segment $\mathrm{E}_L\mathrm{E}_H$. It is worth noting that μ_t^{bc} depends on the current and future capital intensity in the economy via factor prices. Given the specification of preferences and technology, however, μ_t^{bc} does not depend on η itself.

In the presence of binding borrowing constraints, the capital accumulation identity (23) is augmented to:

$$K_{t+1} = L_t \int_{\eta_L}^{\eta_H}\int_{\mu_t^{bc}}^{\mu_H} A_t^p(\mu,\eta)h(\mu,\eta)d\eta d\mu. \tag{44}$$

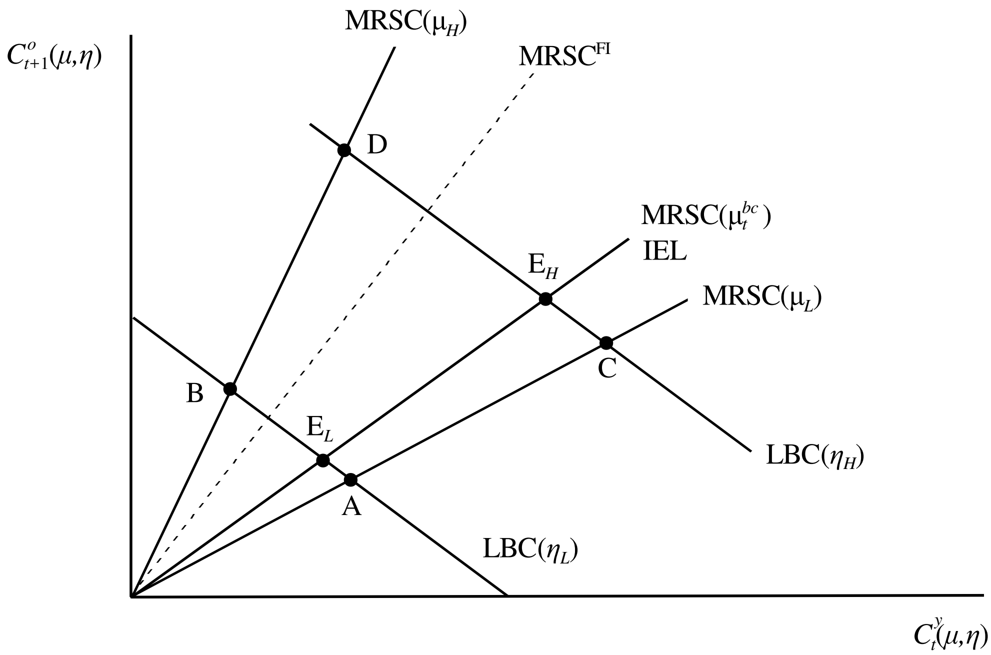
By substituting the demand for annuities (41) into (44) we obtain the fundamental difference equation for the capital intensity:

$$k_{t+1} = \frac{1}{1+n}\frac{L_t}{N_t}\left[\int_{\mu_t^{bc}}^{\mu_H}\left[w_t - \Phi\left(\mu, \frac{1+r_{t+1}}{\bar{\mu}_t^p}\right)\left[w_t + \frac{\lambda\bar{\mu}_t^p w_{t+1}}{1+r_{t+1}}\right]\right] h_\mu(\mu)\Gamma_1(\mu)d\mu\right], \tag{45}$$

where $\Gamma_1(\mu)$ is the conditional mean of η (defined in Lemma 1(vi) above), and the factor prices follow from (20)–(21).

The main features of the steady-state AI equilibrium are reported in column (b) of Table 2. As a result of asymmetric information in the annuity market, output per efficiency unit of labour drops by 1.60% ($\hat{y} = 9.840$) whilst the steady-state capital intensity falls by 5.71% ($\hat{k} = 0.373$). The decrease in the capital intensity causes the annual interest rate to rise by

Figure 4: Consumption-saving choices under asymmetric information



Legend LBC(η_j) is the lifetime budget constraint for an individual with productivity η_j. IEL is the income endowment line and agents are located on the line segment $\mathrm{E}_L\mathrm{E}_H$. MRSC(μ_i) is the consumption Euler equation for an individual with survival rate μ_i facing a pooled annuity rate of interest $\bar{r}^p_{t+1}$. For individuals with $\mu^{bc}_t \leq \mu \leq \mu_H$ optimal consumption is located at the intersection of MRSC(μ_i) and LBC(η_j). All other individuals face borrowing constraints and consume along $\mathrm{E}_L\mathrm{E}_H$.

12 basis points ($\hat{r}^a = 5.11\%$) and the wage rate to fall by 1.60%. So despite the fact that only 5.83% of young individuals face binding borrowing constraints (see $\widehat{BC}$), the macroeconomic effects of information asymmetry are far from trivial in size. The adverse selection index, as defined in (35) above, equals $\widehat{AS} = 1.31$ and the asset-weighted average survival rate of annuitants equals $\hat{\bar{\mu}}^p = 0.66$. Finally, as the welfare indicators at the bottom of Table 2 reveal, under asymmetric information unhealthy individuals are worse off while their healthy cohort members are better off than under the FI case. The information asymmetry redistributes resources from unhealthy to healthy agents.

In Figure 3 we depict with dashed lines the steady-state profiles for youth consumption, old-age consumption, annuity demand, and expected utility. Just as for the FI case these profiles have been averaged over η:

$$\hat{C}^y(\mu) = \left[1 - \mathbb{I}_{AI}(\mu) + \mathbb{I}_{AI}(\mu)\Phi\left(\mu, \frac{1+\hat{r}}{\hat{\bar{\mu}}^p}\right)\left[1 + \frac{\lambda\hat{\bar{\mu}}^p}{1+\hat{r}}\right]\right]\hat{w}\Gamma_1(\mu), \tag{46}$$

$$\hat{C}^o(\mu) = \left[[1 - \mathbb{I}_{AI}(\mu)]\lambda + \mathbb{I}_{AI}(\mu)\left[1 - \Phi\left(\mu, \frac{1+\hat{r}}{\hat{\bar{\mu}}^p}\right)\right]\left[\frac{1+\hat{r}}{\hat{\bar{\mu}}^p} + \lambda\right]\right]\hat{w}\Gamma_1(\mu), \tag{47}$$

$$\hat{A}^p(\mu) = \mathbb{I}_{AI}(\mu)\left[1 - \Phi\left(\mu, \frac{1+\hat{r}}{\hat{\bar{\mu}}^p}\right)\left[1 + \frac{\lambda\hat{\bar{\mu}}^p}{1+\hat{r}}\right]\right]\hat{w}\Gamma_1(\mu), \tag{48}$$

where $\mathbb{I}_{AI}(\mu) = 0$ for $\mu_L \leq \mu < \hat{\mu}^{bc}$ and $\mathbb{I}_{AI}(\mu) = 1$ for $\hat{\mu}^{bc} \leq \mu \leq \mu_H$. In panel (a) we find that youth consumption $\hat{C}^y(\mu)$ is increasing in μ. Interestingly, for μ close to $\hat{\mu}^{bc}$ youth consumption is higher under AI than for the FI case. Young individuals facing borrowing constraints are unable to smooth consumption in the AI case and just consume their endowment income. Net savers featuring a survival probability close to $\hat{\mu}^{bc}$ purchase virtually no annuities at all as the pooling rate is unattractive to them—see panel (c). For higher levels of μ annuity demands are higher and saving for old-age increases. In panel (b) we show that the healthiest agents consume more during old-age under AI compared to FI. In panel (d) we find that the healthiest individuals are actually better off under AI than under FI. The information asymmetry benefits such individuals.

4 Public annuities to the rescue?

In the adverse selection economy studied in the previous section relatively unhealthy annuitants face a disadvantageous pooling rate of interest on their annuities. In essence such individuals are subsiding their healthy cohort members through the annuity market. Following Abel (1987) we now extend our model by introducing a fully-funded mandatory social security system that is run by the government.[5] Such a system is immune to adverse selection

[5]There is one important difference in that Abel (1987) restricts attention to the full information (FI) case in which perfect private annuities are available. In order not to unduly interrupt the flow of the paper, we present the FI results for our model in Appendix C.

because all individuals are forced to participate in it—the government possesses the power to tax. In particular, every individual pays a social security tax θ (such that $0 < \theta < 1$) and receives a retirement pension upon surviving into old-age. We assume that the pension contribution is proportional to wage income. Like the private sector, the government cannot observe an individual's health status though it can measure a person's income. It follows that the pension contribution can be written as $A_t^s(\eta) = \theta w_t(\eta)$. Total pension contributions amount to $A_t^s = \theta\bar{\eta}w_t L_t$ and are invested in the capital market earning a gross rate of return equal to $1+r_{t+1}$. In the next period the returns $R_{t+1} = (1+r_{t+1})A_t^s$ are paid out to surviving agents. Under this funded pension system redistribution takes place between agents of the same birth cohort (from those who die to survivors). Hence, social security plays the role of public annuities. In this section we consider three prototypical types of pension systems. The difference lies in the method in which the returns are distributed to surviving individuals.

- Pension system A: pension receipts during old-age are proportional to contributions made during youth.
- Pension system B: pension contributions of η types are distributed during old-age to surviving η types.
- Pension system C: pension receipts are the same in absolute value for all surviving agents.

4.1 Pension system A

Under system A pension receipts are given by:

$$R_{t+1}^s(\eta) = \zeta\theta w_t(\eta), \tag{49}$$

where ζ is a parameter to be determined below. The clearing condition for the public annuity system is given in this case by:

$$(1+r_{t+1})A_t^s = \int_{\eta_L}^{\eta_H}\int_{\mu_L}^{\mu_H} \mu R_{t+1}^s(\eta)L_t(\mu,\eta)d\mu d\eta. \tag{50}$$

The left-hand side of this expression is the total amount to be distributed to survivors and the right-hand side represents total pension payments. By substituting (49) into (50) and noting that $w_t(\eta) = \eta w_t$ and $L_t(\mu,\eta) = L_t h(\mu,\eta)$ we find the balanced-budget solution for ζ:

$$\zeta = \zeta_A \frac{1+r_{t+1}}{\bar{\mu}}, \qquad \zeta_A \equiv \frac{\bar{\eta}\bar{\mu}}{\text{cov}(\eta,\mu)+\bar{\eta}\bar{\mu}}, \tag{51}$$

where $\bar{\mu}$ is the average survival rate of the population and ζ_A is a constant (featuring $0 < \zeta_A < 1$ because $\text{cov}(\eta,\mu)$ is positive). It follows from (51) that under pension system A the rate of return on social annuities falls short of the actuarially fair social annuity yield, $(1+r_{t+1})/\bar{\mu}$, because health and productivity are positively correlated. Intuitively, the high contributors (featuring a high η) tend to live longer than average.

Just as in the adverse selection economy studied in the previous section individuals can buy private annuities in the pooled annuity market but some agents will face borrowing constraint. Constrained individuals simply consume their endowment incomes in the two periods:

$$C_t^y(\mu,\eta) = (1-\theta)w_t(\eta), \tag{52}$$

$$C_{t+1}^o(\mu,\eta) = \lambda w_{t+1}(\eta) + R_{t+1}^s(\eta). \tag{53}$$

For unconstrained individuals the consolidated budget constraint in the presence of a pooled annuity market is given by:

$$C_t^y(\mu,\eta) + \frac{C_{t+1}^o(\mu,\eta)}{1+\bar{r}_{t+1}^p} = (1-\theta)w_t(\eta) + \frac{\lambda w_{t+1}(\eta) + R_{t+1}^s(\eta)}{1+\bar{r}_{t+1}^p}, \tag{54}$$

where $\bar{r}_{t+1}^p$ is the pooling rate of interest. The pension system reduces current wage income but increases future income. Consumers choose $C_t^y(\mu,\eta)$ and $C_{t+1}^o(\mu,\eta)$ in order to maximize expected lifetime utility (2) subject to the budget constraint (54). The optimal consumption plans and annuity demands are fully characterized by:

$$C_t^y(\mu,\eta) = \Phi\left(\mu, \frac{1+r_{t+1}}{\bar{\mu}_t^p}\right)\left[(1-\theta)w_t(\eta) + \theta\zeta_A w_t(\eta)\frac{\bar{\mu}_t^p}{\bar{\mu}} + \frac{\lambda\bar{\mu}_t^p w_{t+1}(\eta)}{1+r_{t+1}}\right], \tag{55}$$

$$\begin{aligned}\frac{\bar{\mu}_t^p C_{t+1}^o(\mu,\eta)}{1+r_{t+1}} &= \left[1-\Phi\left(\mu, \frac{1+r_{t+1}}{\bar{\mu}_t^p}\right)\right]\left[(1-\theta)w_t(\eta) + \theta\zeta_A w_t(\eta)\frac{\bar{\mu}_t^p}{\bar{\mu}}\right.\\ &\quad \left. + \frac{\lambda\bar{\mu}_t^p w_{t+1}(\eta)}{1+r_{t+1}}\right],\end{aligned} \tag{56}$$

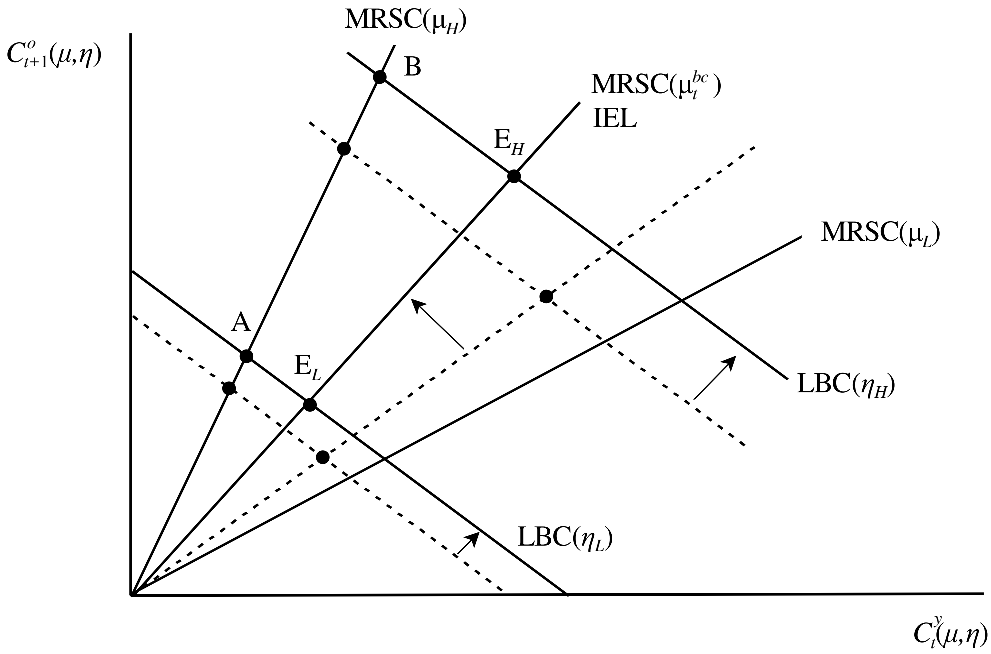
$$\begin{aligned}A_t^p(\mu,\eta) &= \left[1-\Phi\left(\mu, \frac{1+r_{t+1}}{\bar{\mu}_t^p}\right)\right](1-\theta)w_t(\eta)\\ &\quad - \Phi\left(\mu, \frac{1+r_{t+1}}{\bar{\mu}_t^p}\right)\left[\theta\zeta_A w_t(\eta)\frac{\bar{\mu}_t^p}{\bar{\mu}} + \frac{\lambda\bar{\mu}_t^p w_{t+1}(\eta)}{1+r_{t+1}}\right],\end{aligned} \tag{57}$$

where we have used the expression for the pooled annuity rate as given in (32). The social annuity system affects an individual's human wealth at birth (the term in square brackets on the right-hand side of (55)) but it is not a priori clear in which direction. Indeed, the *effective* pension contribution rate is:

$$\theta_t^n \equiv \theta\left(1-\zeta_A\frac{\bar{\mu}_t^p}{\bar{\mu}}\right). \tag{58}$$

On the one hand, with a positive correlation between health and ability ζ_A is such that

Figure 5: Consumption-saving choices under pension system A



Legend LBC(η_j) is the lifetime budget constraint for an individual with productivity η_j. IEL is the income endowment line and agents are located on the line segment E_LE_H. MRSC(μ_i) is the consumption Euler equation for an individual with survival rate μ_i facing a pooled annuity rate of interest $\bar{r}^p_{t+1}$. For individuals with $\mu^{bc}_t \leq \mu \leq \mu_H$ optimal consumption is located at the intersection of MRSC(μ_i) and LBC(η_j). All other individuals face borrowing constraints and consume along E_LE_H. The dashed lines visualize the corresponding schedules for the AI case. Factor prices are held the same for SA and AI to facilitate the comparison.

$0 < \zeta_A < 1$. On the other hand, the survival rate of private annuitants exceeds the population-wide average survival rate, i.e. $\bar{\mu}^p_t/\bar{\mu} > 1$. It thus follows that θ^n_t is ambiguous in sign. In this paper we focus on the case for which θ^n_t is negative so that, ceteris paribus factor prices and the pooled survival rate, human wealth is increased as a result of the public pension system.[6]

The optimal consumption choices can be explained with the aid of Figure 5. To facilitate the comparison with the AI case we keep factor prices and the pooled survival rate at the levels for that case. Hence the diagram shows the partial equilibrium effects on individual choices of the introduction of a pension system. The dashed lines correspond to the AI case. As a result of the public pension system the lifetime budget constraints shift outward (because $\theta^n_t < 0$), more so the higher is η. The income endowment line rotates in a counter-clockwise fashion. Unconstrained individuals increase consumption during youth and old-age.

[6]In the numerical simulations $\zeta_A = 0.9569$ and $\bar{\mu} = 0.5$. Hence the effective pension contribution is negative for any $\bar{\mu}^p_t$ exceeding $\bar{\mu}/\zeta_A = 0.5225$. This condition is easily satisfied. See also Figure 9(c) for an illustration of effective contribution rates under the different pension systems.

In contrast, constrained individuals are forced to consume less during youth. Such agents are bunched along the line segment $E_L E_H$. Just as for the AI case there is a single cut-off value for the survival probability below which agents are facing borrowing constraints:

$$\mu_t^{bc} = \frac{\bar{\mu}_t^p U'((1-\theta)w_t(\eta))}{(1+r_{t+1})\beta U'\left(\lambda w_{t+1}(\eta) + \theta\zeta_A \frac{1+r_{t+1}}{\bar{\mu}} w_t(\eta)\right)}. \tag{59}$$

Because wages and pension receipts are proportional to η and the felicity function is homothetic, it follows from (59) that μ_t^{bc} does not depend on η. As is clear from the diagram, the introduction of public pensions will increase the population fraction of people facing borrowing constraints.

In order to glean the general equilibrium effects of introducing a public pension system we must formulate the capital accumulation identity. Since public and private annuities are invested in the capital markets, the accumulation equation takes the following format:

$$K_{t+1} = L_t \left[A_t^s + \int_{\eta_L}^{\eta_H} \int_{\mu_t^{bc}}^{\mu_H} A_t^p(\mu,\eta) h(\mu,\eta) d\eta d\mu \right]. \tag{60}$$

By substituting the demand for annuities (57) into (60) we obtain the fundamental difference equation for the capital intensity:

$$k_{t+1} = \frac{1}{1+n}\frac{L_t}{N_t}\left[\theta\bar{\eta}w_t + \int_{\mu_t^{bc}}^{\mu_H} \left((1-\theta)w_t - \Phi\left(\mu, \frac{1+r_{t+1}}{\bar{\mu}_t^p}\right) \right.\right.$$
$$\left.\left. \cdot \left[(1-\theta)w_t + \theta\zeta_A w_t \frac{\bar{\mu}_t^p}{\bar{\mu}} + \frac{\lambda\bar{\mu}_t^p w_{t+1}}{1+r_{t+1}} \right] \right) h_\mu(\mu)\Gamma_1(\mu) d\mu \right], \tag{61}$$

where $\Gamma_1(\mu)$ is the conditional mean of η (defined in Lemma 1(vi) above), and the factor prices follow from (20)–(21).

The main features of the steady-state equilibrium with pension system A (labeled SA) are reported in columns (c)-(d) of Table 2. In column (c) the contribution rate equals $\theta = 0.010$ which means that the system is relatively small as the income replacement rate during retirement, $\xi_{SA} \equiv \theta\zeta_A(1+\hat{r})/[(1-\lambda)\bar{\mu}]$, is only about 0.3812. In column (d) the contribution rate equals $\theta = 0.025$ which results in a large pension system, i.e. $\xi_{SA} = 0.9776$. Comparing columns (b) and (d) we find that output per efficiency unit of labour drops by 1.62% ($\hat{y} = 9.680$) whilst the steady-state capital intensity falls by 5.76% ($\hat{k} = 0.351$). As a result of the decrease in the capital intensity, the annual interest rate rises by 11 basis points ($\hat{r}^a = 5.22\%$) whilst the wage rate falls by 1.6%. The proportion of constrained individual rises from 5.83% to 17.66%. The adverse selection index, as defined in (35) above, increases to $\widehat{AS} = 1.39$ and the asset-weighted average survival rate of annuitants rises to $\hat{\bar{\mu}}^p = 0.70$. Despite the fact that the rate of return on capital increases, the return on private annuities decreases slightly because the pooled survival rate $\hat{\bar{\mu}}^p$ increases by more. Finally, as the welfare indicators at the bottom of Table 2 reveal, under pension system A all individuals are worse off compared to the AI case. The pension system crowds out capital and exacerbates the adverse selection problem in the market for private annuities.

In Figure 6 we use solid lines to depict the profiles for youth and old-age consumption, annuity demand, and utility (averaged over η) for the SA case. These are given by:

$$\frac{\hat{C}^y(\mu)}{\hat{w}} = \left[1 - \mathbb{I}_{SA}(\mu) + \mathbb{I}_{SA}(\mu)\Phi\left(\mu, \frac{1+\hat{r}}{\hat{\bar{\mu}}^p}\right)\left[1 - \theta + \theta\zeta_A \frac{\hat{\bar{\mu}}^p}{\bar{\mu}} + \frac{\lambda\hat{\bar{\mu}}^p}{1+\hat{r}}\right]\right]\Gamma_1(\mu), \tag{62}$$

$$\begin{aligned}\frac{\hat{C}^o(\mu)}{\hat{w}} &= [1 - \mathbb{I}_{SA}(\mu)]\left[\lambda + \theta\zeta_A \frac{1+\hat{r}}{\bar{\mu}}\right]\Gamma_1(\mu) + \mathbb{I}_{SA}(\mu)\left[1 - \Phi\left(\mu, \frac{1+\hat{r}}{\hat{\bar{\mu}}^p}\right)\right] \\ &\quad \cdot \left[\left(1 - \theta + \theta\zeta_A \frac{\hat{\bar{\mu}}^p}{\bar{\mu}}\right)\frac{1+\hat{r}}{\hat{\bar{\mu}}^p} + \lambda\right]\Gamma_1(\mu),\end{aligned} \tag{63}$$

$$\frac{\hat{A}^p(\mu)}{\hat{w}} = \mathbb{I}_{SA}(\mu)\left[1 - \theta - \Phi\left(\mu, \frac{1+\hat{r}}{\hat{\bar{\mu}}^p}\right)\left[1 - \theta + \theta\zeta_A \frac{\hat{\bar{\mu}}^p}{\bar{\mu}} + \frac{\lambda\hat{\bar{\mu}}^p}{1+\hat{r}}\right]\right]\Gamma_1(\mu), \tag{64}$$

where $\mathbb{I}_{SA}(\mu) = 0$ for $\mu_L \leq \mu < \hat{\mu}^{bc}$ and $\mathbb{I}_{AI}(\mu) = 1$ for $\hat{\mu}^{bc} \leq \mu \leq \mu_H$. The dashed lines in Figure 6 correspond to the profiles for the AI case. Youth consumption, annuity demand, and lifetime utility are all lower under SA than under AI. Old-age consumption is higher under AI for most borrowing constrained individuals.

4.2 Pension system B

Under pension system B the government uses information on a person's wage income to deduce that individual's innate ability. It uses its knowledge of η by setting pension receipts according to the following rule:

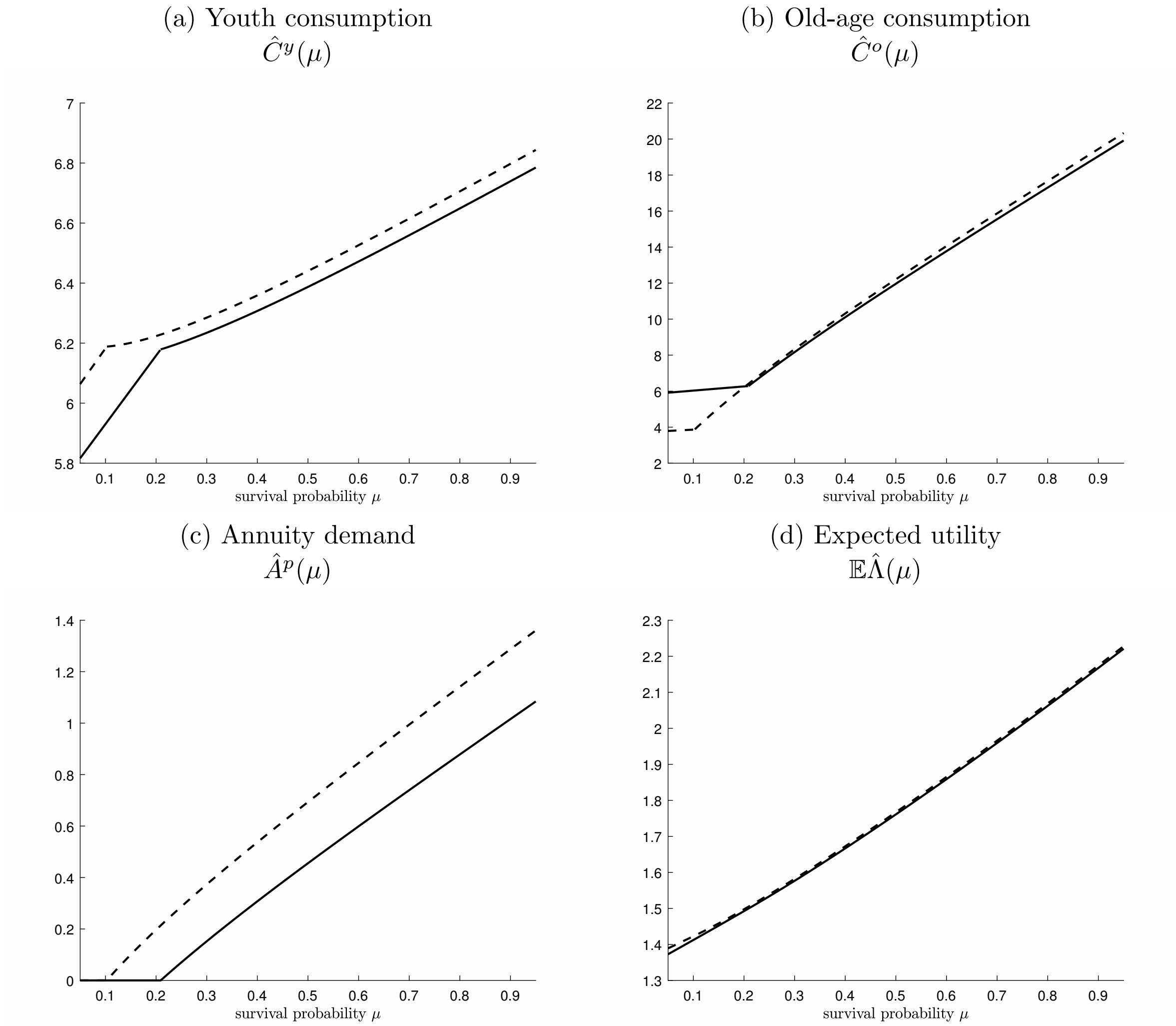
$$R^s_{t+1}(\eta) = \zeta(\eta)\theta w_t(\eta), \tag{65}$$

where $\zeta(\eta)$ is a function to be determined below. For each ability level η, the budget constraint for the public pension system is given by:

$$(1 + r_{t+1})\theta w_t(\eta)\int_{\mu_L}^{\mu_H} L_t(\mu, \eta)d\mu = \int_{\mu_L}^{\mu_H} \mu R^s_{t+1}(\eta)L_t(\mu, \eta)d\mu. \tag{66}$$

The left-hand side of this expression is the total amount to be distributed to type η survivors whilst the right-hand side represents total pension payments to such individuals. Under this system public annuities are such that longevity risk is shared among individuals of the same productivity type. By substituting (65) into (66) and noting that $w_t(\eta) = \eta w_t$ and

Figure 6: Steady-state profiles under pension system A



Legend The solid lines depict the steady-state profiles under pension system A (SA), and the dashed lines visualize the profiles for the asymmetric information (AI) case without pensions. In both cases adverse selection results in a single pooling rate of interest on annuities, $\bar{r}^p_{t+1}$, and agents with poor health face binding borrowing constraints. The SA case has been drawn for a large system featuring $\theta = 0.025$.

$L_t(\mu,\eta) = L_t h(\mu,\eta)$ we find the balanced-budget solution for $\zeta(\eta)$:

$$\zeta(\eta) = \zeta_B(\eta)\,\frac{1+r_{t+1}}{\bar{\mu}}, \qquad \zeta_B(\eta) \equiv \frac{\bar{\mu}}{\bar{\mu}+\xi\sigma_\mu^2(\eta-\bar{\eta})}. \tag{67}$$

For relatively productive individuals (featuring $\eta > \bar{\eta}$) the rate of return on social annuities falls short of the actuarially fair social annuity yield, $(1 + r_{t+1})/\bar{\mu}$, because such people tend to have a relatively high survival rate. In contrast, for relatively unproductive individuals (with $\eta < \bar{\eta}$) the rate of return on social annuities is better than the actuarially fair social annuity yield because such people tend to have a relatively low survival rate.

Individuals facing a binding borrowing constraint consume according to (52)–(53) with $R^s_{t+1}(\eta)$ as stated in (65) and (67). For unconstrained individuals the optimal consumption plans and annuity demands are fully characterized by:

$$C^y_t(\mu,\eta) = \Phi\left(\mu, \frac{1+r_{t+1}}{\bar{\mu}^p_t}\right)\left[(1-\theta)w_t + \theta\zeta_B(\eta)w_t\frac{\bar{\mu}^p_t}{\bar{\mu}} + \frac{\lambda\bar{\mu}^p_t w_{t+1}}{1+r_{t+1}}\right]\eta, \tag{68}$$

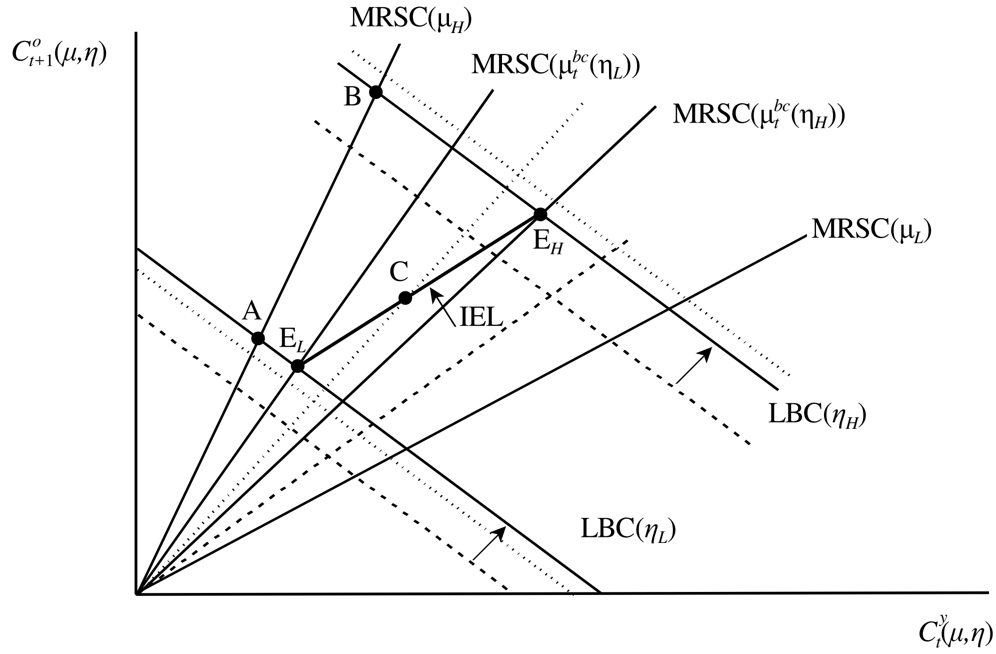
$$\frac{\bar{\mu}^p_t C^o_{t+1}(\mu,\eta)}{1+r_{t+1}} = \left[1-\Phi\left(\mu, \frac{1+r_{t+1}}{\bar{\mu}^p_t}\right)\right]\left[(1-\theta)w_t + \theta\zeta_B(\eta)w_t\frac{\bar{\mu}^p_t}{\bar{\mu}} + \frac{\lambda\bar{\mu}^p_t w_{t+1}}{1+r_{t+1}}\right]\eta, \tag{69}$$

$$A^p_t(\mu,\eta) = \left[1-\Phi\left(\mu, \frac{1+r_{t+1}}{\bar{\mu}^p_t}\right)\right](1-\theta)\eta w_t - \Phi\left(\mu, \frac{1+r_{t+1}}{\bar{\mu}^p_t}\right)\left[\theta\zeta_B(\eta)w_t\frac{\bar{\mu}^p_t}{\bar{\mu}} + \frac{\lambda\bar{\mu}^p_t w_{t+1}}{1+r_{t+1}}\right]\eta, \tag{70}$$

where we have substituted $w_t(\eta) = \eta w_t$ and used the expression for the pooled annuity rate as given in (32).

The optimal consumption choices can be explained with the aid of Figure 7. Just as before we focus on the four extreme types. For purposes of reference the dashed lines in the diagram represent the AI case (without pensions) whilst the thin dotted lines represent the SA case. We keep factor prices constant at their AI levels. Under pension system B the IEL pivots around some point C on the old IEL line for the SA case. Intuitively this is because system B incorporates *explicit* redistribution from high-ability to low-ability individuals and, as a result of the positive correlation between ability and health, *implicit* redistribution from healthy to unhealthy individuals. With asymmetric information in the private annuity market the pooling equilibrium causes a redistribution of resources from unhealthy to healthy individuals, i.e. from people who tend to be poor to individuals who tend to be rich. Pension system A does nothing to redress this phenomenon. In contrast, under system B the high-skilled get a lower return on social annuities than the low-skilled do, so there is some redistribution from healthy to unhealthy individuals via that channel.

Figure 7: Consumption-saving choices under pension system B



Legend LBC(η_j) is the lifetime budget constraint for an individual with productivity η_j. IEL is the income endowment line and agents are located on the line segment $\mathrm{E}_L\mathrm{E}_H$. MRSC(μ_i) is the consumption Euler equation for an individual with survival rate μ_i facing a pooled annuity rate of interest $\bar{r}^{p}_{t+1}$. The dashed and dotted lines visualize the corresponding schedules for the AI and SA cases respectively. Factor prices are held the same for SB and AI to facilitate the comparison. An individual with productivity η_j faces borrowing constraints if $\mu < \mu^{bc}_t(\eta_j)$ and is unconstrained otherwise.

As is marked in the diagram, lowest-ability types experience borrowing constraint for $\mu < \mu_t^{bc}(\eta_L)$ whilst highest-ability individuals experience such constraints for $\mu < \mu_t^{bc}(\eta_H)$, where $\mu_t^{bc}(\eta_H) < \mu_t^{bc}(\eta_L)$. Mathematically, an individual with productivity η experiences a binding borrowing constraint if his/her survival probability falls short of $\mu_t^{bc}(\eta)$:

$$\mu_t^{bc}(\eta) = \frac{\bar{\mu}_t^p U'((1-\theta)\eta w_t)}{(1+r_{t+1})\beta U'\left(\lambda\eta w_{t+1} + \theta\eta\zeta_B(\eta)\ \frac{1+r_{t+1}}{\bar{\mu}} w_t\right)} \tag{71}$$

Despite the fact that the felicity function is homothetic and wages are proportional to η, μ_t^{bc} depends on η because productivity features nonlinearly in $\zeta_B(\eta)$.

In Figure 8 we illustrate the relationship between ability η and the critical survival rate $\mu_t^{bc}(\eta)$. The thin solid line represents the AI case for which $\hat{\mu}^{bc} = 0.1028$ and 5.83% of agents are constrained. The dashed line depicts the situation for the SA case (with $\theta = 0.025$) for which $\hat{\mu}^{bc} = 0.2090$ and 17.66% of agents are constrained. Finally, the thick solid line in Figure 8 illustrates the SB case. As is predicted by the theory there is a downward sloping relationship between η and $\hat{\mu}^{bc}$. For the lowest-ability types the cut-off value equals 0.2590 whereas it is equal to 0.1902 for the highest-ability individuals. So by engaging in redistribution from high-ability to low-ability individuals the policy maker worsens the incidence of borrowing constraints to the latter types.

In Figure 9 we compare some features of pension systems A and B. Panel (a) depicts the fair-rates shares ζ_A (a constant) and $\zeta_B(\eta)$ (downward sloping because of redistribution). Panel (b) shows that pensions receipts are increasing in ability for both systems. In panel (c) we depict the effective pension contribution rate $\theta_t^n(\eta)$. Under system A this is a negative constant, but under system B the effective rate is increasing in ability:

$$\theta_t^n(\eta) \equiv \theta\left(1 - \zeta_B(\eta)\frac{\bar{\mu}_t^p}{\bar{\mu}}\right). \tag{72}$$

For our parameterization $\theta_t^n(\eta)$ remains negative for all ability levels, although barely so for the highest-ability types.

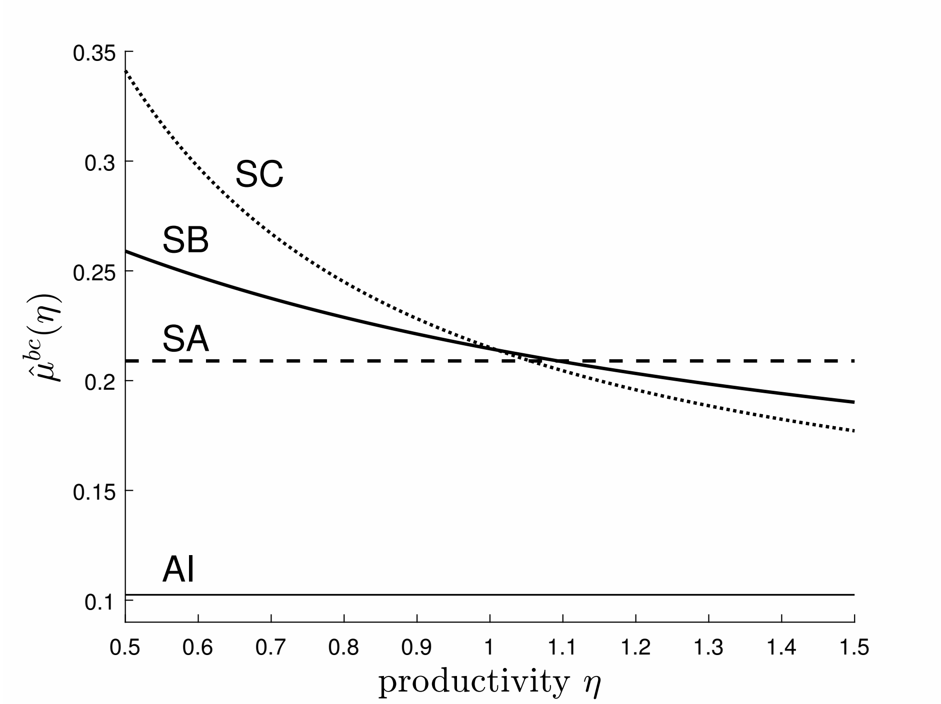
Under pension system B the capital accumulation identity is given by:

$$K_{t+1} = L_t\left[A_t^s + \int_{\eta_L}^{\eta_H}\int_{\mu_t^{bc}(\eta)}^{\mu_H} A_t^p(\mu,\eta)h(\mu,\eta)d\eta d\mu\right], \tag{73}$$

where $\mu_t^{bc}(\eta)$ is determined in (71) and is illustrated in Figure 8. By substituting the demand for annuities (70) into (73) we obtain the fundamental difference equation for the capital intensity:

$$k_{t+1} = \frac{1}{1+n}\frac{L_t}{N_t}\left[\theta\bar{\eta}w_t + \int_{\eta_L}^{\eta_H}\int_{\mu_t^{bc}(\eta)}^{\mu_H}\left((1-\theta)w_t - \Phi\left(\mu, \frac{1+r_{t+1}}{\bar{\mu}_t^p}\right)\right.\right.$$

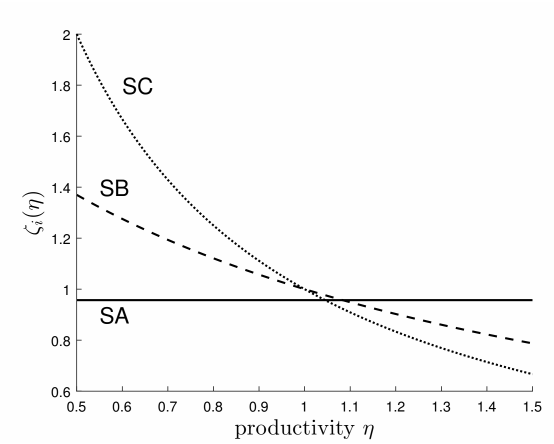
Figure 8: Ability and borrowing constraints



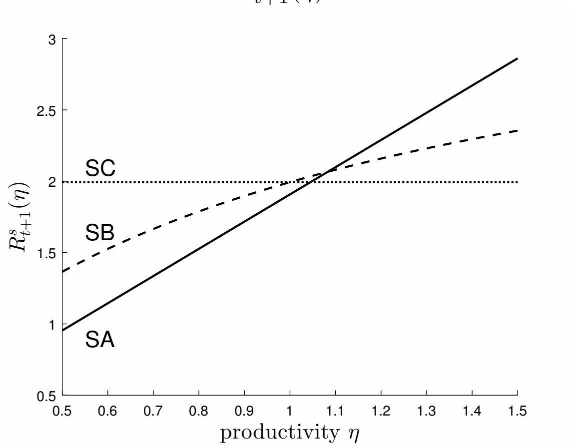
Legend Under pension systems B and C the critical level of the survival rate below which borrowing constraints become active, $\mu_t^{bc}(\eta)$, depends negatively on the individual's productivity η. In Figure 7 the income endowment points no longer lie along a ray from the origin.

Figure 9: Comparing pension systems

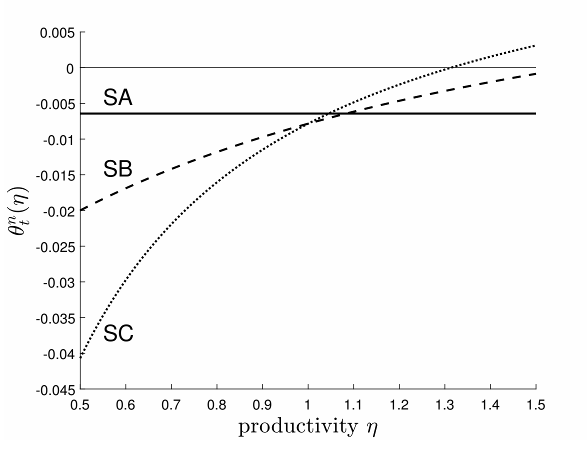
(a) Fair-rate share
$\zeta_i(\eta)$



(b) Pension receipt
$R^s_{t+1}(\eta)$



(c) Effective pension contribution rate
$\theta^n_t(\eta)$



Legend The fair-rate share $\zeta_i(\eta)$ measures the individual's gross yield on social annuities under pension system i expressed as a share of the actuarially fair yield, $(1 + \bar{r}^p_{t+1})/\bar{\mu}$. A negative value for the effective pension contribution rate $\theta^n_t(\eta)$ implies that the pension system makes individuals wealthier in a partial equilibrium sense.

$$\cdot \left[(1-\theta)w_t + \theta\zeta_B(\eta)w_t\frac{\bar{\mu}_t^p}{\bar{\mu}} + \frac{\lambda\bar{\mu}_t^p w_{t+1}}{1+r_{t+1}}\right]\Bigg)\eta h(\mu,\eta)d\mu d\eta\Bigg]. \tag{74}$$

The main features of the steady-state equilibrium with a small and large pension system B (labeled SB) are reported in, respectively, columns (e) and (f) of Table 2. We focus attention at the large pension system featuring $\theta = 0.025$. Comparing columns (b) and (f) we find that output per efficiency unit of labour drops by 1.74% ($\hat{y} = 9.668$) whilst the steady-state capital intensity falls by 6.19% ($\hat{k} = 0.350$). As a result of the decrease in the capital intensity, the annual interest rate rises by 12 basis points ($\hat{r}^a = 5.23\%$) whilst the wage rate falls by 1.74%. The proportion of constrained individual rises from 5.83% to 19.33%. The adverse selection index, as defined in (35) above, increases to $\widehat{AS} = 1.40$, the asset-weighted average survival rate of annuitants rises to $\hat{\bar{\mu}}^p = 0.70$, and the return on private annuities decreases slightly to $\hat{\bar{r}}^p = 9.98$. Finally, as the welfare indicators at the bottom of Table 2 reveal, under pension system B poor-health individuals are better off compared to the AI case as a result of the redistributionary feature of system B. The opposite holds for the healthy agents. Even though the policy maker cannot observe an individual's health status, by including a redistributionary component in the public pension system, the unhealthiest in society are aided somewhat.

In Figure 10 we present the η-averaged profiles for consumption during youth and old-age, annuity demand, and lifetime utility. These profiles are defined as:

$$\begin{aligned}
\frac{\hat{C}^y(\mu)}{\hat{w}} &= (1-\theta)\int_{\eta_L}^{\eta_H}[1-\mathbb{I}_{SB}(\mu,\eta)]\frac{\eta h(\mu,\eta)}{h_\mu(\mu)}d\eta \\
&\quad + \left(1-\theta+\frac{\lambda\hat{\bar{\mu}}^p}{1+\hat{r}}\right)\int_{\eta_L}^{\eta_H}\Phi\left(\mu,\frac{1+\hat{r}}{\hat{\bar{\mu}}^p}\right)\mathbb{I}_{SB}(\mu,\eta)\frac{\eta h(\mu,\eta)}{h_\mu(\mu)}d\eta \\
&\quad + \theta\frac{\hat{\bar{\mu}}^p}{\bar{\mu}}\int_{\eta_L}^{\eta_H}\Phi\left(\mu,\frac{1+\hat{r}}{\hat{\bar{\mu}}^p}\right)\zeta_B(\eta)\mathbb{I}_{SB}(\mu,\eta)\frac{\eta h(\mu,\eta)}{h_\mu(\mu)}d\eta, && (75)\\
\frac{\hat{C}^o(\mu)}{\hat{w}} &= \int_{\eta_L}^{\eta_H}\left(\lambda+\theta\zeta_B(\eta)\frac{1+\hat{r}}{\bar{\mu}}\right)[1-\mathbb{I}_{SB}(\mu,\eta)]\frac{\eta h(\mu,\eta)}{h_\mu(\mu)}d\eta \\
&\quad + \left((1-\theta)\frac{1+\hat{r}}{\hat{\bar{\mu}}^p}+\lambda\right)\int_{\eta_L}^{\eta_H}\left[1-\Phi\left(\mu,\frac{1+\hat{r}}{\hat{\bar{\mu}}^p}\right)\right]\mathbb{I}_{SB}(\mu,\eta)\frac{\eta h(\mu,\eta)}{h_\mu(\mu)}d\eta \\
&\quad + \theta\frac{1+\hat{r}}{\bar{\mu}}\int_{\eta_L}^{\eta_H}\left[1-\Phi\left(\mu,\frac{1+\hat{r}}{\hat{\bar{\mu}}^p}\right)\right]\zeta_B(\eta)\mathbb{I}_{SB}(\mu,\eta)\frac{\eta h(\mu,\eta)}{h_\mu(\mu)}d\eta, && (76)\\
\frac{\hat{A}^p(\mu)}{\hat{w}} &= (1-\theta)\int_{\eta_L}^{\eta_H}\mathbb{I}_{SB}(\mu,\eta)\frac{\eta h(\mu,\eta)}{h_\mu(\mu)}d\eta \\
&\quad - \left(1-\theta+\frac{\lambda\hat{\bar{\mu}}^p}{1+\hat{r}}\right)\int_{\eta_L}^{\eta_H}\Phi\left(\mu,\frac{1+\hat{r}}{\hat{\bar{\mu}}^p}\right)\mathbb{I}_{SB}(\mu,\eta)\frac{\eta h(\mu,\eta)}{h_\mu(\mu)}d\eta \\
&\quad - \theta\frac{\hat{\bar{\mu}}^p}{\bar{\mu}}\int_{\eta_L}^{\eta_H}\Phi\left(\mu,\frac{1+\hat{r}}{\hat{\bar{\mu}}^p}\right)\zeta_B(\eta)\mathbb{I}_{SB}(\mu,\eta)\frac{\eta h(\mu,\eta)}{h_\mu(\mu)}d\eta, && (77)
\end{aligned}$$

where $\mathbb{I}_{SB}(\mu,\eta) = 0$ if $\hat{A}^p(\mu,\eta) < 0$ and $\mathbb{I}_{SB}(\mu,\eta) = 1$ if $\hat{A}^p(\mu,\eta) \geq 0$.[7] The profiles for SB and SA (in Figure 6) are very similar.

4.3 Pension system C

The final case we consider is pension system C under which the government engages in more extreme redistribution from the rich to the poor (than under system B) by providing every surviving individual with the *same* pension payment:

$$R^s_{t+1}(\eta) = \bar{R}^s_{t+1}, \tag{78}$$

where $\bar{R}^s_{t+1}$ is to be determined below. The clearing condition for the public pension system is given in this case by:

$$(1 + r_{t+1})\theta\bar{\eta}w_t L_t = L_t \int_{\mu_L}^{\mu_H} \int_{\eta_L}^{\eta_H} \mu\bar{R}^s_{t+1} h(\mu,\eta) d\eta d\mu, \tag{79}$$

so that $\bar{R}^s_{t+1}$ is given by:

$$\bar{R}^s_{t+1} = \theta\bar{\eta}w_t \frac{1 + r_{t+1}}{\bar{\mu}}. \tag{80}$$

Expressing the pension receipt in terms of the contribution made during youth we find for a person of type η that $R^s_{t+1}(\eta) = \zeta(\eta)\theta w_t(\eta)$ where $\zeta(\eta)$ is given by:

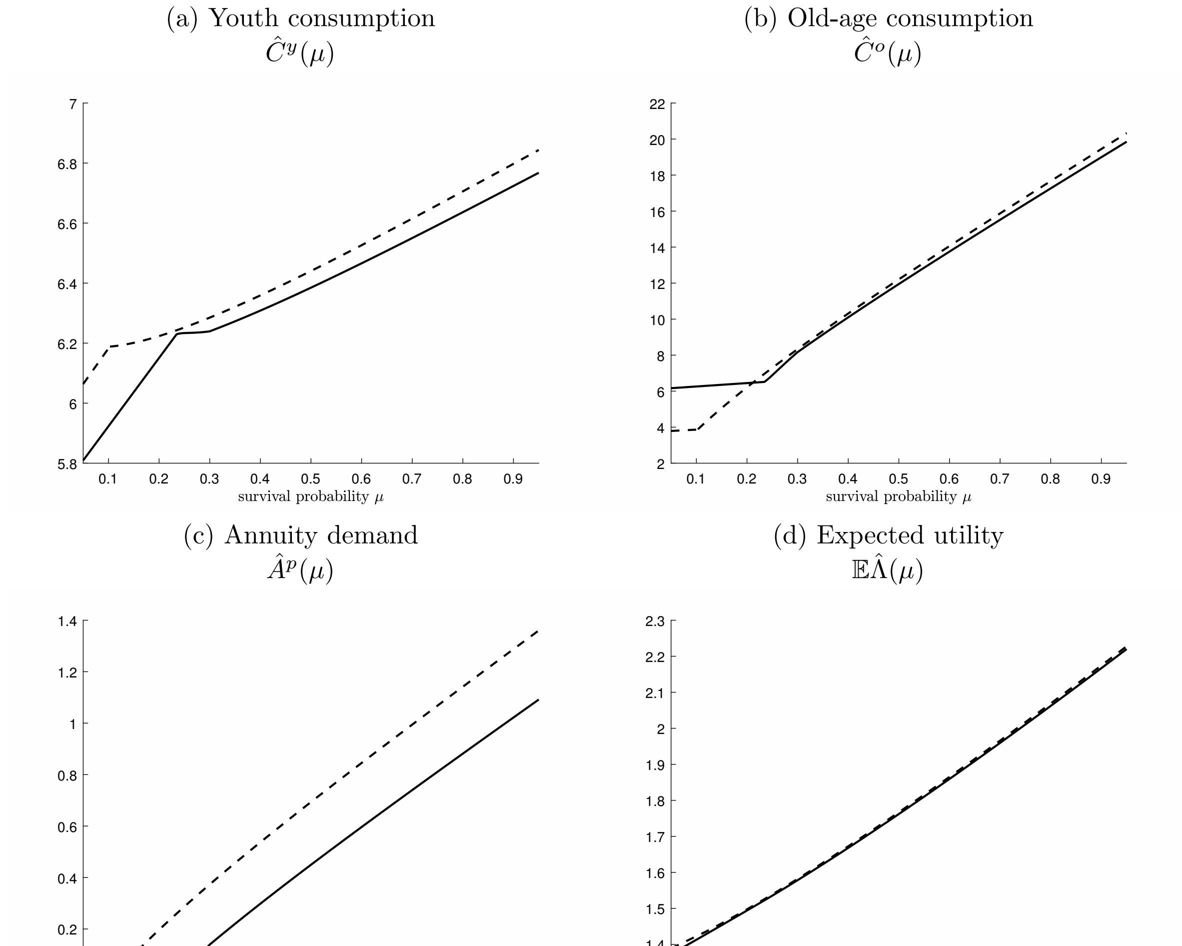
$$\zeta(\eta) = \zeta_C(\eta)\, \frac{1 + r_{t+1}}{\bar{\mu}}, \qquad \zeta_C(\eta) \equiv \frac{\bar{\eta}}{\eta}. \tag{81}$$

See Figure 9 for features of pension system C. It follows from (81) that for individuals with above-average productivity, $\eta > \bar{\eta}$, the rate of return on social annuities falls short of the actuarially fair social annuity yield, $(1 + r_{t+1})/\bar{\mu}$. In contrast, below-average individuals get a better-than actuarially fair rate on the pension contributions. Intuitively these results follow from the fact that the pension system redistributes resources from productive to less productive agents.

Qualitatively system C is very similar to system B (in that both feature redistribution for healthy to unhealthy agents) and the key expressions characterizing system C can be obtained by replacing $\zeta_B(\eta)$ with $\zeta_C(\eta)$ in equations (68)–(77). The main features of system C are the following. First, the comparison of columns (b) and (g) in Table 2 reveals that output and wages fall by 1.83% ($\hat{y} = 9.660$) and the capital intensity drops by 6.49% ($\hat{k} = 0.349$). Out of

[7] Using Figure 7 the indicator function $\mathbb{I}_{SB}(\mu,\eta)$ can be characterized a bit further. For $\mu_L \leq \mu < \hat{\mu}^{bc}(\eta_H)$ all individuals are constrained, i.e. $\mathbb{I}_{SB}(\mu,\eta) = 0$ for all $\eta \in [\eta_L,\eta_H]$. Similarly, for $\hat{\mu}^{bc}(\eta_L) \leq \mu < \mu_H$ all individuals are unconstrained, i.e. $\mathbb{I}_{SB}(\mu,\eta) = 1$ for all $\eta \in [\eta_L,\eta_H]$. Finally, for $\hat{\mu}^{bc}(\eta_H) \leq \mu \leq \hat{\mu}^{bc}(\eta_L)$ we define the critical level of η at which borrowing constraints cease to bind, i.e. $\hat{\eta}^{bc}(\mu)$ is the inverse function of $\hat{\mu}^{bc}(\eta)$ in that domain. Then $\mathbb{I}_{SB}(\mu,\eta) = 0$ for $\eta_L \leq \eta < \hat{\eta}^{bc}(\mu)$ and $\mathbb{I}_{SB}(\mu,\eta) = 1$ for $\hat{\eta}^{bc}(\mu) \leq \eta \leq \eta_H$.

Figure 10: Steady-state profiles under pension system B



Legend The solid lines depict the steady-state profiles under pension system B (SB), and the dashed lines visualize the profiles for the asymmetric information (AI) case without pensions. In both cases adverse selection results in a single pooling rate of interest on annuities, $\bar{r}^p_{t+1}$, and agents with poor health face binding borrowing constraints. The SB case has been drawn for a large system featuring $\theta = 0.025$.

the three pension systems considered, system C features the largest macroeconomic effects. Redistribution is macroeconomically costly. Second, from Figure 9 it is clear that pension system C indeed features the highest degree of redistribution from healthy to unhealthy individuals. Indeed, as can be observed in panel (c) the effective contribution rate $\theta_t^n(\eta)$ becomes positive for the most healthy individuals. Such individuals experience the pension system as a tax burden. Third, as is shown in Figure 8 low-ability types are affected most severely by borrowing constraints under pension system C. Finally, the individual η-averaged profiles for consumption, annuity demands, and utility are depicted in Figure 11. These profiles are very similar to the ones we found for system B.

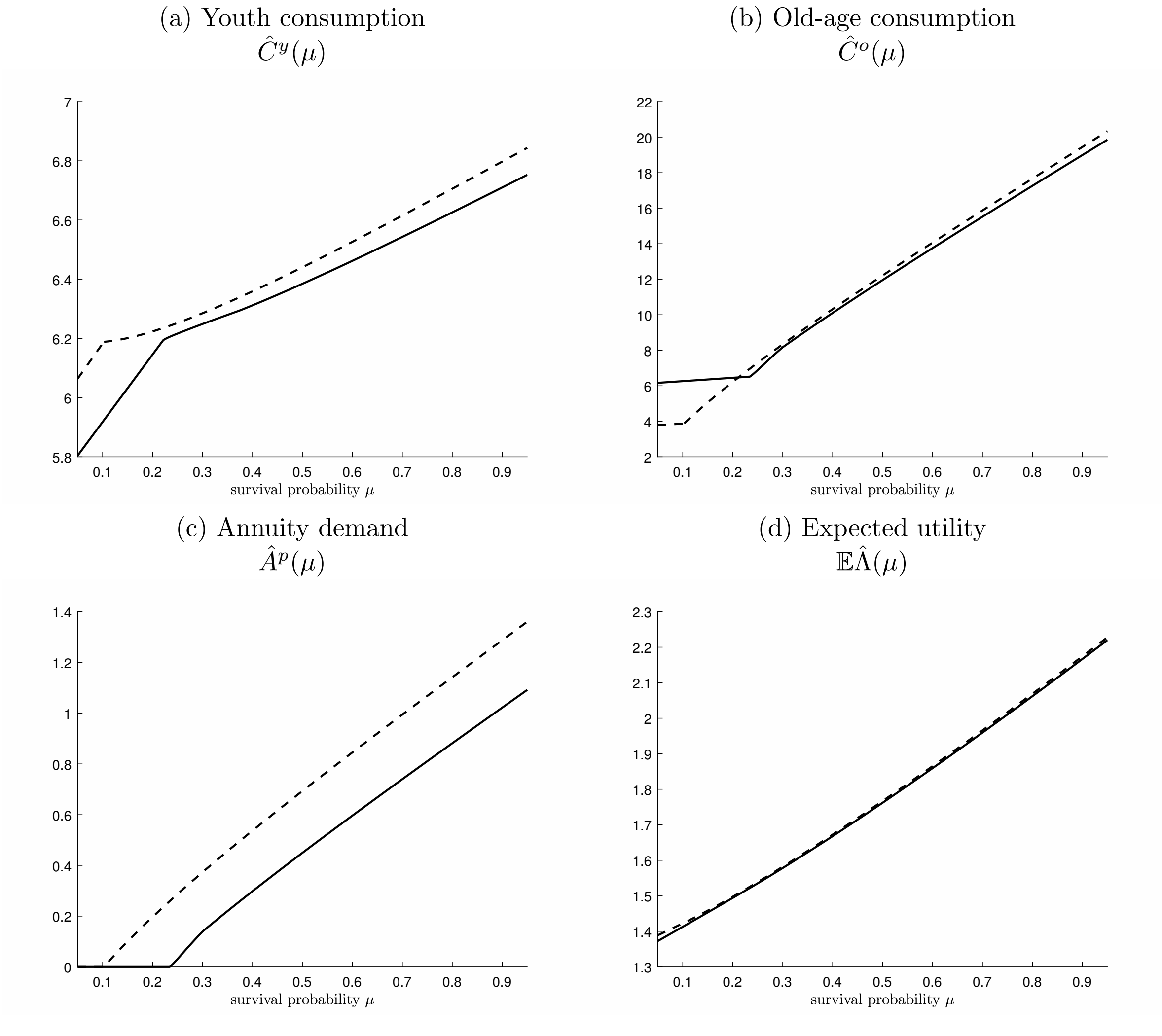
5 Privatizing social security

The key message of the previous section is loud and clear. The mandatory funded pension systems that we have studied are immune to adverse selection by design but they exacerbate the adverse selection problem in the market for private annuities, increase the fraction of borrowing-constrained ('over-annuitized') individuals in the population, and lead to long-run crowding out of capital and substantial output losses. This begs the following question: is it better to privatize social security altogether and to allow individuals to insure against longevity risk in the private annuity market even though this market is not perfect? Referring to Table 2 we find that abolishing the large pension system A (featuring $\theta = 0.025$) would increase output by 1.65% in the long run. In addition, it would increase steady-state welfare of all corner types in the economy, cf. the information contained in columns (d) and (b). At least in the long run, privatization is a 'win-win' scenario.

Of course, comparing steady states gives only part of the answer. What matters is whether or not is possible to abolish the funded pension system in a Pareto improving manner, i.e. is it a 'win-win' scenario to all generations? To answer this question we now study the transitional dynamic effects of abolishing pension system A. The economy is in the steady state for the SA system with $\theta = 0.025$ and the capital intensity is equal to $\hat{k}_{SA} = 0.351$. At shock-time $t = 0$, the pension system is abolished so that young individuals do not pay the pension contribution anymore, i.e. wage income from $t = 0$ onward equals $w_t(\eta)$ and pensions receipts from period $t = 1$ onward are equal to zero, $R_t^s(\eta) = 0$. Of course the old survivors at the time of the shock receive the pension they saved for, i.e. $R_0^s(\eta) > 0$.

Figure 12 depicts some of the key features of the transition process. Panel (a) shows that the capital intensity is predetermined at impact but thereafter rises monotonically to settle at the new steady-state level associated with the AI equilibrium, $\hat{k}_{AI} = 0.373$. Panel (b) show the percentage change in youth-consumption for healthy and unhealthy individuals with the lowest skill level. Interestingly, the healthy individuals decrease their consumption whilst the unhealthy increase it. The response of the latter group of people is easy to understand: these

Figure 11: Steady-state profiles under pension system C



Legend The solid lines depict the steady-state profiles under pension system C (SC), and the dashed lines visualize the profiles for the asymmetric information (AI) case without pensions. In both cases adverse selection results in a single pooling rate of interest on annuities, $\bar{r}^p_{t+1}$, and agents with poor health face binding borrowing constraints. The SC case has been drawn for a large system featuring $\theta = 0.025$.

individuals were facing severe borrowing constraints in the SA system (and will continue to do so to a lesser degree in the AI equilibrium). Because the pension system is abolished (and $\theta = 0$) they can increase their consumption during youth and reduce the degree of overannuitization. Note that in panel (c) the overannuitization faced by the unhealthy is illustrated by the dramatic fall in old-age consumption for period $t = 1$ (when the surviving shock-time young are old) and beyond. Finally, in panel (d) we show that there is a strong increase in the demand for private annuities by the healthy agents.[8] There is virtually no transitional dynamics in μ_t^{bc} which falls from $\hat{\mu}^{bc} = 0.2090$ to $\mu_0^{bc} = 0.1026$ and thereafter settles at $\hat{\mu}^{bc} = 0.1025$. It follows that all agents featuring $\mu < 0.1025$ face borrowing constraints during youth no matter when they are born.

In Figure 13 we illustrate the effects on lifetime welfare for the four corner types in the economy, i.e. (μ_L, η_L), (μ_L, η_H), (μ_H, η_L), and (μ_H, η_H). Regardless of when they are born and irrespective of their productivity level, the unhealthiest individuals are better off as a result of the pension abolishment. Expected lifetime utility rises over time so for all corner types the gain is higher the later they are born. Interestingly, healthy agents born at the time of the shock are worse off than they would have been under the SA system. Privatizing social security is not a 'win-win' scenario to all generations.

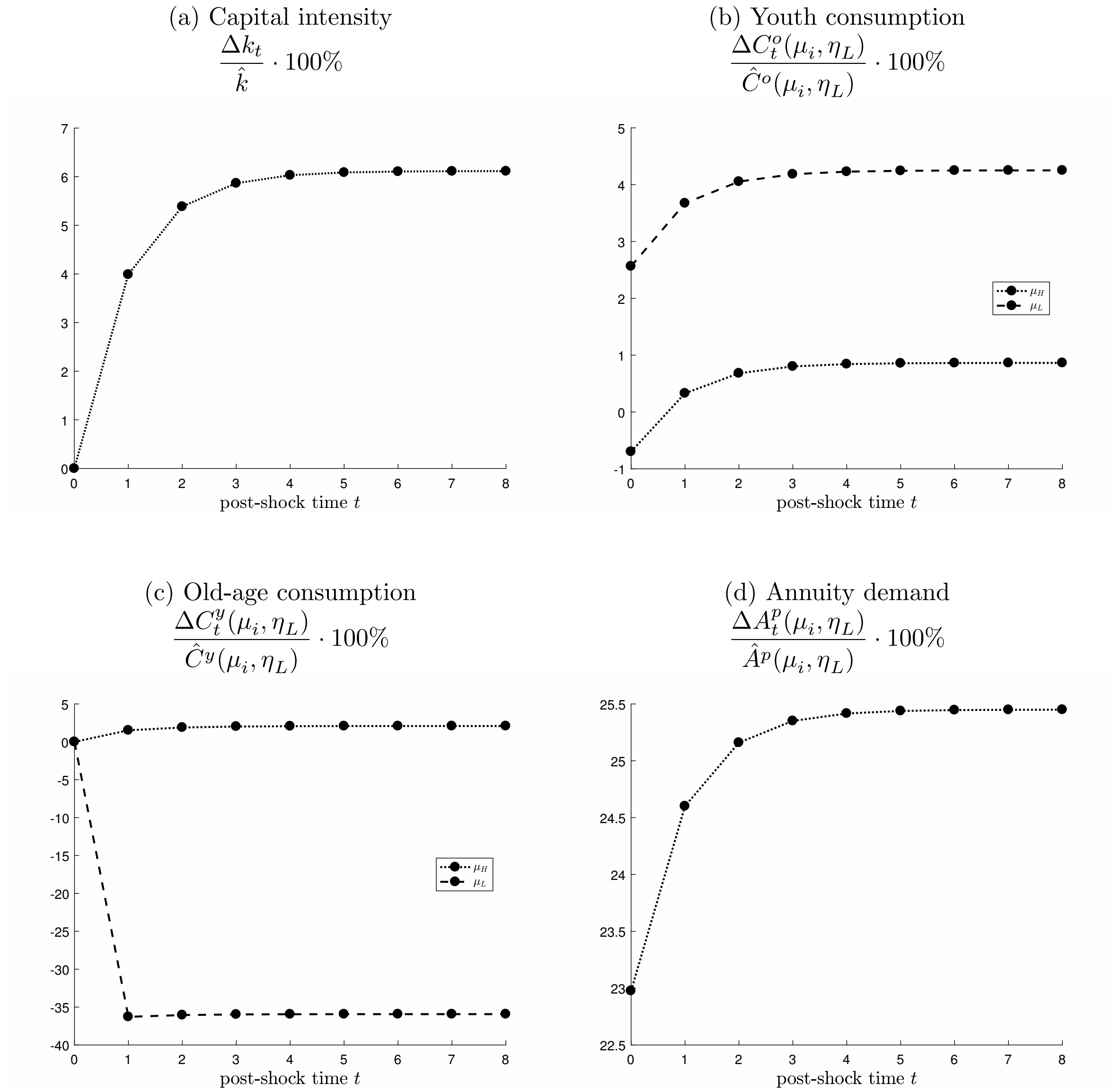
6 Conclusion

In our paper we have developed an overlapping generations model which features adverse selection in the private annuity market and endogenously determined borrowing constraints in the capital market. Consumers are assumed to be heterogeneous in two dimensions—working ability and health status—which in the absence of perfect information leads to adverse selection in the private annuity market. Furthermore, they are restricted from borrowing against their anticipated future wage income due to the borrowing constraints. We demonstrate numerically that the informational asymmetry matters quantitatively in that, compared to the world with perfect information, it causes first-order reductions in output per efficiency unit of labour and the capital intensity . Starting from the benchmark model with adverse selection we introduce a fully-funded social security system and study its impact on capital accumulation and individual welfare under three different pension benefit rules.

We find that the social security system affects both capital accumulation and the proportion of individuals that are facing borrowing constraints. Capital crowding out increases and borrowing constraints become more prevalent the larger is the pension system. Intuitively a social security system causes more consumers to be over-annuitized and to face borrowing constraints. They cannot undo the effects of social security by transacting in their private accounts because any attempt to go short on annuities (demanding life-insured loans) would

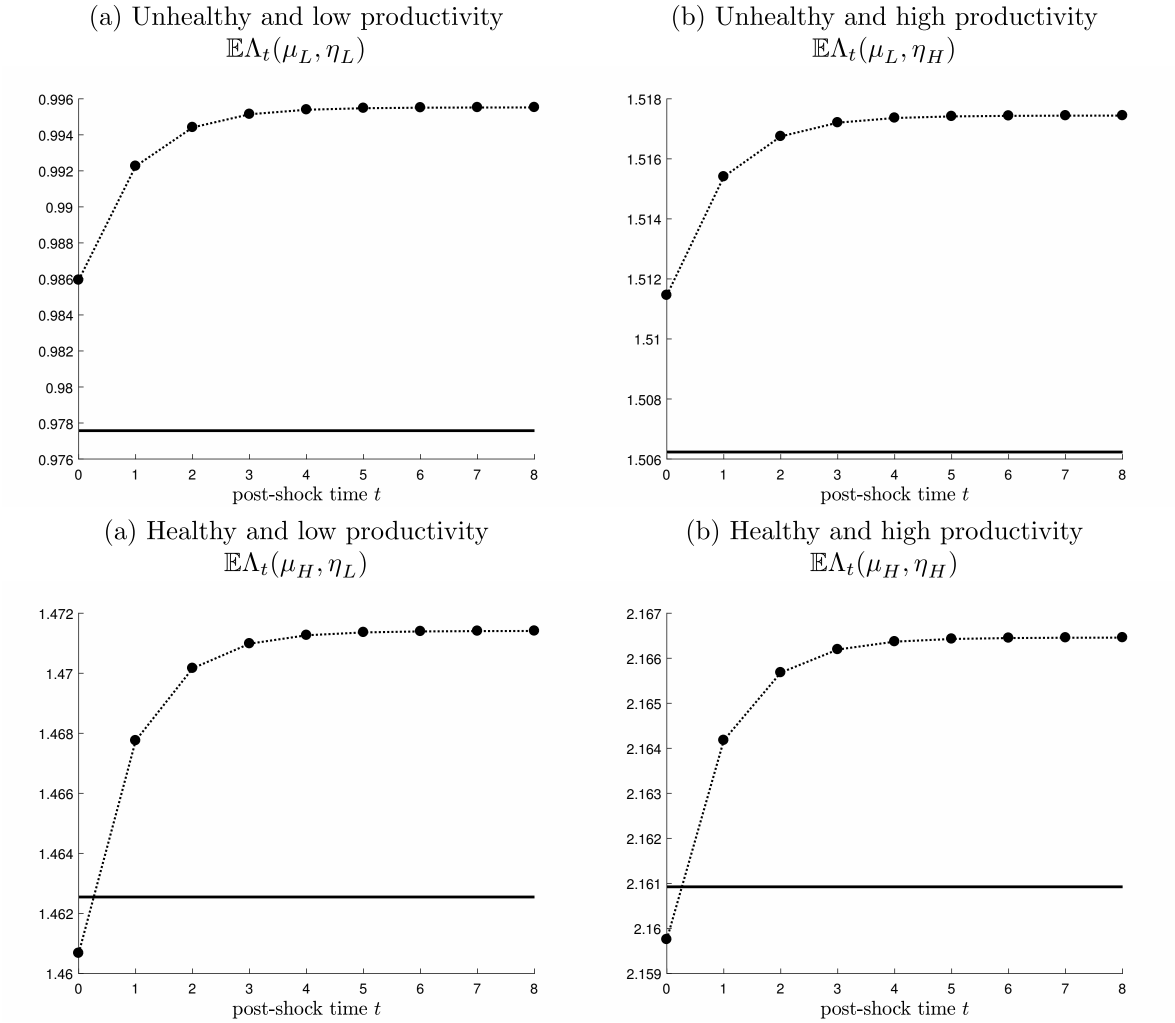
[8] Since youth-consumption, private annuity demand, and old-age consumption are linear in η for both SA and AI systems, it follows that the information in panels (b)-(d) is the same for all values of η.

Figure 12: Abolishing pension system A



Legend At time $t = 0$ pension system A with a contribution rate of $\theta = 0.025$ is abolished permanently. The system is initially in the steady state featuring a capital intensity $\hat{k}_{SA} = 0.351$. Panel (a): over time the economy converges monotonically to the steady-state for the AI case with $\hat{k}_{AI} = 0.373$. Panels (b)-(c) show the percentage change in, respectively, youth and old-age consumption for an individual of type (μ_i, η_L). Panel (d) depicts the percentage change in annuity demand of a person of type (μ_H, η_L). See also Table 2.

Figure 13: Lifetime utility of corner types



Legend The solid lines depict the steady-state lifetime utility levels attained by the different corner types under pension system A (SA) with $\theta = 0.025$. The abolishment of the pension system occurs at time $t = 0$ and affects lifetime utility of different types over time. Unhealthy agents benefit from the policy initiative no matter when they are born. Healthy individuals born at the time of the shock are worse off as a result of it.

reveal their health status to the insurance companies in a world with asymmetric information.

The welfare effects of social security depend both on the pension recipient's type and on the specific form of the pension benefit rule. Provided the rule incorporates some implicit or explicit redistribution from healthy to unhealthy individuals, the latter group will actually benefit from the existence of the social security system in the steady state. In contrast, if pension benefits are proportional to an individual's contributions during youth and the proportionality factor is the same for everybody then the pension system makes everybody worse off in the long run.

A comparison of steady-state equilibria is not a guarantee that the privatization of social security is Pareto improving for all generations. For example, the simulations have shown that the abolition of a public pension system featuring a proportional benefit rule will harm shock-time healthy individuals. Even though all other generations and types are better off as a result, the privatization does not constitute a 'win-win' scenario.

In this paper we have intentionally ignored the role of an intentional bequest motive and its effect on capital accumulation. Of course, the intention to leave bequests to one's offspring does affect an individual's attitude toward private annuities. Indeed, with an operative bequest motive, the rational individual will no longer fully annuitize his/her assets. Despite the high return on private annuities the individual will put aside a certain amount of unannuitized savings to pass on to their offspring upon death. In future work we intend to generalize the heterogeneous-agent model developed here by including an intentional bequest motive and to study the effects of social security with this extended framework.

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Appendix A

In this appendix we show some important results regarding the bivariate uniform distribution for μ and η that is employed in this paper (see equation (1) for the density function). First we show how to derive it by using the Farly-Morgenstern Family approach. In doing so we impose that the marginal distribution of μ (denoted by $h_\mu(\mu)$) is uniform in the interval $[\mu_L, \mu_H]$ whilst the one for η (denoted by $h_\eta(\eta)$) is uniform in the interval $[\eta_L, \eta_H]$. It follows that:

$$h_\mu(\mu) = \begin{cases} 1/(\mu_H - \mu_L) & \text{for } \mu_L \leq \mu \leq \mu_H \\ 0 & \text{otherwise,} \end{cases} \tag{A.1}$$

and:

$$h_\eta(\eta) = \begin{cases} 1/(\eta_H - \eta_L) & \text{for } \eta_L \le \eta \le \eta_H \\ 0 & \text{otherwise.} \end{cases} \tag{A.2}$$

For future reference we define the unconditional means as:

$$\bar{\mu} \equiv \int_{\mu_L}^{\mu_H} \mu h_\mu(\mu)\, d\mu = \frac{\mu_L + \mu_H}{2}, \qquad \bar{\eta} \equiv \int_{\eta_L}^{\eta_H} \eta h_\eta(\eta)\, d\eta = \frac{\eta_L + \eta_H}{2}. \tag{A.3}$$

Next we define the corresponding cumulative distribution functions as:

$$H_\mu(\mu) \equiv \int_{\mu_L}^{\mu} h_\mu(s) ds = \frac{\mu - \mu_L}{\mu_H - \mu_L}, \qquad H_\eta(\eta) \equiv \int_{\eta_L}^{\eta} h_\eta(s) ds = \frac{\eta - \eta_L}{\eta_H - \eta_L}. \tag{A.4}$$

Rice (2007, pp. 77-78) shows that for any parameter α such that $|\alpha| < 1$ a bivariate distribution $H(\mu, \eta)$ possessing uniform marginal distributions is obtained by computing:

$$H(\mu, \eta) = H_\mu(\mu) H_\eta(\eta) \left[1 + \alpha \left[1 - H_\mu(\mu)\right]\left[1 - H_\eta(\eta)\right]\right]. \tag{A.5}$$

Because $\lim_{\mu \to \mu_H} H_\mu(\mu) = 1$ and $\lim_{\eta \to \eta_H} H_\eta(\eta) = 1$ we find that the marginal distributions resulting from (A.5) are $H(\mu_H, \eta) = H_\eta(\eta)$ and $H(\mu, \eta_H) = H_\mu(\mu)$.

By using the expression from (A.4) in (A.5) we find that:

$$H(\mu, \eta) = \frac{(\mu - \mu_L)(\eta - \eta_L)}{(\mu_H - \mu_L)(\eta_H - \eta_L)} \left[1 + \alpha \frac{(\mu_H - \mu)(\eta_H - \eta)}{(\mu_H - \mu_L)(\eta_H - \eta_L)}\right]. \tag{A.6}$$

It follows from (A.6) that the density function, $h(\mu, \eta)$, is given by:

$$h(\mu, \eta) \equiv \frac{\partial^2 H(\mu, \eta)}{\partial \mu \partial \eta} = \frac{1 + \xi(\mu - \bar{\mu})(\eta - \bar{\eta})}{(\mu_H - \mu_L)(\eta_H - \eta_L)}, \tag{A.7}$$

where we have used the fact that $2\bar{\mu} = \mu_L + \mu_H$, $2\bar{\eta} \equiv \eta_L + \eta_H$, and define the parameter:

$$\xi \equiv \frac{4\alpha}{(\mu_H - \mu_L)(\eta_H - \eta_L)}. \tag{A.8}$$

The distribution function (A.6) can thus be written as:

$$H(\mu, \eta) = \frac{(\mu - \mu_L)(\eta - \eta_L)}{(\mu_H - \mu_L)(\eta_H - \eta_L)} \left[1 + \frac{\xi}{4} (\mu_H - \mu)(\eta_H - \eta)\right]. \tag{A.9}$$

Second, we compute some locational parameters for the bivariate uniform distribution.

The unconditional means are stated in (A.3). For the unconditional variances we find:

$$\sigma_\mu^2 \equiv \text{var}(\mu) \equiv \text{E}[\mu - \bar{\mu}]^2 = \int_{\mu_L}^{\mu_H} \mu^2 h_\mu(\mu)\, d\mu - \bar{\mu}^2 = \frac{(\mu_H - \mu_L)^2}{12}, \tag{A.10}$$

$$\sigma_\eta^2 \equiv \text{var}(\eta) \equiv \text{E}[\eta - \bar{\eta}]^2 = \int_{\eta_L}^{\eta_H} \eta^2 h_\eta(\eta)\, d\eta - \bar{\eta}^2 = \frac{(\eta_H - \eta_L)^2}{12}. \tag{A.11}$$

The following lemma is useful.

Lemma A.1 *The following density functions can be derived:*

$$\Gamma_1(\mu) \equiv \frac{\int_{\eta_L}^{\eta_H} \eta h(\eta, \mu)\, d\eta}{\int_{\eta_L}^{\eta_H} h(\eta, \mu)\, d\eta} = \bar{\eta} + \xi \sigma_\eta^2 (\mu - \bar{\mu}),$$

$$\Gamma_2(\eta) \equiv \frac{\int_{\mu_L}^{\mu_H} \mu h(\eta, \mu)\, d\mu}{\int_{\mu_L}^{\mu_H} h(\eta, \mu)\, d\mu} = \bar{\mu} + \xi \sigma_\mu^2 (\eta - \bar{\eta}).$$

Proof The derivation of proceeds as follows.

$$\begin{aligned}
\Gamma_1(\mu) &\equiv \frac{\int_{\eta_L}^{\eta_H} \eta h(\eta, \mu)\, d\eta}{h_\mu(\mu)} \\
&= \frac{1}{\eta_H - \eta_L} \int_{\eta_L}^{\eta_H} \left[[1 - \xi(\mu - \bar{\mu})\bar{\eta}]\eta + \xi(\mu - \bar{\mu})\eta^2 \right] d\eta \\
&= \frac{1}{\eta_H - \eta_L} \left[[1 - \xi(\mu - \bar{\mu})\bar{\eta}] \frac{\eta_H^2 - \eta_L^2}{2} + \xi(\mu - \bar{\mu}) \frac{\eta_H^3 - \eta_L^3}{3} \right].
\end{aligned} \tag{A.12}$$

Note that for $\chi = \eta$ or $\chi = \mu$ we can write:

$$\begin{aligned}
\chi_H^2 - \chi_L^2 &= (\chi_H - \chi_L)(\chi_H + \chi_L) = 2\bar{\chi}(\chi_H - \chi_L), \\
\chi_H^3 - \chi_L^3 &= (\chi_H - \chi_L)\left(\chi_H^2 + \chi_L \chi_H + \chi_L^2\right) = (\chi_H - \chi_L)\left[(2\bar{\chi})^2 - \chi_L \chi_H\right], \\
\bar{\chi}^2 - \chi_L \chi_H &= \frac{(\chi_H - \chi_L)^2}{4}.
\end{aligned} \tag{A.13}$$

Using these results for $\chi = \eta$, the term in square brackets on the right-hand side of (A.12) can be simplified to:

$$\begin{aligned}
[\cdot] &= [1 - \xi(\mu - \bar{\mu})\bar{\eta}]\bar{\eta}(\eta_H - \eta_L) + \xi(\mu - \bar{\mu}) \frac{(\eta_H - \eta_L)\left[(2\bar{\eta})^2 - \eta_L \eta_H\right]}{3} \\
&= (\eta_H - \eta_L) \left[[1 - \xi(\mu - \bar{\mu})\bar{\eta}]\bar{\eta} + \frac{\xi(\mu - \bar{\mu})\left[4\bar{\eta}^2 - \eta_L \eta_H\right]}{3} \right] \\
&= (\eta_H - \eta_L) \left[\bar{\eta} + \frac{\xi(\mu - \bar{\mu})\left[\bar{\eta}^2 - \eta_L \eta_H\right]}{3} \right]
\end{aligned}$$

$$= (\eta_H - \eta_L) \left[\bar{\eta} + \frac{\xi(\mu - \bar{\mu}) (\eta_H - \eta_L)^2}{12} \right]. \tag{A.14}$$

By using (A.14) and noting the definition of σ_η^2 in (A.12) we obtain the result to be proved. The derivation for Γ_2 proceeds along similar lines. ■

After some straightforward but tedious manipulations we find:

$$\begin{aligned}
\mathrm{E}(\mu\eta) &= \int_{\mu_L}^{\mu_H} \mu\eta h(\mu, \eta) d\mu \\
&= \int_{\mu_L}^{\mu_H} \mu \frac{\bar{\eta} + \xi\sigma_\eta^2(\mu - \bar{\mu})}{\mu_H - \mu_L} d\mu \\
&= \frac{1}{\mu_H - \mu_L} \left[(\bar{\eta} - \xi\sigma_\eta^2\bar{\mu}) \frac{\mu_H^2 - \mu_L^2}{2} + \xi\sigma_\eta^2 \frac{\mu_H^3 - \mu_L^3}{3} \right] \\
&= (\bar{\eta} - \xi\sigma_\eta^2\bar{\mu})\bar{\mu} + \xi\sigma_\eta^2 \frac{(2\bar{\mu})^2 - \mu_H\mu_L}{3} \\
&= \bar{\mu}\bar{\eta} + \xi\sigma_\eta^2 \frac{\bar{\mu}^2 - \mu_H\mu_L}{3} \\
&= \bar{\mu}\bar{\eta} + \xi\sigma_\mu^2\sigma_\eta^2,
\end{aligned} \tag{A.15}$$

where we have used the results from (A.13) for $\chi = \mu$. Hence, the covariance between η and μ is given by:

$$\mathrm{cov}(\mu, \eta) \equiv \mathrm{E}(\mu\eta) - \bar{\mu}\bar{\eta} = \xi\sigma_\mu^2\sigma_\eta^2, \tag{A.16}$$

and the correlation coefficient between η and μ is:

$$\mathrm{cor}(\mu, \eta) \equiv \frac{\mathrm{cov}(\mu, \eta)}{\sqrt{\mathrm{var}(\mu)\,\mathrm{var}(\eta)}} = \xi\sigma_\mu\sigma_\eta. \tag{A.17}$$

If $\xi = 0$ then μ and η are uncorrelated.

Appendix B

In the presence of binding borrowing constraints, $\mu_L < \mu_t^{bc} < \mu_H$, we define the average survival rate of annuitants by:

$$\bar{\mu}_t^{an} \equiv \frac{\int_{\mu_t^{bc}}^{\mu_H} \mu h_\mu(\mu) d\mu}{\int_{\mu_t^{bc}}^{\mu_H} h_\mu(\mu) d\mu}.$$

In this appendix we prove that $\bar{\mu}_t^p > \bar{\mu}_t^{an} > \bar{\mu}$ and $AS_t > 1$. The proof of $\bar{\mu}_t^{an} > \bar{\mu}$ is obvious. To show that $\bar{\mu}_t^p > \bar{\mu}_t^{an}$ is less trivial. The proof for this result proceeds along the lines of Heijdra and Reijnders (2012, fn. 7). Individual annuity demand can be written in a separable form as:

$$A_t^p(\mu, \eta) \equiv A_t^p(\mu)\eta,$$

with:

$$A_t^p(\mu) \equiv 1 - \Phi\left(\mu, \frac{1+r_{t+1}}{\bar{\mu}_t^p}\right)\left[w_t + \frac{\lambda \bar{\mu}_t^p w_{t+1}}{1+r_{t+1}}\right].$$

Since $\partial\Phi(\cdot)/\partial\mu < 0$ it follows readily that $\partial A_t^p(\mu)/\partial\mu > 0$. The expression for the pooling rate can be rewritten as:

$$\bar{\mu}_t^p \int_{\mu_t^{bc}}^{\mu_H} Z_t(\mu) h_\mu(\mu) d\mu = \int_{\mu_t^{bc}}^{\mu_H} Z_t(\mu)\mu h_\mu(\mu) d\mu,$$

where $Z_t(\mu)$ is defined as follows:

$$Z_t(\mu) \equiv A_t^p(\mu)\Gamma_1(\mu).$$

Note that $Z_t(\mu)$ is increasing in μ (because both $A_t^p(\mu)$ and $\Gamma_1(\mu)$ are) so that $\text{cov}(Z_t(\mu), \mu) > 0$. Define the following average:

$$\bar{Z}_t \equiv \frac{\int_{\mu_t^{bc}}^{\mu_H} Z_t(\mu) h_\mu(\mu) d\mu}{\int_{\mu_t^{bc}}^{\mu_H} h_\mu(\mu) d\mu}.$$

By definition we have:

$$\begin{aligned}
\int_{\mu_t^{bc}}^{\mu_H} Z_t(\mu) h_\mu(\mu) d\mu &= \int_{\mu_t^{bc}}^{\mu_H} \bar{Z}_t h_\mu(\mu) d\mu \\
\int_{\mu_t^{bc}}^{\mu_H} Z_t(\mu)\mu h_\mu(\mu) d\mu &= \int_{\mu_t^{bc}}^{\mu_H} \left[Z_t(\mu) - \bar{Z}_t + \bar{Z}_t\right]\left[\mu - \bar{\mu}_t^{an} + \bar{\mu}_t^{an}\right] h_\mu(\mu) d\mu \\
&= \bar{\mu}_t^{an} \int_{\mu_t^{bc}}^{\mu_H} \bar{Z}_t h_\mu(\mu) d\mu + \text{cov}(Z_t(\mu), \mu)
\end{aligned}$$

It follows that $\bar{\mu}_t^p$ can be written as:

$$\bar{\mu}_t^p = \bar{\mu}_t^{an} + \frac{\text{cov}(Z_t(\mu), \mu)}{\int_{\mu_t^{bc}}^{\mu_H} \bar{Z}_t h_\mu(\mu) d\mu} > \bar{\mu}_t^{an},$$

where the inequality follows from the fact that $\text{cov}(Z_t(\mu), \mu) > 0$. Hence we have established that $\bar{\mu}_t^p > \bar{\mu}_t^{an} > \bar{\mu}$ and thus $AS_t > 1$. ■

Appendix C

In this appendix we briefly discuss the micro- and macroeconomic effects of public pensions under perfect information. This was also the subject matter of Abel (1987). In our discussion we focus the attention on pension system A. We conclude this appendix with a brief evaluation of the quantitative results for systems B and C.

Under pension system A the income endowment points are given by:

$$(1-\theta)\eta w_t, \qquad \lambda\eta w_{t+1} + \theta\zeta_A \frac{1+r_{t+1}}{\bar{\mu}}\eta w_t. \tag{C.1}$$

For given factor prices the endowments are linear in η. In terms of Figure C.1, the income endowment line IEL is a ray from the origin and individuals are distributed on the line segment $\mathrm{E}_L\mathrm{E}_H$. Note that IEL is a counter-clockwise rotation of the income endowment line without pensions (the dashed line in the figure). The budget constraints of an individual with characteristics μ and η are given by:

$$C_t^y(\mu,\eta) + A_t^p(\mu,\eta) = (1-\theta)\eta w_t, \tag{C.2}$$

$$C_{t+1}^o(\mu,\eta) = \lambda\eta w_{t+1} + \frac{1+r_{t+1}}{\mu}A_t^p(\mu,\eta) + \theta\zeta_A\frac{1+r_{t+1}}{\bar{\mu}}\eta w_t, \tag{C.3}$$

where we have used the expression for the full-information annuity rate of interest from (4). Under full information there is no sign restriction on annuity demand. Indeed, if an individual chooses $A_t^p(\mu,\eta) < 0$ then he/she purchases a life-insured loan (at the actuarially fair borrowing rate). As a result the lifetime budget constraint of the individual is:

$$C_t^y(\mu,\eta) + \frac{\mu C_{t+1}^o(\mu,\eta)}{1+r_{t+1}} = \left[1-\theta+\theta\zeta_A\frac{\mu}{\bar{\mu}}\right]\eta w_t + \frac{\lambda\eta\mu w_{t+1}}{1+r_{t+1}}. \tag{C.4}$$

The effective pension contribution rate is defined as:

$$\theta_t^n \equiv \theta\left(1-\zeta_A\frac{\mu}{\bar{\mu}}\right) \gtreqless 0 \quad \text{for} \quad \mu \lesseqgtr \frac{\bar{\mu}}{\zeta_A}. \tag{C.5}$$

Note that θ_t^n gets larger the unhealthier is the individual so that—in stark contrast to the asymmetric information case—the unhealthiest individuals actually face a positive contribution rate. The public pension system provides such individuals with a highly disadvantageous social annuity rate based on the average survival rate in the population (rather than their own). Under full information, pension system A thus redistributes resources from unhealthy to healthy individuals.

Optimal consumption (during youth and old age) and private annuity demand are given by:

$$C_t^y(\mu,\eta) = \Phi\left(\mu, \frac{1+r_{t+1}}{\mu}\right)\left[\left[1-\theta+\theta\zeta_A\frac{\mu}{\bar{\mu}}\right]w_t + \frac{\lambda\mu w_{t+1}}{1+r_{t+1}}\right]\eta, \tag{C.6}$$

$$\frac{\mu C_{t+1}^o(\mu,\eta)}{1+r_{t+1}} = \left[1-\Phi\left(\mu, \frac{1+r_{t+1}}{\mu}\right)\right]\left[\left[1-\theta+\theta\zeta_A\frac{\mu}{\bar{\mu}}\right]w_t + \frac{\lambda\mu w_{t+1}}{1+r_{t+1}}\right]\eta, \tag{C.7}$$

$$A_t^p(\mu,\eta) = (1-\theta)\eta w_t - \Phi\left(\mu, \frac{1+r_{t+1}}{\mu}\right)\left[(1-\theta)w_t + \theta\zeta_A\frac{\mu w_t}{\bar{\mu}} + \frac{\lambda\mu w_{t+1}}{1+r_{t+1}}\right]\eta. \tag{C.8}$$

In Figure C.1 the choices of the four corner types are illustrated. For a pension system of realistic size, IEL lies to the right of MRSC and all individuals purchase private annuities, i.e. the nation's capital stock is not fully owned by the public pension system. For the lowest-ability individuals consumption occurs at points A (for μ_L) and B (for μ_H). For the highest-ability agents the consumption points are at C (for μ_L) and D (for μ_H). Hence, the introduction of a pension system A of realistic size does not cause the market for life-insured loans to become active. (For a large pension system, all agents buy life-insured loans and the national capital stock is owned by the public pension system.)

The fundamental difference equation for the capital intensity is given by:

$$k_{t+1} = \frac{1}{1+n}\frac{L_t}{N_t}\left[\theta\bar{\eta}w_t + \int_{\eta_L}^{\eta_H}\int_{\mu_L}^{\mu_H} A_t^p(\mu,\eta)h(\mu,\eta)d\mu d\eta\right]. \tag{C.9}$$

By substituting (C.8) into (C.9) and simplifying we find:

$$k_{t+1} = \frac{1}{1+n}\frac{L_t}{N_t}\left[\bar{\eta}w_t - \int_{\mu_L}^{\mu_H}\left[(1-\theta)w_t + \theta\zeta_A\frac{\mu w_t}{\bar{\mu}} + \frac{\lambda\mu w_{t+1}}{1+r_{t+1}}\right]\right.$$
$$\left.\Phi\left(\mu, \frac{1+r_{t+1}}{\mu}\right)h_\mu(\mu)\Gamma_1(\mu)d\mu\right], \tag{C.10}$$

where factor prices follow from (20)–(21).

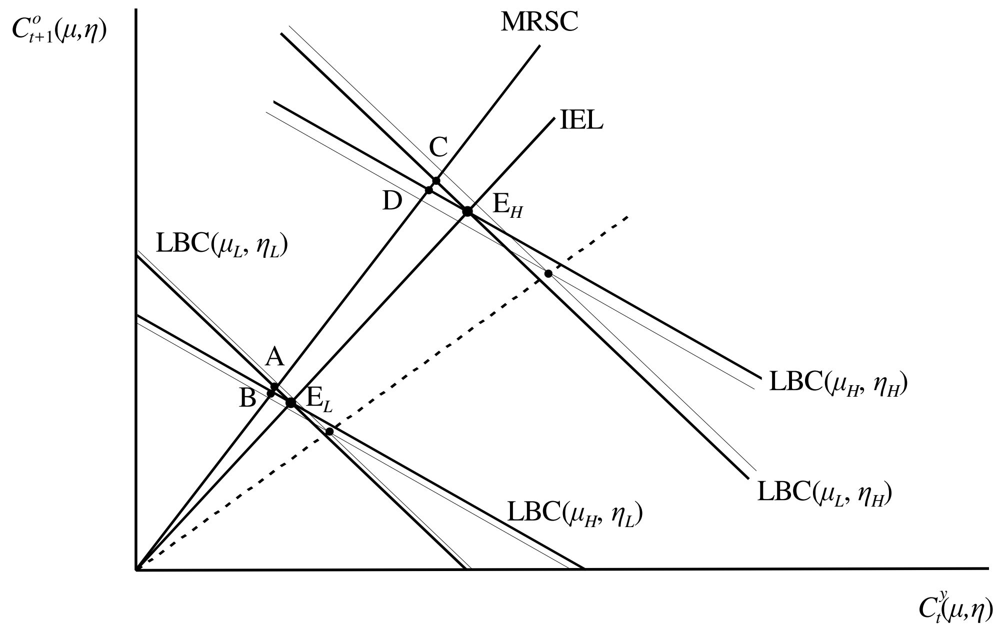
In column (b) of Table C.1 we report some key features of the steady-state equilibrium under pension system A. Comparing columns (b) and (a) we find that output per efficiency unit of labour and the wage rate both increase slightly (by 0.28%) whilst the steady-state capital intensity is increased somewhat (by 1.04%). As a result of the increase in the capital intensity, the annual interest rate falls by 2 basis points ($\hat{r}^a = 4.98\%$). Finally, as the welfare indicators at the bottom of Table C.1 reveal, under pension system A healthy (unhealthy) individuals are better (worse) off compared to the FI case. The pension system slightly stimulates capital accumulation but redistributes resources from unhealthy to healthy individuals.

In columns (c) and (d) of Table C.1 we also report the quantitative results for, respectively, pension systems B and C. The analytical expressions characterizing these equilibria are obtained by replacing ζ_A in (C.1)–(C.9) by, respectively, $\zeta_B(\eta)$ and $\zeta_C(\eta)$ given in (67) and (81) above. The expression for k_{t+1} is slightly more complicated because annuity demand is no longer linear in η under systems B and C:

$$k_{t+1} = \frac{1}{1+n}\frac{L_t}{N_t}\left[\bar{\eta}w_t - \int_{\eta_L}^{\eta_H}\int_{\mu_L}^{\mu_H}\left[(1-\theta)w_t + \theta\zeta_i(\eta)\frac{\mu w_t}{\bar{\mu}} + \frac{\lambda\mu w_{t+1}}{1+r_{t+1}}\right]\right.$$
$$\left.\Phi\left(\mu, \frac{1+r_{t+1}}{\mu}\right)\eta h(\mu,\eta)d\mu d\eta\right], \tag{C.11}$$

for $i \in \{B, C\}$. Despite the fact that systems B and C incorporate some (implicit or explicit) redistribution from healthy to unhealthy individuals, the unhealthy continue to be worse off under both systems when compared to the FI case without pensions.

Figure C.1: Consumption-saving choices under full information and pension system A



Legend LBC(μ_i, η_j) is the lifetime budget constraint for an individual with survival probability μ_i and productivity level η_j. The thin lines represent the FI case without pensions. IEL is the income endowment line and agents are located on the line segment E_LE_H. MRSC is the consumption Euler equation under perfect information with actuarially fair annuities at the individual level. Optimal consumption for individual (μ_i, η_j) is located at the intersection of MRSC and LBC(μ_i, η_j). Provided the pension system is of a realistic size, IEL lies to the right of MRSC and all individuals purchase private annuities.

Table C.1: Pensions under full information

	(a) FI	(b) SA_A $\theta = 0.025$	(c) SA_B $\theta = 0.025$	(d) SA_C $\theta = 0.025$
$\hat{y}$	10.000	10.028	10.027	10.025
$\hat{k}$	0.395	0.400	0.399	0.399
%Q1	12.34	9.25	8.47	7.55
%Q2	19.81	14.84	14.37	14.07
%Q3	28.73	21.53	21.68	21.99
%Q4	39.12	29.31	30.41	31.31
%SAS		25.06	25.08	25.09
$\hat{r}$	6.04	5.99	5.99	5.99
$\hat{r}^a$	5.00%	4.98%	4.98%	4.98%
$\hat{w}$	7.250	7.271	7.269	7.268
$\widehat{BC}$	0.00%	0.00%	0.00%	0.00%
$\hat{c}^y$	5.357	5.368	5.367	5.367
%Q1	15.99	15.90	15.98	16.07
%Q2	22.10	22.05	22.10	22.13
%Q3	28.06	28.07	28.06	28.02
%Q4	33.85	33.98	33.86	33.76
$\hat{c}^o$	4.087	4.099	4.099	4.098
%Q1	12.23	12.17	12.26	12.37
%Q2	19.74	19.70	19.77	19.81
%Q3	28.75	28.76	28.75	28.72
%Q4	39.28	39.37	39.22	39.10
$\mathbb{E}\hat{\Lambda}(\mu_L, \eta_L)$	1.014	1.002	1.003	1.004
$\mathbb{E}\hat{\Lambda}(\mu_H, \eta_L)$	1.433	1.450	1.463	1.484
$\mathbb{E}\hat{\Lambda}(\mu_L, \eta_H)$	1.529	1.522	1.521	1.521
$\mathbb{E}\hat{\Lambda}(\mu_H, \eta_H)$	2.143	2.153	2.149	2.146

Note Here %Qj denotes the share accounted for by skill quartile j (averaged over all survival rates) of the variable directly above it. %SAS is the share owned by the social annuity system. $\mathbb{E}\hat{\Lambda}(\mu_i, \eta_j)$ gives expected utility for an agent with health type μ_i and skill type η_i.