

# Keeping up with the Ageing Joneses\*

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## Abstract

In this paper we consider the implications of relative consumption externalities in the Blanchard-Yaari overlapping generations framework. Unlike most of the macroeconomic literature that studies this question, the differences between agents, and, thus, in their relative position, persist in equilibrium. We show in our fixed employment model that consumption externalities lower consumption and the capital stock in long-run equilibrium, a result in sharp contrast to the recent findings of Liu and Turnovsky (2005). In addition, we solve for the intertemporal path of the economy to investigate its response to demographic shocks, specifically, to permanent changes in the birth and death rates.

*JEL* Codes: D91, E21

**Key Words:** Relative Consumption, Overlapping Generations, Demographic Shocks

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# 1 Introduction

Social scientists have long suggested that the drive for social position, or status, is a crucial motivation in economic decision making. In modern economics this idea has received general analytical treatments by authors such as Layard (1980) and Frank (1985, 1997), while empirical support for the importance of social position for economic well-being is found, for example, in the research of Easterlin (1974, 1995) and Oswald (1997).

Since the 1990s, this concept has also attracted the attention of macroeconomists, who have explored the implications of a preference for status on dynamic, aggregate behavior. Researchers focusing on the effects of status preference for macroeconomic equilibrium and growth include Rauscher (1997), Grossmann (1998), Fisher and Hof (2000), Dupor and Liu (2003), and Liu and Turnovsky (2005). These researchers assume that the quest for status—frequently referred to in this context as “Keeping-up-with-the Joneses”—is reflected in reduced-form specifications of individual preferences that depend on a benchmark level of consumption, such as the average, or aggregate, level of consumption in an economy.<sup>1</sup> Among the questions these authors consider is whether, and under what circumstances, status preferences of this type cause agents to “over-consume” and work “too hard”, compared to a hypothetical social optimum. In other words, does a welfare-reducing “rat race” result if individuals compare their own consumption to some economy-wide, benchmark level? For example, Liu and Turnovsky (2005), employing a standard representative agent (RA) setting, show that the long-run effects of consumption externalities depend on whether work effort is an endogenous variable: if, on the one hand, employment is fixed, then the steady state of the economy is independent of benchmark consumption, while, on the other, if work effort is endogenous, then consumption externalities lead to excessive long-run consumption and capital accumulation, as well as too much employment.<sup>2</sup>

Consumption externalities have also been used by authors such as Abel (1990) and Galí (1994), to study asset pricing, while Ljungqvist and Uhlig (2000) employ the “Catching-up-

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<sup>1</sup>Another branch of the macroeconomic literature in this area assumes that social standing depends on relative wealth, rather than on relative consumption. See, for example, the recent work of Corneo and Jeanne (1997, 2001a, b), Futagami and Shibata (1998), Fisher (2004), Van Long and Shimomura (2004a, b), and Fisher and Hof (2005a, b).

<sup>2</sup>A similar result is found in Dupor and Liu (2003) and is attributable to the fact that consumption externalities raise the marginal rate of substitution for leisure above its Pareto optimal level.

with-the-Joneses” version of status preferences in a simple business cycle model.<sup>3</sup> More recently, this literature has been extended—particularly in terms of an analysis of the economy’s transitional dynamics—by Alvarez-Cuadrado *et al.* (2004), and Turnovsky and Monteiro (2007), who incorporate a time non-separable preference structure based on the fundamental work of Ryder and Heal (1973) on habit formation.<sup>4</sup>

While these studies have contributed many insights to our understanding of the aggregate implications of status preferences, the RA framework employed by all these researchers is, nevertheless, restrictive: since all agents are identical, all differences between them are eliminated in the symmetric macroeconomic equilibrium. In other words, no one “wins” the rat race in this context. In view of the fact, however, that status inherently concerns economic differences among individuals, it is important, in our view, to develop a macroeconomic model of social position in which these differences persist over time. Moreover, some crucial effects of consumption externalities might be lost in a symmetric economic equilibrium.

A natural starting point to model agent heterogeneity is the overlapping generations (OLG) framework in which individuals differ in age and, thus, in their consumption levels and asset holdings. In particular, the economic positions of agents differ from the corresponding economy-wide averages in this setting. To our knowledge, only the recent study of Abel (2005), who uses a discrete-time Diamond (1965) approach, considers the effects of benchmark consumption in an OLG setting.<sup>5</sup>

In this paper we employ, in contrast, the continuous-time Blanchard (1985)-Yaari (1965) framework. Agents in the Blanchard-Yaari (BY) approach are finite-lived and identical except with respect to their “vintage”, i.e., their dates of birth. Furthermore, cohorts of individuals are not linked by a system of intergenerational transfers or bequests. Since agents do not know the date of death, although they know its probability distribution, they face

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<sup>3</sup>Under the Catching-up-with-the-Joneses specification, which is also employed by Abel (1990), the benchmark level of consumption is a weighted average of past consumption. In this context Ljungqvist and Uhlig (2000) show that the optimal consumption tax is countercyclical. In our model, the benchmark is the current consumption of all surviving generations.

<sup>4</sup>Both Alvarez-Cuadrado *et al.* (2004), and Turnovsky and Monteiro (2007) employ an endogenous growth framework. Moreover, Turnovsky and Monteiro (2007) show that the results of Liu and Turnovsky (2005), regarding the conditions under which consumption externalities distort the economy’s long run, extend to the time non-separable preference setting.

<sup>5</sup>Abel (2005) derives the balanced-growth optimal capital tax and transfer policy.

the dilemma that they can die leaving behind their unconsumed wealth to generations they do not care about. In these circumstances there is an incentive for insurance companies, assumed to be perfectly competitive, to offer annuities to the currently living. Specifically, the firms pay “reverse insurance” premiums to the living in exchange for receipt of their financial wealth in case of their death. Since insurance is “fair” in this setting, premiums—the annuity rate of return—equal the probability of death plus the rate of return of alternative assets.<sup>6</sup> This OLG setting has crucial implications for the dynamics of aggregate consumption, because the intergenerational “turnover” from the asset-rich old to the asset-poor young lowers the growth rate of economy-wide consumption, even though all population cohorts face the same interest rate.

An important analytical advantage of the BY setting is that it allows us to calculate detailed dynamic responses to macroeconomic disturbances in the context of finite lives. The BY framework has been employed to study a wide variety of public policy and aggregate macroeconomic shocks in both closed and open economy contexts.<sup>7</sup> Of immediate interest for our purposes are the recent applications of Heijdra and Ligthart (2006) and Bettendorf and Heijdra (2006), who study the dynamic implications of various demographic shocks. While Heijdra and Ligthart (2006) conduct their analysis in a general macroeconomic context with an endogenous employment decision, Bettendorf and Heijdra (2006) model the effects of demographic change on the pension system of an open economy that consumes and produces traded and non-traded goods. In this paper we follow these authors in modeling the impact of demographic shocks, focusing on the adjustment of aggregate consumption and the capital stock in our model of consumption externalities.<sup>8</sup> Specifically, we consider the effects of demographic shocks characteristic of advanced, contemporary societies: a decline in the birth rate—a “baby bust”—and a fall in the mortality rate—a “longevity boost”. In

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<sup>6</sup>Agents are also allowed to go into debt in the BY model. Insurance companies act as lenders in this case and require that an agent pay the annuity rate of return in exchange for paying-off the debt in the event the borrower dies.

<sup>7</sup>In the context of tax and environmental policy see, for example, Bovenberg and Heijdra (1998). Representative applications of the BY framework in the open economy context include Frenkel and Razin (1986), Buitier (1987), Obstfeld and Rogoff (1995), and Heijdra and Romp (2008).

<sup>8</sup>As in Heijdra and Ligthart (2006) and Bettendorf and Heijdra (2006), the demographic disturbances modeled in this paper are time-dependent, but cohort independent. For research that considers the implications of more realistic, cohort-specific demographic shocks, see the recent work of Heijdra and Romp (2006, 2007).

addition, we calculate the implications of an increase in the parameter determining the degree, or intensity, of status preference. Regarding the latter, a key finding of the paper is that the result of Liu and Turnovsky (2005)—that the long-run equilibrium is independent of consumption externalities if employment is fixed—does not hold in the BY framework. We show, in fact, that negative consumption externalities lower both aggregate consumption and the capital stock in the long run, even though labor is exogenously supplied in our model. Moreover, consumption externalities also have important lessons for ageing societies. Clearly, the intergenerational turnover from old to young is less pronounced as the population ages. In the context of the BY model, this implies faster aggregate consumption growth. Nevertheless, our findings imply that the more agents care about relative consumption, the more important is intergenerational turnover even if the population, on average, becomes older. Our model, then, provides a framework in which the “rat race” can be analyzed in an ageing society.

The order of material in the remainder of the paper is as follows: the next section, section 2, analyzes the household and firm sectors. The OLG equilibrium is derived in section 3, which includes a phase diagram describing the macroeconomic dynamics. The log-linearized solution is calculated in section 4. In section 5 we use our model solution to conduct the following macroeconomic experiments: i) an increase in the relative consumption parameter, ii) a decline in the birth rate, and iii) a fall in the mortality rate. Our analytical results regarding the economy’s comparative dynamics are also supplemented in section 5 by numerical simulations. Section 6 briefly outlines our conclusions and suggestions for future work. Finally, a mathematical appendix contains some results used in the main text.

## **2 The Macroeconomy**

### **2.1 Households**

We begin this section with a description of household preferences and then proceed to an analysis of their intertemporal choices and constraints. As indicated above, we analyze an economy in which individuals of particular “vintages” care about their own consumption compared to the average prevailing level of consumption across generations. In other words, while young and old differ, they all aspire to the same level of consumption. For simplicity,

we assume that agents supply a fixed amount of labor. The lifetime utility at time  $t$  of an agent born at time  $v$  (with  $v \leq t$ ) is then given by:

$$\Lambda(v, t) = \int_t^\infty U[\bar{c}(v, \tau), c(\tau)] e^{(\rho+\beta)(t-\tau)} d\tau, \quad (1)$$

where  $\bar{c}(v, \tau)$  is the individual level of consumption,  $c(\tau)$  is the economy-wide level of consumption,  $\rho$  is the rate of time preference, and  $\beta$  is the instantaneous death probability, which we assume to be independent of age. The latter assumption is crucial in yielding a tractable analytical solution of the model. For simplicity, we use the following logarithmic felicity function:

$$U[\cdot] \equiv \ln \bar{x}(v, \tau), \quad (2)$$

where the subfelicity function  $\bar{x}(v, \tau)$  is defined as follows:

$$\bar{x}(v, \tau) \equiv \frac{\bar{c}(v, \tau) - \alpha c(\tau)}{1 - \alpha}, \quad \alpha < 1. \quad (3)$$

The key parameter  $\alpha$  in (3) scales the importance of relative consumption. Dupor and Liu (2003) and Liu and Turnovsky (2005) provide useful taxonomies regarding the distinct forms of status preferences. If average consumption lowers (resp. increases) utility,  $U[\cdot] / \partial c < 0$  (resp.  $\partial U[\cdot] / \partial c > 0$ ), agents are “jealous” (resp. “admiring”). In our model jealousy obtains if  $\alpha > 0$ , while if  $\alpha < 0$ , then agents admire the consumption of others. In contrast, agents “Keep up with the Joneses” (KUJ) if the marginal utility of own consumption increases with average consumption, i.e.,  $\partial^2 U[\cdot] / \partial \bar{c} \partial c > 0$ . Given the specification of preferences in (2)–(3), agents are characterized by KUJ, since  $\partial^2 U[\cdot] / \partial \bar{c} \partial c = c / (\bar{c} - \alpha c)^2 > 0$ , whether or not they are jealous or admiring.<sup>9</sup> Observe, in addition, that the specification of  $\bar{x}(v, \tau)$  satisfies the condition stated in Liu and Turnovsky (2005), Proposition 3, for consumption externalities to have no effect on economic outcomes in the context of the RA, fixed employment framework. Thus, our use of (3) does not bias our results in favor of consumption externalities.

As indicated above, agents receive annuity income on their real asset holdings  $\bar{a}(v, \tau)$ —consisting of (reverse) insurance premiums  $\beta \bar{a}(v, \tau)$  plus interest payments  $r(\tau) \bar{a}(v, \tau)$ —together with real wage income from their exogenous labor supply. Their flow budget identity corresponds to:

$$\dot{\bar{a}}(v, \tau) = [r(\tau) + \beta] \bar{a}(v, \tau) + w(\tau) - \bar{c}(v, \tau), \quad (4)$$

<sup>9</sup>Liu and Turnovsky (2005) emphasize that if  $U[\cdot]$  is non-separable in individual and aggregate consumption then, in general, jealousy and KUJ are not independent.

where  $r(\tau) + \beta$  is the annuity rate of interest, and  $w(\tau)$  is the age-independent wage rate. Note that while each household supplies a single unit of labor, the (real) wage rate  $w(\tau)$  is, in general, not constant over time.

As indicated above, we consider the implications of consumption externalities and demographic shocks in the overlapping generations framework, using an analytical solution of the model as well as numerical results. Applying standard methods, the following individual optimality condition is obtained for the time profile of subfelicity:

$$\frac{\dot{\bar{x}}(v, \tau)}{\bar{x}(v, \tau)} = r(\tau) - \rho. \quad (5)$$

Note that the mortality probability  $\beta$  does not appear in (5): at the *individual* level it cancels out, since it equally affects the effective time preference rate,  $\rho + \beta$ , and the annuity rate of return,  $r(\tau) + \beta$ . Nevertheless, the probability of death, as we show below, not only affects the level of individual consumption, but also the dynamics of its aggregate counterpart.

To solve for the household's intertemporal budget constraint, we integrate, subject to  $\bar{a}(v, t)$  and the standard NPG condition  $\lim_{t \rightarrow \infty} \bar{a}(v, \tau) e^{-R(t, \tau)} = 0$ , the household budget identity (4) and obtain:

$$\int_t^\infty [(1 - \alpha) \bar{x}(v, \tau) + \alpha c(\tau)] e^{-R(t, \tau)} d\tau = \bar{a}(v, t) + h(t), \quad (6)$$

where  $R(t, \tau) \equiv \int_t^\tau [r(s) + \beta] ds$  is the annuity interest factor and  $h(t) = \int_t^\infty w(\tau) e^{-R(t, \tau)} d\tau$  is age-independent human wealth. According to (6), the present discounted value of a weighted average of individual subfelicity and economy-wide per capita consumption equals the sum of the individual's financial and human wealth. Solving (5) for  $\bar{x}(v, \tau)$  and substituting the resulting expression, equal to  $\bar{x}(v, \tau) = \bar{x}(v, t) e^{R(t, \tau) - (\rho + \beta)(\tau - t)}$ ,  $\tau \geq t$ , into (6), we next obtain, noting the definition of  $R(t, \tau)$ , an expression that is useful in determining the aggregate Euler equation:

$$(1 - \alpha) \bar{x}(v, t) + \alpha (\rho + \beta) \Gamma(t) = (\rho + \beta) [\bar{a}(v, t) + h(t)], \quad (7)$$

where  $\Gamma(t) \equiv \int_t^\infty c(\tau) e^{-R(t, \tau)} d\tau$ . Substituting the expression  $(1 - \alpha) \bar{x}(v, t) = \bar{c}(v, t) - \alpha c(t)$  from equation (3) into the intertemporal household budget constraint (7), we obtain the following relationship between individual,  $\bar{c}(v, t)$ , and average,  $c(t)$ , consumption:

$$\bar{c}(v, t) = (\rho + \beta) [\bar{a}(v, t) + h(t)] + \alpha [c(t) - (\rho + \beta) \Gamma(t)]. \quad (8)$$

In the absence of a consumption externality ( $\alpha = 0$ ), individuals condition their consumption solely on  $\bar{a}(v, t) + h(t)$ , with  $\rho + \beta$  representing the propensity to consume out of total wealth. With a non-zero consumption externality, however, individual consumption is also directly affected by the future time path of economy-wide, per capita consumption.

## 2.2 Firms

We next turn to the firm sector of economy, which is kept as simple as possible in order to focus on the implications of consumption externalities in the OLG framework.<sup>10</sup> The production sector is characterized by a large number of firms that produce an identical good under conditions of perfect competition. Net<sup>11</sup> output,  $Y(t)$ , is produced according to a Cobb-Douglas technology with labor,  $L(t)$ , and physical capital,  $K(t)$ , as homogeneous factor inputs that are rented from households:

$$Y(t) = F[K(t), L(t)] = Z_0 K^\varepsilon L^{1-\varepsilon}, \quad y(t) = Z_0 k(t)^\varepsilon, \quad 0 < \varepsilon < 1, \quad (9)$$

where  $y(t) \equiv Y(t) / L(t)$  is per-capita output,  $k(t) \equiv K(t) / L(t)$  is the capital-labor ratio, and  $Z_0$  is exogenous total factor productivity. Clearly, the production function possesses the standard features of positive but diminishing marginal products in both factors. By assumption, there are no adjustment costs associated with investment. The following first-order conditions emerge from the standard maximizing problem of the firm

$$F_K[k(t), 1] = \varepsilon y(t) / k(t) = r(t), \quad F_L[k(t), 1] = (1 - \varepsilon)y(t) = w(t). \quad (10)$$

and imply that the marginal products of capital and labor equal their respective factor costs,  $r(t)$  and  $w(t)$ .<sup>12</sup>

## 3 Aggregation and Macroeconomic Equilibrium

In this part of the paper we state and describe the overall OLG macroeconomic equilibrium. We allow for constant population growth  $n$  and distinguish between the birth rate,  $\eta$ , and

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<sup>10</sup>Similar to Liu and Turnovsky (2005), the present model can be extended to incorporate production as well as consumption externalities. We leave this task for future work.

<sup>11</sup>In other words,  $Y(t)$  is measured taking into account the physical depreciation of the capital stock.

<sup>12</sup>It is straightforward to incorporate government consumption, taxes, and public debt into this framework. A task for future work is an analysis of the role of taxes and fiscal deficits in smoothing the economy's adjustment to demographic shocks.



the death rate,  $\beta$ , so that  $n \equiv \eta - \beta$ . The relative cohort weights evolve, in turn, according to:

$$l(v, t) \equiv \frac{L(v, t)}{L(t)} = \eta e^{\eta(v-t)}, \quad t \geq v. \quad (11)$$

This expression then permits us to calculate the per-capita average values of consumption and financial assets

$$c(t) \equiv \int_{-\infty}^t l(v, t) \bar{c}(v, t) dv, \quad a(t) \equiv \int_{-\infty}^t l(v, t) \bar{a}(v, t) dv, \quad (12)$$

where the former represents the status benchmark for the individual. To derive the macroeconomic equilibrium, we must derive the expressions for the aggregate evolution of consumption and financial assets. Since physical assets are the only form of savings,  $k(t) \equiv a(t)$ , the latter can easily be converted, using the optimality conditions of the firm given in (10), into the aggregate capital accumulation equation. To focus on the economic interpretation of our results, we leave this task for the mathematical appendix and here simply state the OLG equilibrium:

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho - \frac{\eta(\rho + \beta)}{1 - \alpha} \cdot \frac{k(t)}{c(t)}, \quad (13)$$

$$\dot{k}(t) = y(t) - c(t) - (\eta - \beta)k(t), \quad (14)$$

$$r(t) = \varepsilon y(t)/k(t), \quad w(t) = (1 - \varepsilon)y(t), \quad (15)$$

$$y(t) = Z_0 k(t)^\varepsilon, \quad 0 < \varepsilon < 1. \quad (16)$$

The dynamics of aggregate consumption and capital are governed by (13)–(14), with  $k(t)$  the predetermined and  $c(t)$  the “jump” variable that responds to new information. Observe that (14) is stated in terms of the demographic variables, since  $n \equiv \eta - \beta$ .<sup>13</sup> Equations (15)–(16) restate, respectively, the optimality conditions of the firm and the per-capita production function.

While equations (14)–(16) are standard, equation (13) describing the evolution of  $c(t)$  is the key relationship of our model and requires further explanation. Observe that the second term of (13) is a function not only of the birth and death rates, but also of the consumption externality, which, as indicated, is parameterized by  $\alpha$ . This “correction” term, characteristic

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<sup>13</sup>The standard RA model is recovered by setting  $\eta = 0$  and  $\beta = -n$  in (13)–(14). Intuitively, in the RA case there are no new disconnected agents, and population growth shows up in the form of a negative  $\beta$ , i.e. an increase in the size of the dynastic family.

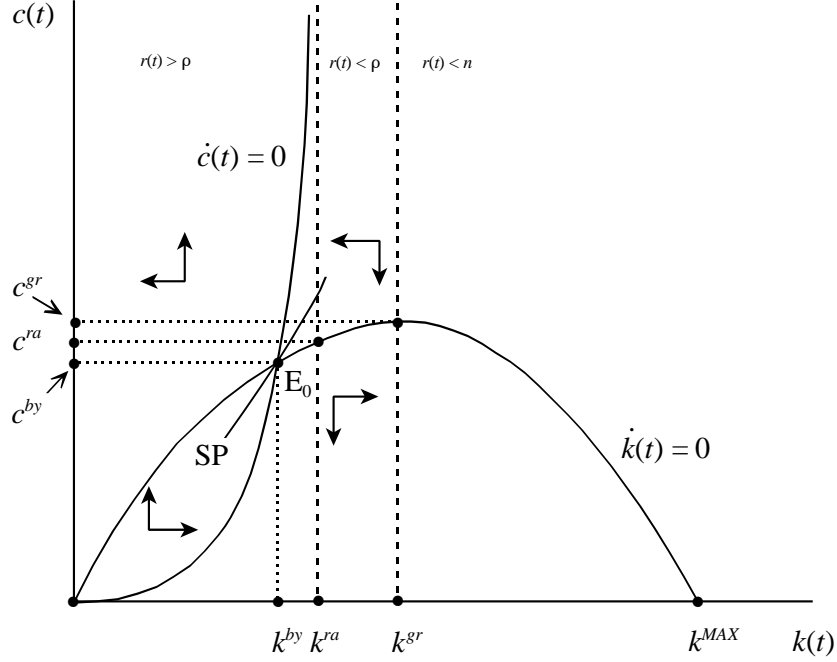


Figure 1: Phase Diagram

of the BY framework, is known as the intergenerational turnover term and takes into account the fact that older cohorts, enjoying greater *levels* of consumption due to greater stocks of wealth, are succeeded by new individuals, who start life without financial assets. This is made explicit by an equivalent representation of (13) derived in the appendix and given by:

$$\dot{c}(t) = [r(t) - \rho] c(t) - \frac{\eta}{1 - \alpha} \cdot [c(t) - \bar{c}(t, t)],$$

where the difference between aggregate and newborn consumption,  $c(t) - \bar{c}(t, t)$ , represents the intergenerational turnover term. As a consequence, the growth of consumption for the economy as a whole is *less* than the growth of consumption for each individual, even though each individual faces the same interest rate. Below, we detail implications of the consumption externality for the OLG steady state.

We close this section with a description of the phase diagram of our OLG model. It follows from (13)–(16), that the  $\dot{c}(t) = 0$  and  $\dot{k}(t) = 0$  are given by:

$$c(t) = \frac{\eta(\rho + \beta)}{\rho(1 - \alpha)} \cdot \frac{k(t)}{(k(t)/k^{ra})^{\varepsilon-1} - 1} \equiv \Phi(k(t)), \quad (17)$$

$$c(t) = Z_0 k(t)^\varepsilon - (\eta - \beta) k(t), \quad (18)$$

where  $k^{ra} = (\varepsilon Z_0 / \rho)^{1/(1-\varepsilon)}$  is the long-run value of the capital stock in the standard RA framework. The corresponding slopes of these relationships equal:

$$\left. \frac{dc(t)}{dk(t)} \right|_{\dot{c}(t)=0} = \frac{\Phi(k(t))}{k(t)} \left[ 1 + \frac{(1-\varepsilon)(k(t)/k^{ra})^{\varepsilon-1}}{(k(t)/k^{ra})^{\varepsilon-1} - 1} \right] > 0, \quad \text{for } 0 \leq k(t) \leq k^{ra},$$

$$\left. \frac{dc(t)}{dk(t)} \right|_{\dot{k}(t)=0} = \varepsilon Z_0 k(t)^{\varepsilon-1} - (\eta - \beta) \geq 0, \quad \text{as } \varepsilon Z_0 k(t)^{\varepsilon-1} \geq \eta - \beta,$$

and are illustrated in Figure 1. In Figure 1 the (unique) intersection of the  $\dot{c}(t) = 0$  and  $\dot{k}(t) = 0$  isoclines determines the (initial) long-run values,  $k^{by}$  and  $c^{by}$ , of the capital stock and consumption, where “by” denotes our Blanchard-Yaari framework, corresponding to point  $E_0$  in Figure 1, in which relative consumption matters. For convenience and to compare the initial BY equilibrium with the one that results after the shock to status preference, we set  $\alpha = 0$  in Figure 1. Nevertheless, it is straightforward to show that if  $0 < \alpha < 1$ , the resulting equilibrium in Figure 1 lies to the *left* of  $E_0$ , while if  $\alpha < 0$ , then it lies to the *right* of  $E_0$ . Observe in Figure 1 that we also illustrate the corresponding long-run values of capital and consumption in the RA setting, which depicts the standard result that the long-run values of these variables in the RA framework exceed their BY counterparts,  $k^{ra} > k^{by}$  and  $c^{ra} > c^{by}$  (although both fall short of the corresponding Golden-Rule values,  $k^{gr}$  and  $c^{gr}$ ).<sup>14</sup> Observe, in addition, that while the  $\dot{k}(t) = 0$  locus is independent of preferences—including the agent’s attitude toward status—the  $\dot{c}(t) = 0$  locus is a function, among others, of the relative consumption parameter  $\alpha$ .<sup>15</sup> In contrast, both the  $\dot{c}(t) = 0$  and  $\dot{k}(t) = 0$  isoclines are functions of the demographic parameters  $\eta$  (the birth rate) and  $\beta$  (the death rate). We employ log-linearized versions of this diagram in section 5 to analyze the short and long-run effects of status preference and demographic disturbances.

<sup>14</sup>Figure 1 is drawn under the (reasonable) assumption that  $\rho > n$ . In the OLG model this assumption is not necessary, i.e. a saddle-point stable equilibrium materializes to the right of  $k^{gr}$  if  $\rho < n$  (dynamic inefficiency). In contrast, in the RA model,  $\rho > n$  is a necessary condition for saddle-point stability. Throughout the paper we focus the discussion on the dynamically consistent case, i.e. we assume that  $r^{by} > n$ .

<sup>15</sup>If, however, labor supply is endogenous, then the  $\dot{k}(t) = 0$  isocline depends on preferences.

## 4 Log-Linearization and Model Solution

### 4.1 Log-Linearization

In order to calculate the solution of the model, we must linearize the macroeconomic equilibrium derived above in (13)–(16), employing the following notation:  $\tilde{y}(t) \equiv dy(t)/y$ ,  $\tilde{k}(t) \equiv dk(t)/k$ ,  $\tilde{c}(t) \equiv dc(t)/c$ ,  $\tilde{r}(t) \equiv dr(t)/r$ ,  $\tilde{w}(t) \equiv dw(t)/w$ ,  $\tilde{\eta} \equiv d\eta/\eta$ ,  $\tilde{\beta} \equiv d\beta/\beta$ ,  $\tilde{\alpha} \equiv d\alpha/(1-\alpha)$ ,  $\dot{\tilde{k}}(t) \equiv d\dot{k}(t)/k$ , and  $\dot{\tilde{c}}(t) \equiv d\dot{c}(t)/c$ . The log-linearized equilibrium then corresponds to:

$$\dot{\tilde{c}}(t) = (r - \rho) \left[ \tilde{c}(t) - \tilde{k}(t) - \tilde{\alpha} - \tilde{\eta} - \frac{\beta}{\rho + \beta} \tilde{\beta} \right] + r \tilde{r}(t), \quad (19)$$

$$\dot{\tilde{k}}(t) = \frac{r}{\varepsilon} [\tilde{y}(t) - \omega_C \tilde{c}(t)] - n \tilde{k}(t) - \eta \tilde{\eta} + \beta \tilde{\beta}, \quad (20)$$

$$\tilde{y}(t) = \varepsilon \tilde{k}(t), \quad \tilde{r}(t) = \tilde{y}(t) - \tilde{k}(t), \quad \tilde{w}(t) = \tilde{y}(t), \quad (21)$$

where  $n \equiv \eta - \beta$ ,  $\omega_C \equiv c/y$  and  $\omega_A \equiv rk/y = \varepsilon$ .

### 4.2 Model Solution

Solving equations (19)–(21), the dynamic system for the capital stock and consumption can be written as follows:

$$\begin{bmatrix} \dot{\tilde{c}}(t) \\ \dot{\tilde{k}}(t) \end{bmatrix} = \Delta \begin{bmatrix} \tilde{c}(t) \\ \tilde{k}(t) \end{bmatrix} - \begin{bmatrix} \gamma_c \\ \gamma_k \end{bmatrix}, \quad (22)$$

where the Jacobian matrix and the vector of (time invariant) exogenous shocks are given, respectively, by:<sup>16</sup>

$$\Delta \equiv \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}, \quad \begin{bmatrix} \gamma_c \\ \gamma_k \end{bmatrix} \equiv \begin{bmatrix} (r - \rho) \left[ \tilde{\alpha} + \tilde{\eta} + \frac{\beta}{\rho + \beta} \tilde{\beta} \right] \\ \eta \tilde{\eta} - \beta \tilde{\beta} \end{bmatrix}, \quad (23)$$

where  $\delta_{11} = r - \rho > 0$ ,  $\delta_{12} = -r(1 - \varepsilon) - (r - \rho) < 0$ ,  $\delta_{21} = -r\omega_C/\varepsilon < 0$ ,  $\delta_{22} = r - n > 0$ . The system described by (22) is saddle-point stable, with  $\det \Delta < 0$  and the corresponding eigenvalues given by  $-\lambda_1 < 0$  and  $\lambda_2 > 0$ .<sup>17</sup> Evaluating the dynamic system in steady-state equilibrium, it is straightforward to calculate the long-run effects of permanent status

<sup>16</sup>Note that both the birth and death rates enter negatively in the  $\dot{\tilde{c}}(t)$  equation. In contrast, while the birth rate  $\eta$  is a negative shift parameter in the  $\dot{\tilde{k}}(t)$  equation, the death rate  $\beta$  enters positively.

<sup>17</sup>We do not require additional parametric assumptions to obtain the saddlepoint property. See the appendix for the expression for  $\det \Delta$ .

preference and demographic shocks on consumption and physical capital:

$$\begin{bmatrix} \tilde{c}(\infty) \\ \tilde{k}(\infty) \end{bmatrix} = \Delta^{-1} \begin{bmatrix} \gamma_c \\ \gamma_k \end{bmatrix} \quad (24)$$

The next step in analyzing the adjustment of the economy is to compute the initial response of consumption,  $\tilde{c}(0)$ . To do so, we calculate the Laplace transform of (22), assuming that physical capital evolves from an initial predetermined stock,  $\tilde{k}(0) = 0$ . The specific procedure is outlined in Heijdra and van der Ploeg (2002, pp. 684–690) and yields the following (equivalent) expressions for  $\tilde{c}(0)$ :

$$\tilde{c}(0) = \mathcal{L}\{\gamma_c, \lambda_2\} + \frac{\delta_{12}}{\lambda_2 - \delta_{22}} \mathcal{L}\{\gamma_k, \lambda_2\} = \mathcal{L}\{\gamma_c, \lambda_2\} + \frac{\lambda_2 - \delta_{11}}{\delta_{21}} \mathcal{L}\{\gamma_k, \lambda_2\}, \quad (25)$$

where  $\mathcal{L}\{\gamma_i, s\} \equiv \int_0^\infty \gamma_i(t) e^{-st} dt$  is the Laplace transform of the time path for  $\gamma_i(t)$ . Finally, the transitional solution paths for consumption and capital correspond to:

$$\begin{bmatrix} \tilde{c}(t) \\ \tilde{k}(t) \end{bmatrix} = \begin{bmatrix} \tilde{c}(0) \\ 0 \end{bmatrix} e^{-\lambda_1 t} + \begin{bmatrix} \tilde{c}(\infty) \\ \tilde{k}(\infty) \end{bmatrix} [1 - e^{-\lambda_1 t}], \quad (26)$$

where we have used the fact that the Laplace transform of the shock terms take the form  $\mathcal{L}\{\gamma_i, s\} = \gamma_i/s$  for  $i = c, k$ , since, as indicated, we consider only unanticipated, permanent, time-invariant disturbances to the status preference and demography parameters. Below, we also illustrate the dynamic effects on post-shock newborn agents. See the appendix for the derivation of the consumption of the newborn,  $\tilde{c}(t, t)$ .

## 5 Comparative Dynamics

Employing the results stated in the previous section, we now analyze the dynamic response of aggregate consumption and physical capital to status preference, birth rate, and mortality rate disturbances. We illustrate the transitional adjustment of the economy by using phase diagrams based on the dynamic system (22) and supplement these findings with numerical simulations of the transitional paths given in (26). We initially set the benchmark parameters for this exercise as follows:  $\alpha = 0.0$ ,  $\beta = 0.02$ ,  $\eta = 0.03$ ,  $\varepsilon = 0.2$ , and  $\rho = 0.04$ .<sup>18</sup>

<sup>18</sup>Using our initial benchmark parameterization, the initial equilibrium value of the capital-output ratio,  $k/y$ , equals 4.18, a value not too far from empirical estimations, since  $y$  represents *net* output. The corresponding initial interest is  $r = \varepsilon y/k = 0.0479$ . Solving the non-linear model we choose  $Z_0 = 0.7512$  to set  $y = 1$  in initial equilibrium.

## 5.1 Change in the Status Parameter

Letting  $\tilde{\alpha} > 0$  and  $\tilde{\eta} = \tilde{\beta} = 0$  in (24), the long-run effect of an increase in the parameter describing the importance of status equals:

$$\tilde{c}(\infty) = (r - \rho)(r - n) \cdot \frac{\tilde{\alpha}}{\det \Delta} < 0, \quad \tilde{k}(\infty) = (r - \rho) \frac{r\omega_C}{\varepsilon} \cdot \frac{\tilde{\alpha}}{\det \Delta} < 0, \quad (27)$$

where  $\det \Delta < 0$  and  $r > n$ . According to (27), a rise in  $\alpha$  lowers both the level of consumption and the physical capital stock in the steady-state equilibrium. This result stands in sharp contrast to the findings of Liu and Turnovsky (2005) in the context of the RA model with exogenous employment. There, (see Proposition 1) the steady state of the economy was independent of consumption externalities. Our OLG relationships in (27) show, on the other hand, that an increase in the preference weight for relative consumption permanently lowers economic activity, even if work effort is given, as it is in our model.<sup>19</sup> A change in  $\alpha$  leads to an adjustment in the long-run equilibrium because it affects the importance of the generational turnover term in the Euler equation given above in (13). Higher values of  $\alpha$ , corresponding to greater degrees of KUJ, increase the importance of the generational turnover effect, which tends to lower the long-run values of consumption and physical capital.

Evaluating either expression in (25) for  $\tilde{\alpha} > 0$  and  $\tilde{\eta} = \tilde{\beta} = 0$ , we calculate the initial jump in consumption:

$$\tilde{c}(0) = \frac{r - \rho}{\lambda_2} \cdot \tilde{\alpha} > 0, \quad (28)$$

which is unambiguously positive. The adjustment of consumption and the capital stock can be illustrated in the phase diagram in Figure 2(a). The rise in  $\alpha$  causes the  $\dot{\tilde{c}}(t) = 0$  isocline to shift to the left, while the  $\dot{\tilde{k}}(t) = 0$  locus is unaffected by this exogenous disturbance. This leads to a shift in the long-run equilibrium from  $E_0$  to  $E_1$ , which corresponds, as indicated, to a decline on  $\tilde{c}(\infty)$  and  $\tilde{k}(\infty)$ . The initial increase in consumption is depicted by the jump in the dynamic system from point  $E_0$  to point A in Figure 2(a), with  $(\tilde{c}(t), \tilde{k}(t))$  proceeding down the (new) saddle path SP from point A to point  $E_1$ . The phase diagram analysis is confirmed by the numerical simulation of the adjustment paths of the model, which is calculated for an increase in  $\alpha$  from its benchmark value of 0.0 to 0.5. In other words, we

<sup>19</sup>Clearly, if  $r = \rho$ , our results in (27) collapse to the RA case.

simulate an increase in KUJ.<sup>20</sup> The numerical simulations in the time domain are illustrated in Figures 2(b)–(c) and depict, respectively, the initial jump along with the long-run decline (the solid locus) in aggregate consumption,  $\tilde{c}(t)$ , along with the gradual decumulation of physical capital,  $\tilde{k}(t)$ .

Moreover, in Figure 2(b) we illustrate (the dashed locus) the effects of this shock on newborn consumption,  $\tilde{c}(t, t)$ . Interestingly, despite the fact that per-capita consumption falls in the long run, consumption by newborns remains above its initial steady-state value, both during transition and in the new steady state. This distinction between the individual response of cohorts, such as newborns, alive at the time of the shock and that of cohorts not yet born is, in fact, the key to understanding the long-run aggregate dynamics of the economy. Using the relationship (8) between individual,  $\bar{c}(v, t)$ , and average,  $c(t)$ , consumption and the aggregate response of the economy derived above, we can demonstrate that the rise in KUJ increases the consumption — and depresses the savings — of each living generation by the *same amount* at  $t = 0$ .<sup>21</sup> This is due to the fact that human wealth, which falls due to the decline in wages and the rise in the interest rate — both caused by the intertemporal fall in the capital stock — is cohort-independent. The time profile of consumption of the pre-shock generations also “steepens” because of the corresponding rise in the interest rate. The key mechanism driving the fall in aggregate consumption is generational turnover in which post-shock newborns gradually replace pre-shock cohorts. Yet post-shock newborns are poorer than pre-shock newborns, because they are only endowed with human wealth, which, as indicated, falls over time. In other words, aggregate consumption has fallen because “poor” post-shock generations replace their “rich” pre-shock counterparts.

We supplement our numerical simulations with Figures 2(d)–(f), which depict the steady-state effects of the increase in  $\alpha$  in the age domain,  $u$ , where the solid line denotes the pre-shock relationship ( $\alpha = 0.0$ ), while the dashed line illustrates its post-shock counterpart

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<sup>20</sup>Jealousy alone is insufficient to generate these results. For example, if felicity is given by  $\log(\bar{c}) - \alpha c$ , which is characterized by jealousy, then the same Euler equation for the standard BY model emerges. Hence, jealousy does not affect the macroeconomy at all. Of course, there will be welfare effects. Moreover, we can show that the long-run, though not the transitional, effects of consumption externalities disappear for the multiplicative felicity function studied by Liu and Turnovsky (2005, p. 1110, eq. (14b)).

<sup>21</sup>Space constraints prevent us from giving the full analytical details here; they are available from the authors on request.

( $\alpha = 0.5$ ). From the steady-state equilibrium, we show in the appendix that the expressions for  $\bar{k}(u)$  and  $\bar{c}(u)$  correspond to:<sup>22</sup>

$$\bar{k}(u) = \frac{\rho + \eta - r}{r - \rho} \cdot k \cdot \left[ e^{(r-\rho)u} - 1 \right], \quad \bar{c}(u) = \frac{\rho + \beta}{r - \rho} \cdot k \cdot \left[ (\rho + \eta - r) e^{(r-\rho)u} + \frac{\alpha\eta}{1 - \alpha} \right], \quad (29)$$

where  $r$  and  $k$  are evaluated at their steady states. Consistent with our above results, we find in Figures 2(d)–(e) that capital holdings  $\bar{k}(u)$  and goods consumption  $\bar{c}(u)$  — for the *same* individual — shifts from youth to old age. Figure 2(f), depicting the growth rate of consumption across ages,  $\dot{\bar{c}}(u)/\bar{c}(u)$ , shows that *prior* to the shock, the growth rate of consumption is the same at all ages and equal to  $r - \rho$ . Subsequent to the shock, this results holds only symptomatically for the very old, of which there are nearly none. Otherwise, there is a marked positive relationship between age and  $\dot{\bar{c}}(u)/\bar{c}(u)$ , reflecting the effects of KUJ in the BY setting.

Finally, an additional feature of the status parameter is that there are bounds it must satisfy in order for the (feasibility) conditions to hold at the individual level. In the appendix we show that the bounds on  $\alpha$  are given by:

$$-\frac{(1 - \varepsilon)(\rho + \eta)}{\eta} < \frac{\alpha}{1 - \alpha} < \frac{1 - \varepsilon}{\varepsilon} \cdot \frac{\rho + \eta}{\rho + \beta'}$$

a result that can be interpreted as follows. The upper bound ensures that newborns will start saving,  $\dot{\bar{k}}(0) > 0$ . Intuitively,  $\alpha$  cannot be too close to unity, i.e., the “catching-up” motive cannot be too strong, because, otherwise, the economy is unable to sustain equilibrium capital accumulation. The lower bound guarantees that  $\bar{c}(0) > 0$ . Hence,  $\alpha$  cannot be too negative. In other words, agents must not admire others’ consumption so much that they wish to consume a negative (infeasible) amount themselves.

## 5.2 Fall in the Birth Rate

Here, we set  $\tilde{\eta} < 0$  and  $\tilde{\alpha} = \tilde{\beta} = 0$  in (24), with the long-run multipliers corresponding to:

$$\tilde{c}(\infty) = \left[ (r - \rho)(r + \beta) + \eta r(1 - \varepsilon) \right] \cdot \frac{\tilde{\eta}}{\det \Delta} > 0, \quad (30)$$

$$\tilde{k}(\infty) = (r - \rho) \left[ \frac{r\omega_C}{\varepsilon} + \eta \right] \cdot \frac{\tilde{\eta}}{\det \Delta} > 0. \quad (31)$$

<sup>22</sup>Clearly, both profiles are upward sloping in the age domain, i.e.,  $\bar{k}'(u) > 0$  and  $\bar{c}'(u) > 0$ .



Clearly, the results in (30)–(31) imply that a permanent drop in the birth rate results in a greater level of consumption and physical capital. As indicated above, this demographic shock affects both the  $\dot{\tilde{c}}(t) = 0$  and the  $\dot{\tilde{k}}(t) = 0$  isoclines of the phase diagram in Figure 3(a), such that the former isocline shifts to the right, while the latter locus shifts to the left, responses that result in a new steady-state equilibrium at point  $E_1$ , corresponding to higher values of  $\tilde{c}(\infty)$  and  $\tilde{k}(\infty)$ .

For this shock the initial jump in consumption, employing (25), equals:

$$\tilde{c}(0) = -\frac{\tilde{\eta}}{\lambda_2} \cdot \left[ \frac{(r - \rho)(r + \beta - \lambda_2) + \eta r(1 - \varepsilon)}{\lambda_2 - (r - n)} \right], \quad (32)$$

which is ambiguous in sign. In Figure 3(a) we illustrate the case in which  $\tilde{c}(0)$  rises to point A on the new stable saddle path SP. Along SP, both consumption and the capital stock increase to the new, long-run equilibrium. In the numerical simulation of this shock—which depicts the dynamic response to a fall in the birth rate  $\eta$  (from 3% to 1.5% per annum) we illustrate in Figures 3(b)–(c), respectively, the initial positive jump in aggregate consumption,  $\tilde{c}(0) > 0$ , and show that paths of consumption and the capital stock (the solid curves) track those derived from our analytical model.<sup>23</sup> The consumption of newborns (the dashed relationship) rises monotonically in Figure 3(b), although the increase in  $\tilde{c}(t, t)$  falls short of the average. Figures 3(d)–(f) depict the implications of a “baby bust” in the age dimension, where, as before, the solid loci illustrate the pre-shock relationship, while the dashed curves are their post-shock counterparts. Figures 3(d)–(f) show that the differences between the capital holdings and goods consumption are less pronounced between young and old after the fall in  $\eta$ . Interestingly, the growth rate of consumption falls across all ages if the birth rate drops.

In sum, a fall in the birth rate  $\eta$  has dynamic implications that are opposite to those of an increase in the relative consumption parameter  $\alpha$ , a result that follows from the fact that a lower birth rate implies that the generational turnover effect of the BY framework is less pronounced. More importantly, however, our previous results have shown that if agents care strongly about relative consumption ( $\alpha$  close to unity), then the generational turnover effect is crucial, even if the economy experiences a declining birth rate.

<sup>23</sup>The numerical simulations in sections 5.2 and 5.3 assume KUJ, i.e., we set  $\alpha = 0.5$ . This results in a lower capital-output ratio,  $k/y = 3.712$  and a higher interest rate,  $r = 0.0539$ . The technology parameter is set to  $Z_0 = 0.764$  to yield  $y = 1$  in initial equilibrium.

### 5.3 Decline in the Mortality Rate

To determine the steady-state implications of a permanent decline in the mortality rate, we set  $\tilde{\beta} < 0$  and  $\tilde{\alpha} = \tilde{\eta} = 0$  in (24) and compute the following long-run multipliers, given by:

$$\tilde{c}(\infty) = [(r - \rho)(r - n) - (\rho + \beta)[r(1 - \varepsilon) + r - \rho]] \cdot \frac{\beta}{\rho + \beta} \cdot \frac{\tilde{\beta}}{\det \Delta}, \quad (33)$$

$$\tilde{k}(\infty) = (r - \rho) \left[ \frac{r\omega_C}{\varepsilon} - (\rho + \beta) \right] \cdot \frac{\beta}{\rho + \beta} \cdot \frac{\tilde{\beta}}{\det \Delta} > 0. \quad (34)$$

While the effect on consumption is ambiguous in sign, the capital stock increases as a result of a longevity boost.<sup>24</sup> The initial jump in consumption, equal to:

$$\tilde{c}(0) = -\frac{\tilde{\beta}}{\lambda_2} \frac{\beta}{\rho + \beta} \left[ -(r - \rho) + \frac{(\rho + \beta)[r(1 - \varepsilon) + (r - \rho)]}{\lambda_2 - (r - n)} \right], \quad (35)$$

can be either positive or negative. The long-run ambiguity is due to the fact that both isoclines in the phase diagram in Figure 4(a) shift to the right subsequent to a fall in the mortality rate. In Figure 4(a) we illustrate the plausible case in which the rightward shift in the  $\dot{c}(t) = 0$  isocline dominates the rightward shift in the  $\dot{k}(t) = 0$  locus, which results in a new equilibrium at point  $E_1$  with lower long-run consumption  $\tilde{c}(\infty)$ , but higher physical capital  $\tilde{k}(\infty)$ . The latter response is quite intuitive, since the fall in the mortality encourages agents, since they live longer, to accumulate more assets. Turning to our numerical simulations for the case in which  $\beta$  falls (from 2% to 1% per annum), we show in Figures 4(b)–(c), respectively, the initial decline in consumption and its partial recovery (below its initial value) in the transition to the steady state, along with the continuous accumulation of capital. Figure 4(b) shows that while newborn consumption also falls as a result of the longevity boost, its decline is less severe than average. We close our numerical analysis with Figures 4(d)–(f), which depict the long-run effects of drop in  $\beta$  in the age domain. As is evident, the decline in  $\beta$  has a negligible effects on the distribution of  $\bar{k}(u)$  and  $\bar{c}(u)$  across ages — in sharp contrast to implications of a decline in  $\eta$  — although the effect on the growth rate of consumption in terms of narrowing the difference the young and the old is greater.

<sup>24</sup>The term in square brackets on the right-hand side of (34) is positive. Note first that  $r\omega_C/\varepsilon = c/k$ . It can be shown readily that  $c/k > \rho + \beta \Leftrightarrow r < \rho + \eta/(1 - \alpha)$ . The latter condition always holds for the feasible range of  $\alpha$  values.

## 6 Conclusions and Extensions

The goal of this paper is to merge two recent strands in the macroeconomic literature: the OLG framework and the work that seeks to investigate the implications of the quest for status. Our principle motivation in adopting the OLG approach with demographic variables is to develop a model of consumption externalities in which differences between individuals do not disappear in equilibrium. In other words, we wish to investigate the properties of a model in which agents are *not* “too equal”, as they are in the RA setting. Employing the BY version of the basic OLG framework, we are able to overturn the recent result of Liu and Turnovsky (2005) regarding the long-run implications of consumption externalities: the latter permanently affect the steady state of the economy, even if employment is fixed. Indeed, if agents as a whole “Keep up with the Joneses” more intensively, then the rat race in our model leads to a long-run decline in aggregate consumption and the capital stock.

Rather than reiterating the rest of our findings, let us briefly indicate some possible extensions of this model. One is to introduce distortionary taxation with endogenous labor supply and to consider the resulting welfare implications in a setting with consumption externalities. The recent work of Calvo and Obstfeld (1988) in calculating welfare effects in an OLG framework would be of assistance in this task. Furthermore, extending the model to an open economy context would permit us to consider the effects of a preference for relative consumption on, for example, the current account dynamics. Finally, incorporating production externalities as well as consumption externalities would allow us to compare and contrast the implications of these two, distinct distortions in an OLG setting.

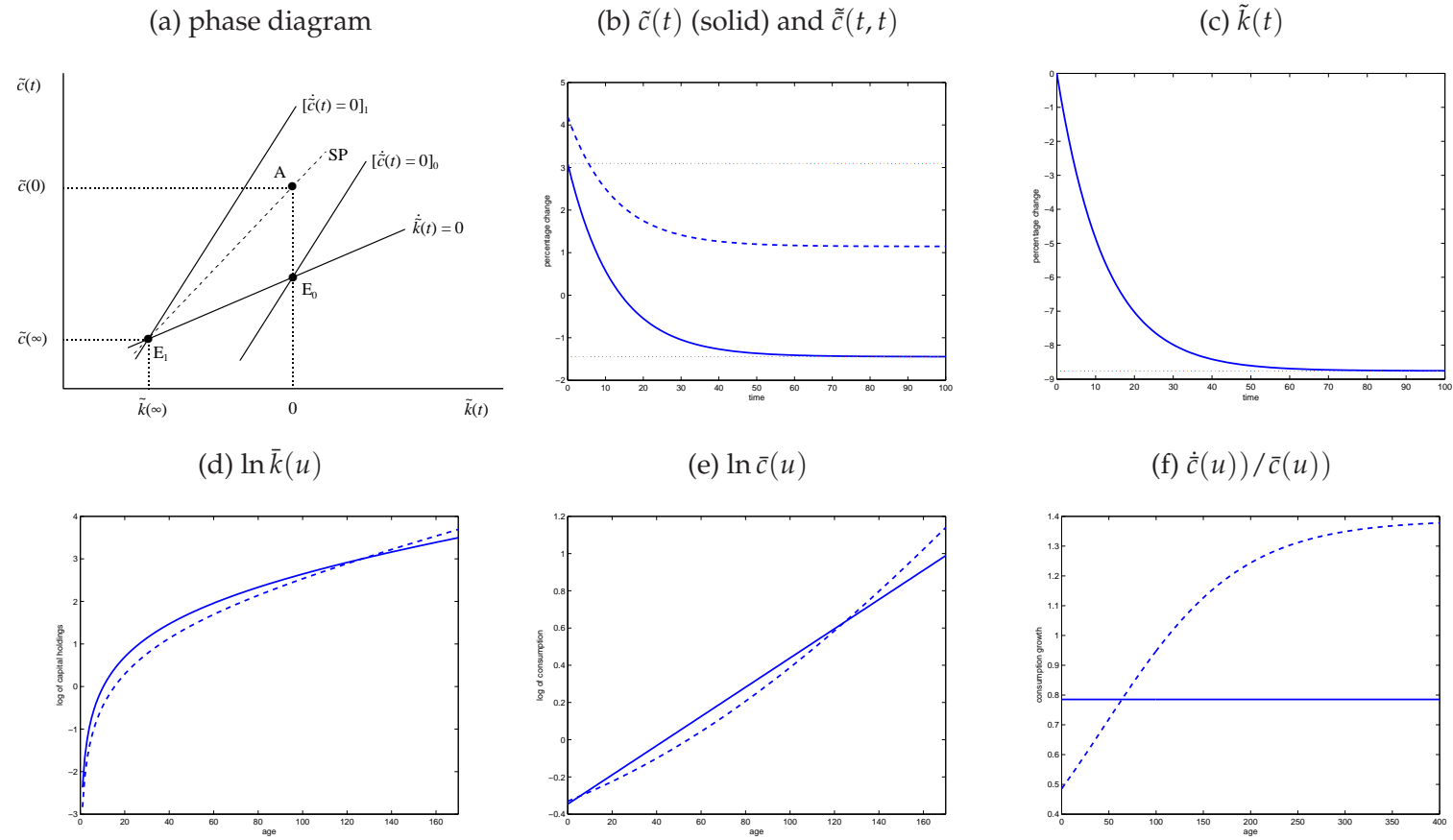


Figure 2: Increase in  $\alpha$  from 0 to 0.5

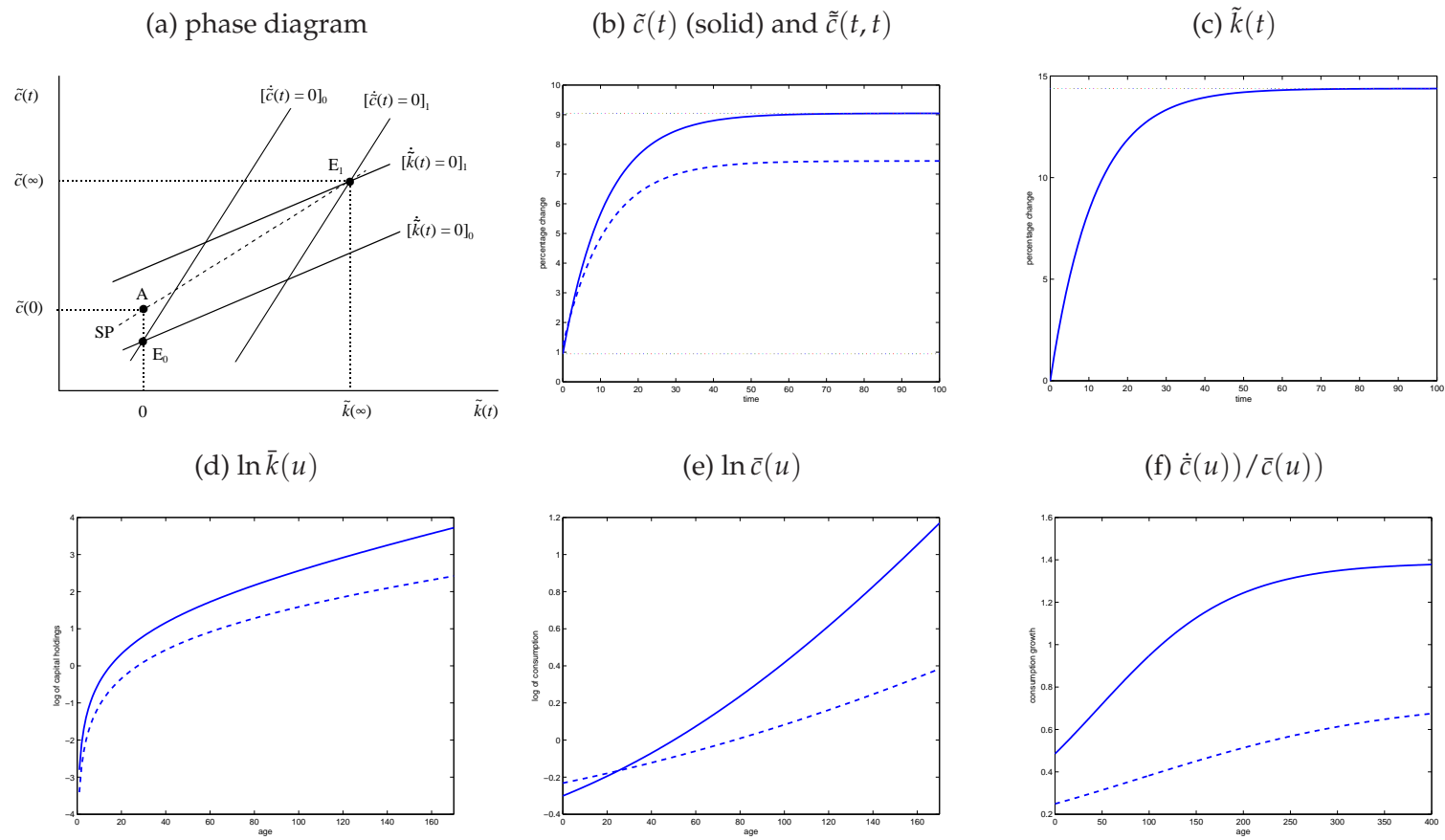


Figure 3: Decrease in  $\eta$  from 3% to 1.5% per annum

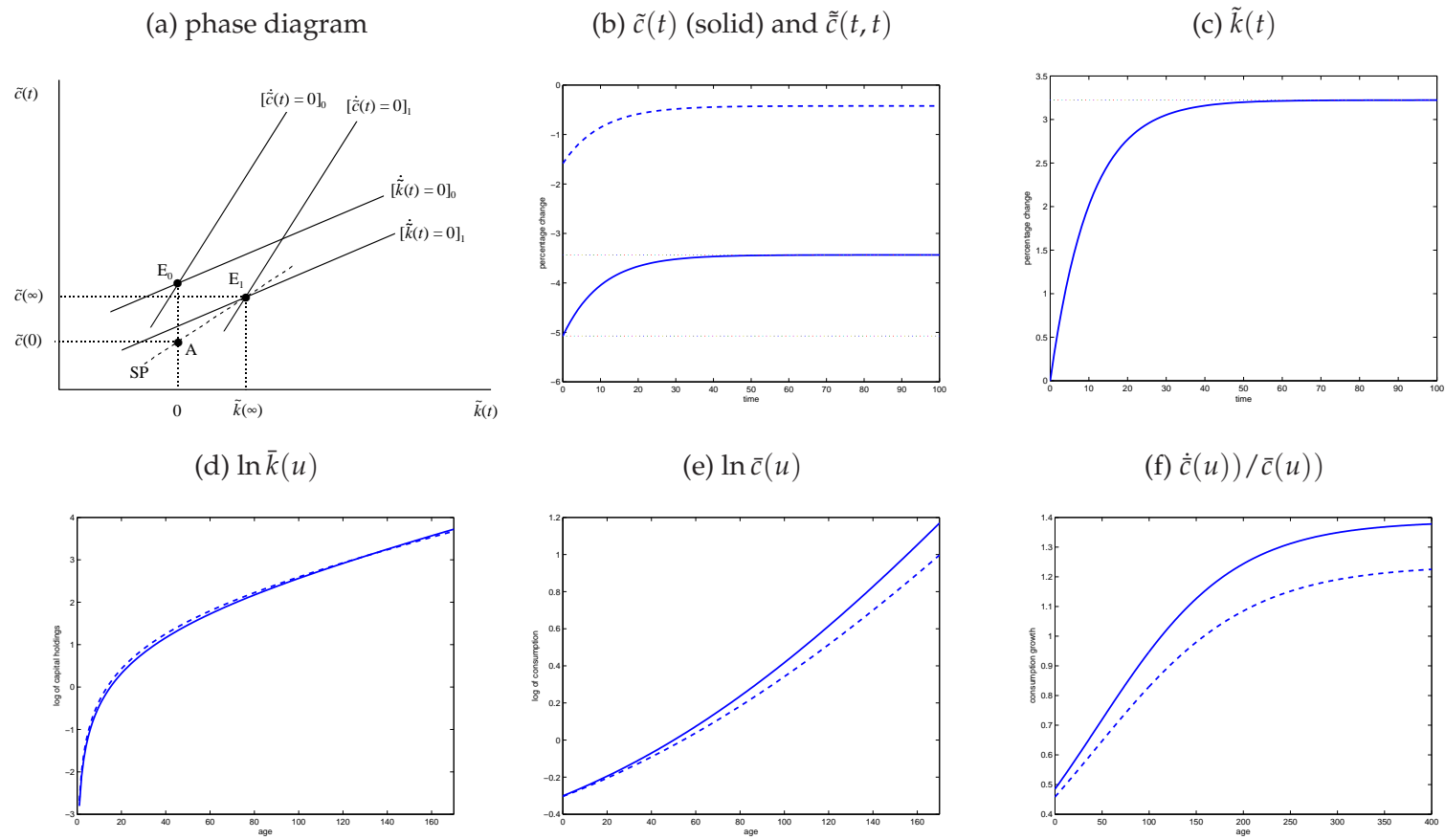


Figure 4: Decrease in  $\beta$  from 2% to 1% per annum

## Appendix

### A.1 Derivation of Equations (13)–(14) for $\dot{c}(t) / c(t)$ and $\dot{k}(t)$

To derive the aggregate Euler equation, we differentiate the expression for  $c(t)$  in (12) with respect to time to calculate  $\dot{c}(t)$ :

$$\begin{aligned}\dot{c}(t) &\equiv l(t, t) \bar{c}(t, t) + \int_{-\infty}^t l(v, t) \dot{\bar{c}}(v, t) dv + \int_{-\infty}^t \dot{l}(v, t) \bar{c}(v, t) dv \\ &= \eta \bar{c}(t, t) + \int_{-\infty}^t l(v, t) \dot{\bar{c}}(v, t) dv - \eta \int_{-\infty}^t l(v, t) \bar{c}(v, t) dv \\ &= \int_{-\infty}^t l(v, t) \dot{\bar{c}}(v, t) dv - \eta [c(t) - \bar{c}(t, t)],\end{aligned}\tag{A.1}$$

where we have substituted for (11)–(12) to obtain (A.1). Since  $\dot{\bar{c}}(v, t) \equiv (1 - \alpha) \dot{\bar{x}}(v, t) + \alpha \dot{c}(t)$  from (3), the first term on the right-hand side of (A.1) is simplified in the following way:

$$\begin{aligned}\int_{-\infty}^t l(v, t) \dot{\bar{c}}(v, t) dv &= (1 - \alpha) \int_{-\infty}^t l(v, t) \dot{\bar{x}}(v, t) dv + \alpha \int_{-\infty}^t l(v, t) \dot{c}(t) dv \\ &= (1 - \alpha) [r(t) - \rho] \int_{-\infty}^t l(v, t) \bar{x}(v, t) dv + \alpha \dot{c}(t) \int_{-\infty}^t l(v, t) dv \\ &= (1 - \alpha) [r(t) - \rho] x(t) + \alpha \dot{c}(t),\end{aligned}\tag{A.2}$$

where we substitute for  $\dot{\bar{x}}(v, t)$  using (5) to obtain the second equality of (A.2). The third equality of (A.2) follows from the definition of  $x(t)$  and the fact that cohort weights sum-up to unity. Thus, substitution of (A.2) into (A.1) and noting that  $x(t) = c(t)$  holds in aggregate, yields the following economy-wide differential equation for consumption:

$$\dot{c}(t) = [r(t) - \rho] c(t) - \frac{\eta}{1 - \alpha} \cdot [c(t) - \bar{c}(t, t)],\tag{A.3}$$

where the term  $c(t) - \bar{c}(t, t)$ , representing intergenerational turnover, is the difference between average consumption and the consumption of new agents. To derive a more convenient expression for  $c(t) - \bar{c}(t, t)$ , we employ (8) to find:

$$c(t) = (\rho + \beta) [a(t) + h(t)] + \alpha [c(t) - (\rho + \beta) \Gamma(t)],\tag{A.4}$$

$$\bar{c}(t, t) = (\rho + \beta) h(t) + \alpha [c(t) - (\rho + \beta) \Gamma(t)],\tag{A.5}$$

where  $\bar{a}(t, t) = 0$ , since new cohorts are born without financial wealth. Combining (A.4)–(A.5), we obtain  $[c(t) - \bar{c}(t, t)] = (\rho + \beta) a(t)$ , which permits us to rewrite (A.3) as:

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho - \frac{\eta(\rho + \beta)}{1 - \alpha} \cdot \frac{a(t)}{c(t)}.\tag{A.6}$$

Using the cohort weights (11), aggregate financial assets given in (12) evolve according to:

$$\begin{aligned}
\dot{a}(t) &\equiv l(t, t) \bar{a}(t, t) + \int_{-\infty}^t l(v, t) \dot{\bar{a}}(v, t) dv + \int_{-\infty}^t \dot{l}(v, t) \bar{a}(v, t) dv \\
&= \int_{-\infty}^t l(v, t) \left[ [r(t) + \beta - \eta] \bar{a}(v, t) + w(t) - \bar{c}(v, t) \right] dv \\
&= [r(t) - n] a(t) + w(t) - c(t),
\end{aligned} \tag{A.7}$$

where, to obtain the second and third equalities in (A.7), we substitute, respectively, for the household's flow budget identity (4) and use the fact that  $n \equiv \eta - \beta$ .

## A.2 Additional Results for Sections 4 and 5

### A.2.1 Characteristic Equation

The characteristic roots of  $\Delta$  are  $-\lambda_1 < 0$  (stable) and  $\lambda_2 > 0$  (unstable), where  $\det \Delta$  is given by:

$$|\Delta| \equiv \frac{r}{\varepsilon} [(r - \rho)(\varepsilon - \omega_C) - r(1 - \varepsilon)\omega_C] - (r - \rho)n.$$

From the fact that  $y = c + nk$  (goods market clearing) can be rewritten as  $n = [1 - \omega_C] \frac{r}{\varepsilon}$  and using  $r = \varepsilon y/k$ ,  $|\Delta|$  can be expressed as:

$$|\Delta| = -\frac{r(1 - \varepsilon)}{\varepsilon} [r - \rho + r\omega_C] < 0,$$

confirming the saddlepoint property. We next state the following lemma regarding the solutions to the characteristic polynomial,  $\Psi(s) \equiv \det(sI - \Delta)$ .

**Lemma A.1** The characteristic roots  $-\lambda_1$  and  $\lambda_2$  are solutions to  $\Psi(s) = 0$ . Since the model is saddle-point stable ( $\det \Delta < 0$ ), we find: (i)  $\lambda_2 > \delta_{22} \equiv r - n$ ;

**Proof:** Clearly,  $\Psi(0) = \det \Delta < 0$  (by saddle-point stability) and  $\Psi(\lambda_2) = 0$  (by definition). It follows that  $\lambda_2 > \delta_{22} \iff \Psi(\delta_{22}) < 0$ . By substitution we find that  $\Psi(\delta_{22}) = -\delta_{12}\delta_{21} < 0$ .  $\square$

### A.2.2 Necessary Bounds on $\alpha$

To obtain the necessary bounds on  $\alpha$ , we first determine the conditions on  $\alpha$  for which  $r < \rho + \eta$ . From the steady-state condition:

$$(r - \rho)[y - nk] = \frac{\eta(\rho + \beta)k}{1 - \alpha},$$



and using  $r = \varepsilon y/k$ , it is straightforward to derive the following polynomial determining the steady-state interest rate  $r$ :

$$\Phi(s) \equiv [s - \rho] [s - \varepsilon(\eta - \beta)] - \frac{\varepsilon\eta(\rho + \beta)}{1 - \alpha}, \quad (\text{A.8})$$

where, by definition,  $\Phi(s) = 0$  for  $s = r$ . Evaluating  $\Phi(0)$  for  $\alpha > 0$ , it is easy to show  $\Phi(0) < 0$ , which means there is a single positive root for which  $\Phi(r) = 0$ . Clearly, then,  $r < \rho + \eta \Leftrightarrow \Phi(\rho + \eta) > 0$ . Substituting  $r_1 = \rho + \eta$  in (A.8) and simplifying the resulting expression, we find:

$$\frac{\Phi(r_1)}{\eta} = (1 - \varepsilon)(\rho + \eta) - \varepsilon(\rho + \beta) \frac{\alpha}{1 - \alpha},$$

which implies:

$$r < \rho + \eta \quad \Leftrightarrow \quad \frac{\alpha}{1 - \alpha} < \frac{1 - \varepsilon}{\varepsilon} \cdot \frac{\rho + \eta}{\rho + \beta}. \quad (\text{A.9})$$

Observe that (A.9) always holds if  $\alpha < 0$ . Moreover, if population growth is zero ( $\eta = \beta$ ), the condition collapses to  $1 - \varepsilon - \alpha > 0$ .

To find the lower bound on  $\alpha$ , we determine the circumstances in which  $r < r_2 \equiv \rho + \eta + \frac{\alpha\eta}{1 - \alpha}$  when  $\alpha < 0$ . We find:

$$\Phi(r_2) = \frac{\eta}{1 - \alpha} \left[ (1 - \varepsilon)(\rho + \eta) + \frac{\alpha\eta}{1 - \alpha} \right],$$

so that:

$$r < \rho + \eta + \frac{\alpha\eta}{1 - \alpha} \quad \Leftrightarrow \quad \frac{\alpha}{1 - \alpha} > -\frac{(1 - \varepsilon)(\rho + \eta)}{\eta}. \quad (\text{A.10})$$

Combining (A.9) and (A.10), we obtain the necessary condition for the bounds on  $\alpha$ .  $\square$

### A.2.3 Derivation of Equation (29)

To derive equation (29), we first note the following steady-state relationships:

$$\begin{aligned} (r - \eta + \beta)k &= c - w, \\ (1 - \alpha)(r - \rho)c &= \eta(\rho + \beta)k, \\ \dot{\bar{k}}(u) &= (r + \beta)\bar{k}(u) + w - \bar{c}(u), \\ \bar{c}(u) - c &= (\rho + \beta)[\bar{k}(u) - k], \end{aligned}$$

where  $u$  is the age domain,  $u \geq 0$ , and  $\rho < r < \rho + \eta$ . Clearly, we can rewrite  $\dot{\bar{k}}(u)$  as:

$$\dot{\bar{k}}(u) = (r - \rho)\bar{k}(u) + (\eta + \rho - r)k.$$

Solving for  $\bar{k}(u)$  and  $\bar{c}(u)$ , we obtain the expressions stated in (29), on which the simulations in Figures 2(d)–(f), 3(d)–(f), and 4(d)–(f) are based.

#### A.2.4 Consumption of Newborns

Finally, we can express, using (8), the consumption of newborns as  $\bar{c}(t, t) = c(t) - (\rho + \beta)k(t)$ .

Log-linearizing this relationship, we obtain:

$$\tilde{c}(t, t) = \frac{c}{\bar{c}(0)}\tilde{c}(t) - \frac{k}{\bar{c}(0)}[(\rho + \beta)\tilde{k}(t) + \beta\tilde{\beta}],$$

where  $c$ ,  $\bar{c}(0)$ , and  $k$  are the initial steady-state values for, respectively, per-capita consumption, newborn consumption, and the per-capita capital stock.

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