Environmental macroeconomics with multiple equilibria

(b) A stochastic model in discrete time

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Outline

Disagreement amoung experts

"The window within which we may limit global temperature increases to 2 ^{o}C above preindustrial times is still open, but is closing rapidly. Urgent and strong action in the next two decades [...] is necessary if the risks of dangerous climate change are to be radically reduced."

Nicholas Stern, *Why Are We Waiting?* (2015, p. 32)

"...we are entering the Climate Casino. By this, I mean that economic growth is producing unintended but perilous changes in the climate and earth systems [which] will lead to unforeseeable and probably dangerous consequences. We are rolling the climatic dice, the outcome will produce surprises, and some of them are likely to be perilous. But we have just entered the Climate casino, and there is still time to turn around and walk back out."

William Nordhaus, *The Climate Casino* (2013, pp. 3-4)

"...I am a climate lukewarmer. That means I think recent global warming is real, mostly man-made and will continue but I no longer think it is likely to be dangerous and I think its slow and erratic progress so far is what we should expect in the future."

Matt Ridley, *The Times* newspaper (January 19, 2015)

Principles of model-based environmental policy analysis

(P1) Generations are the relevant units of analysis. Brundtland Report (UN, 1987): "Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs." $\longrightarrow$ overlapping generations model with **disconnected** generations

(P2) Abrupt environmental changes are possible $\longrightarrow$ tipping points and **non-linear** environmental dynamics

(P3) Both the economy and the ecological system are inherently stochastic $\longrightarrow$ **stochastic** environmental dynamics

(P4) Environmental quality has strong public good features though individuals seem to care weakly for it $\longrightarrow$ preferences include a **warm glow** motive for private abatement

Overview

- Objective of the second hour today: to study economy / environment interactions in a stochastic and non-linear overlapping generations framework
- Mode of attack:
  - Diamond-Samuelson overlapping generations model with a "warm-glow" motive for private abatement
  - Nonlinear and stochastic ecological dynamics
  - Nonlinear system of stochastic difference equations solved numerically
  - Unmanaged market economy: long-lasting high-pollution epochs
  - Optimally managed economy: stochastic first-best social optimum
  - The virtues of ad hoc second-best rules
  - How robust are these results?

Consumers (1)

- Cohort of L identical individuals born in period t
- Each individual i $(= 1, 2, \ldots, L)$ lives for two periods
- Expected lifetime utility function:

$$\mathbb{E}_t[\Lambda_t^{y,i}] \equiv \ln c_t^{y,i} + \chi \ln m_t^i + \beta \left[\ln c_{t+1}^{o,i} + \zeta \mathbb{E}_t \left[Q_{t+1} \right] \right] \quad \text{(S1)}$$

  - $c_t^{y,i}$ is youth consumption
  - m_t^i is private environmental abatement
  - $c_{t+1}^{o,i}$ is old-age consumption
  - Q_{t+1} is future environmental quality (a stochastic variable)
  - β is the utility discount factor $(0 < \beta < 1)$ whilst χ and ζ are utility weights $(\chi > 0$ and $\zeta > 0)$

Consumers (2)

- Budget identities:

$$c_t^{y,i} + s_t^i + m_t^i = w_t - \tau_t \tag{S2a}$$

$$c_{t+1}^{o,i} = (1 + r_{t+1}) s_t^i \tag{S2b}$$

  - s_t^i is saving
  - w_t is the wage rate
  - τ_t is the lump-sum tax
  - r_{t+1} is the future real interest rate
  - individuals work and are taxed only during youth
  - both s_t^i and m_t^i both constitute investment opportunities

Consumers (3)

- Individuals know the environmental transition equation:

$$Q_{t+1} = H(Q_t) - \phi_0 - D_t + \varepsilon_{t+1} \tag{S3a}$$

  - $H(Q_t)$ captures the regenerative capacity of the environment ($H'(Q_t) > 0$)
  - D_t is the pollution flow resulting from economic activities
  - ε_{t+1} is a lognormally distributed random variable (with mean ϕ_0 and standard deviation ν)

- Individuals know the determinants of the pollution flow:

$$D_t = \xi Y_t e^{-\gamma M_t - \eta G_t} \tag{S3b}$$

  - Y_t is aggregate output
  - G_t is public abatement
  - $M_t \equiv m_t^i + M_t^{\neg i}$ is total private abatement ($M_t^{\neg i} \equiv \sum_{j \neq i}^{L} m_t^j$)
  - ξ, γ, and η are constant positive parameters
  - public abatement more effective than private abatement: $\eta > \gamma$

Consumers (4)

- Young individual i chooses $c_t^{y,i}$, m_t^i, $c_{t+1}^{o,i}$, and s_t^i in order to maximize expected utility (S1):
  - subject to the budget identities (S2)
  - subject to the environmental transition function (S3)
  - and taking as given $M_t^{\neg i}$ and G_t
- Key first-order conditions:

$$\frac{1}{c_t^{y,i}} = \frac{\beta(1+r_{t+1})}{c_{t+1}^{o,i}} \tag{S4a}$$

$$\frac{1}{c_t^{y,i}} = \frac{\chi}{m_t^i} + \beta\zeta \frac{\partial \mathbb{E}_t\left[\ln Q_{t+1}\right]}{\partial m_t^i} \tag{S4b}$$

with:

$$\frac{\partial \mathbb{E}_t\left[\ln Q_{t+1}\right]}{\partial m_t^i} = \mathbb{E}_t\left[\frac{\gamma\xi Y_t e^{-\gamma(m_t^i+M_t^{\neg i})-\eta G_t}}{H(Q_t)-\phi_0-\xi Y_t e^{-\gamma(m_t^i+M_t^{\neg i})-\eta G_t}+\varepsilon_{t+1}}\right]$$

Firms

- Perfectly competition and constant returns to scale
- Technology:

$$Y_t = \Omega K_t^{\alpha} N_t^{1-\alpha} \tag{S5}$$

  - Y_t is homogeneous output
  - K_t is the capital stock
  - N_t is employment
  - α is the efficiency parameter of capital $(0 < \alpha < 1)$
  - Ω is the aggregate level of technology in the economy
- Factor demands:

$$w_t = (1 - \alpha)\, \Omega k_t^{\alpha} \tag{S6a}$$

$$r_t + \delta = \alpha \Omega k_t^{\alpha - 1} \tag{S6b}$$

  - $k_t \equiv K_t / N_t$ is the capital intensity
  - w_t is the real wage rate; r_t is the real interest rate
  - $\delta > 0$ is the depreciation rate

Loose ends (symmetry imposed)

- Output per worker:

$$y_t = f\left(k_t\right) \equiv \Omega k_t^{\alpha} \tag{S7a}$$

- Labour market equilibrium:

$$N_t = L \tag{S7b}$$

- Goods market equilibrium:

$$Y_t = L\left(c_t^o + c_t^y\right) + Lm_t + I_t + G_t \tag{S7c}$$

- Investment:

$$K_{t+1} = I_t + \left(1-\delta\right) K_t \tag{S7d}$$

Environmental regeneration function

- A standard discrete-time SLD regeneration function would be:

$$P_{t+1} = (1-\pi)P_t + \frac{P_t^2}{P_t^2+1} + D_t + \phi_0 - \varepsilon_{t+1}, \qquad \frac{1}{2} < \pi < \frac{3\sqrt{3}}{8}$$

with $Q_t \equiv \bar{Q} - P_t$ (disadvantage: only one parameter)

- A more flexible function is used:

$$Q_{t+1} = H(Q_t) - \phi_0 - D_t + \varepsilon_{t+1} \tag{S8}$$

with:

$$H(Q_t) \equiv \phi_5 Q_t^5 + \phi_4 Q_t^4 + \phi_3 Q_t^3 + \phi_2 Q_t^2 + (1+\phi_1)Q_t + \phi_0$$

  - ϕ_i parameters are chosen such that the fundamental difference equation for Q_t is S-shaped – see **Figure 3(c)**
  - ...and, for a given net dirt flow, features two stable steady states – see **Figure 3(d)**

Figure 3(c): Nonlinear $H(Q_t)$

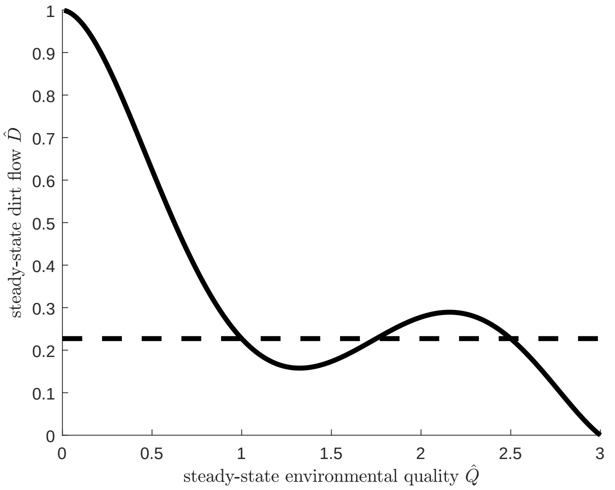
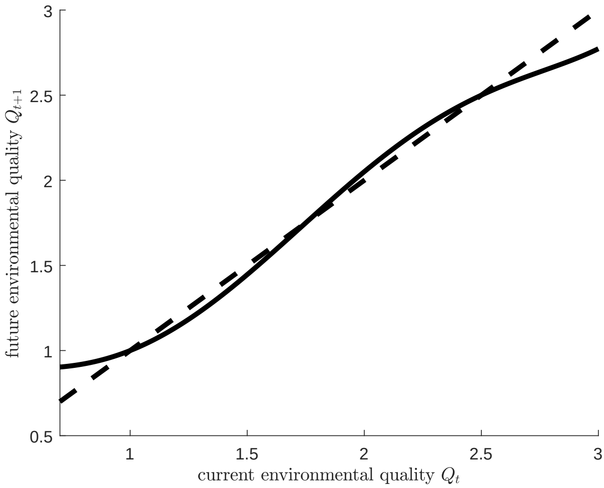


Figure 3(d): Nonlinear FDE for Q_t



The unregulated market and the environment

- What if the government does nothing ($G_t = \tau_t = 0$)?
- The **Stochastic Unmanaged Market Economy** (SUME) is described by:

$$\frac{\chi}{m_t} + \beta\zeta\mathcal{M}(m_t, k_t, Q_t) = \frac{1+\beta}{(1-\alpha)\Omega k_t^\alpha - m_t} \tag{S9a}$$

$$c_t^y = \frac{(1-\alpha)\Omega k_t^\alpha - m_t}{1+\beta} \tag{S9b}$$

$$k_{t+1} = (1-\alpha)\Omega k_t^\alpha - m_t - c_t^y \tag{S9c}$$

$$Q_{t+1} = H(Q_t) - \phi_0 - \xi L \Omega k_t^\alpha e^{-\gamma L m_t} + \varepsilon_{t+1} \tag{S9d}$$

where $\mathcal{M}(m_t, k_t, Q_t)$ is an auxiliary function:

$$\mathcal{M}(m_t, k_t, Q_t) \equiv \mathbb{E}_t\left[\frac{\gamma\xi L\Omega k_t^\alpha e^{-\gamma L m_t}}{H(Q_t) - \phi_0 - \xi L\Omega k_t^\alpha e^{-\gamma L m_t} + \varepsilon_{t+1}}\right] \tag{S9e}$$

Model solution (1)

1. For given (k_t, Q_t) solve (S9a) for m_t by simulating lognormally distributed random variables and conducting quasi Monte Carlo integration to compute the $\mathcal{M}(m_t, k_t, Q_t)$ function $\longrightarrow$ yields the 'policy function' for private abatement:

$$m_t = \mathbf{m}(k_t, Q_t) \tag{S10a}$$

2. Substitute $\mathbf{m}(k_t, Q_t)$ into (S9b)–(S9d) to get the remaining 'policy functions':

$$\mathbf{c}_t^y(k_t, Q_t) \equiv \frac{(1-\alpha)\Omega k_t^\alpha - \mathbf{m}(k_t, Q_t)}{1+\beta} \tag{S10b}$$

$$\mathbf{k}^+(k_t, Q_t) \equiv (1-\alpha)\Omega k_t^\alpha - \mathbf{m}(k_t, Q_t) - \mathbf{c}_t^y(k_t, Q_t) \tag{S10c}$$

$$\mathbf{Q}^+(k_t, Q_t) \equiv H(Q_t) - \phi_0 - \xi L \Omega k_t^\alpha e^{-\gamma L \mathbf{m}(k_t, Q_t)} \tag{S10d}$$

Model solution (2)

3. Dynamics of the capital intensity is deterministic:

$$k_{t+1} = \mathbf{k}^{+}(k_t, Q_t) \tag{S10e}$$

4. Dynamics of environmental quality is stochastic:

$$Q_{t+1} = \mathbf{Q}^{+}(k_t, Q_t) + \varepsilon_{t+1} \tag{S10f}$$

5. By generating quasi-random numbers for ε_τ (for $\tau = t + 1$, $t + 2, \ldots$) the dynamic paths for all variables can be simulated

- ▷ **Figures 6(a)–(f)** depict the long-run view ▸ Show
- ▷ **Figures 7(a)–(f)** zoom in on a bad epoch ▸ Show

Table 2: Structural parameters

Economic parameters				
β	discount factor		0.3083	
L	young cohort size		100.0000	
ρ_a	annual time preference (percent)		4.0000	
α	capital share parameter		0.3000	
Ω	production function constant	c	1.7190	
δ_a	annual capital depreciation rate (percent)	c	4.2468	
δ	capital depreciation factor	c	0.7280	
Environmental parameters				
χ	taste parameter for private abatement	c	4.8584	10^{-3}
ζ	taste parameter for future environmental quality		25.0000	
γ	environmental dirt-private-abatement parameter	c	7.5807	10^{-2}
η	environmental dirt-public-abatement parameter	c	8.4230	10^{-2}
ξ	environmental dirt-output parameter	c	2.3190	10^{-3}
θ_a	annual rate of environmental regeneration (percent)		2.0000	
θ	environmental regeneration factor		0.4545	
$\bar{Q}$	maximum environmental quality		3.0000	

Note See Supplementary Material (Appendix A) for details on the parameterization approach. The parameters labeled 'c' are calibrated as is explained in the appendix. The remaining parameters are postulated a priori. The values for δ, θ, and $\beta \equiv 1/(1+\rho)$ follow from, respectively, δ_a, θ_a, and ρ_a, by noting that each model period represents 30 years.

Table 3: Allocation and welfare

		(a) ME_c	(b) ME_d	(c) DSO_l	(d) DSO_n
$\hat{Q}$	environmental quality	2.5000	1.0005	2.7604	2.7570
$\hat{k}$	capital intensity	0.1643	0.1643	0.0642	0.0642
$\hat{r}$	interest factor	1.0976	1.0979	2.7986	2.7986
$\hat{r}^a$	annual interest rate (percent)	2.5000%	2.5005%	4.5492%	4.5492%
$\hat{y}$	output per worker	1.0000	0.9999	0.7541	0.7541
$\hat{w}$	wage rate	0.7000	0.6999	0.5279	0.5279
$\hat{m}$	private abatement	0.2665 10^{-2}	0.2786 10^{-2}	1.5780 10^{-2}	1.5826 10^{-2}
$\hat{c}^y$	youth consumption	0.5330	0.5329	0.3248	0.3257
$\hat{c}^o$	old-age consumption	0.3447	0.3447	0.3248	0.3257
$\hat{g}$	public abatement	0.0000	0.0000	0.0420	0.0401
$\hat{D}$	net dirt flow	0.2273	0.2270	0.1089	0.1106
$\hat{\Lambda}^y$	life-time utility	6.0763	−0.9826	6.3352	6.3294

Note With a linear environmental regeneration function $H(Q_t)$ the unmanaged market economy settles in the unique steady state labeled ME_c. If $H(Q_t)$ is nonlinear there is also a heavily polluted steady state for the unmanaged economy labeled ME_d. DSO_l and DSO_n denote the deterministic first-best social optimum for, respectively, the linear and nonlinear regeneration function.

Figure 6(a): Capital intensity k_t

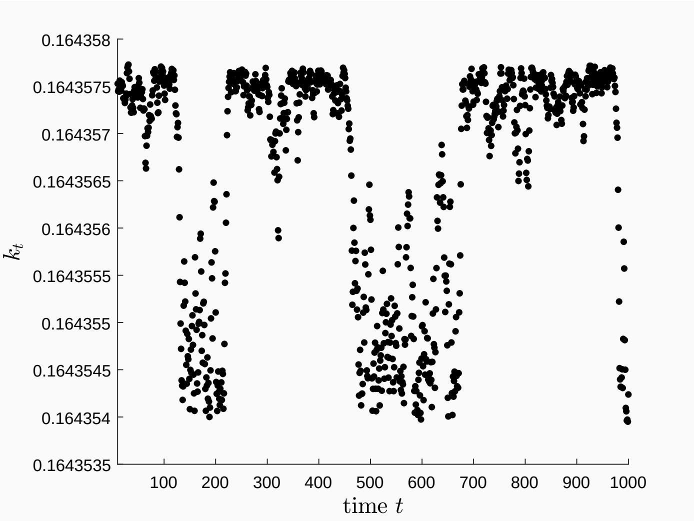


Figure 6(b): Environmental quality Q_t

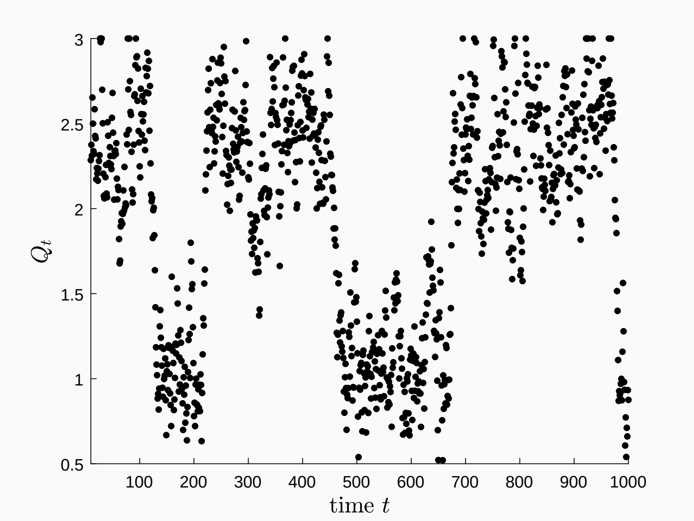


Figure 6(c): Private abatement m_t

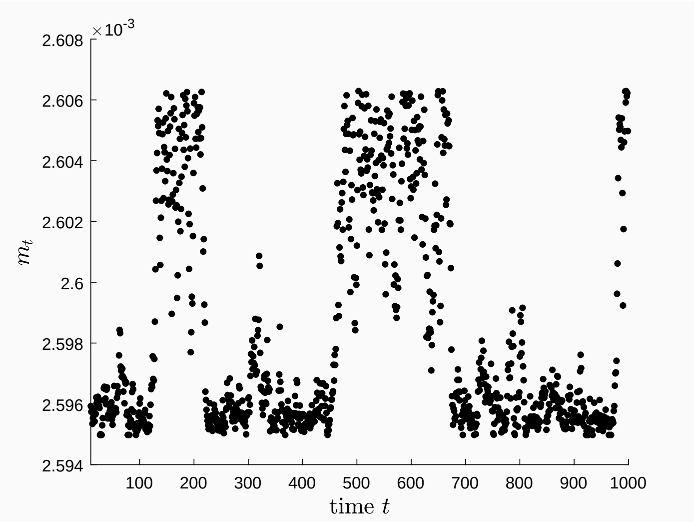


Figure 6(d): Youth consumption c_t^y

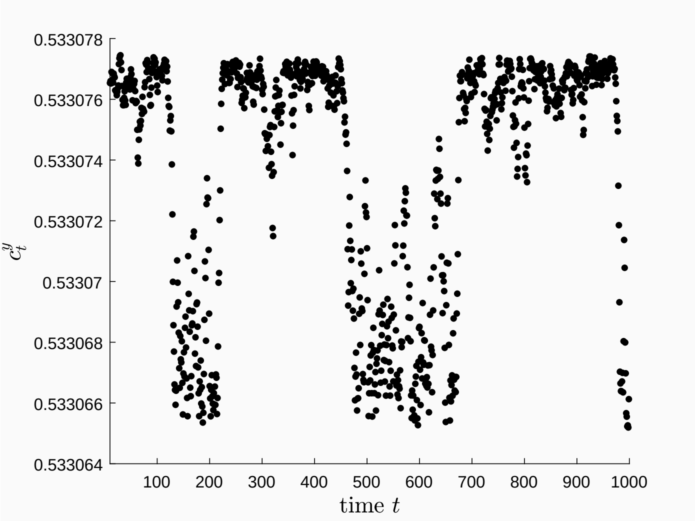


Figure 6(e): Net dirt flow D_t

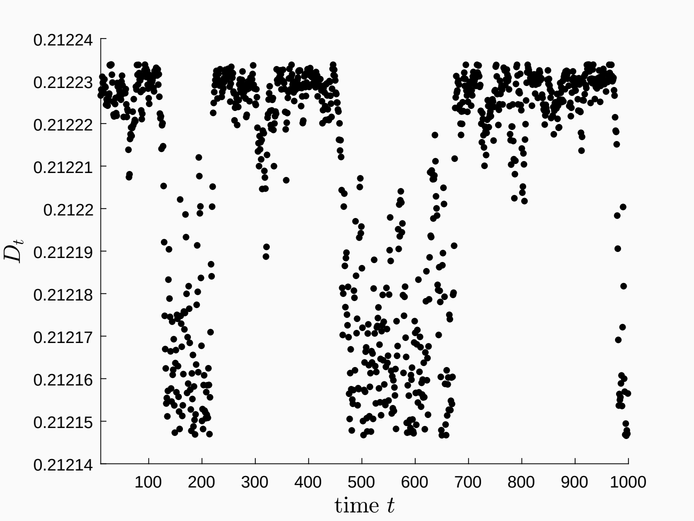


Figure 6(f): Old-age consumption c_t^o

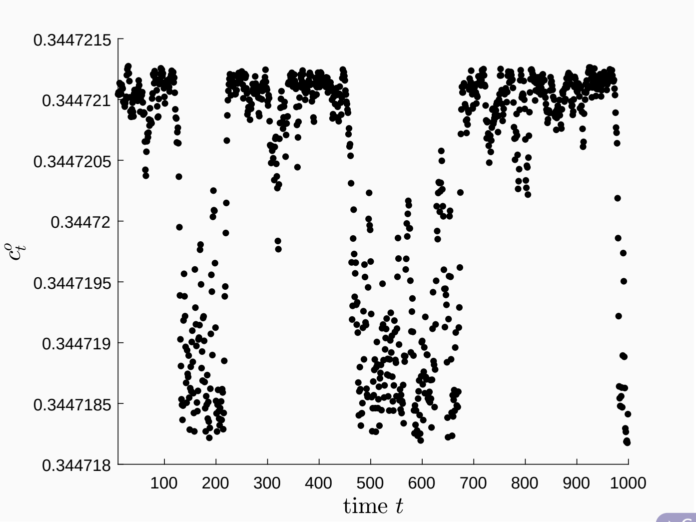


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Figure 7(a): Capital intensity k_t

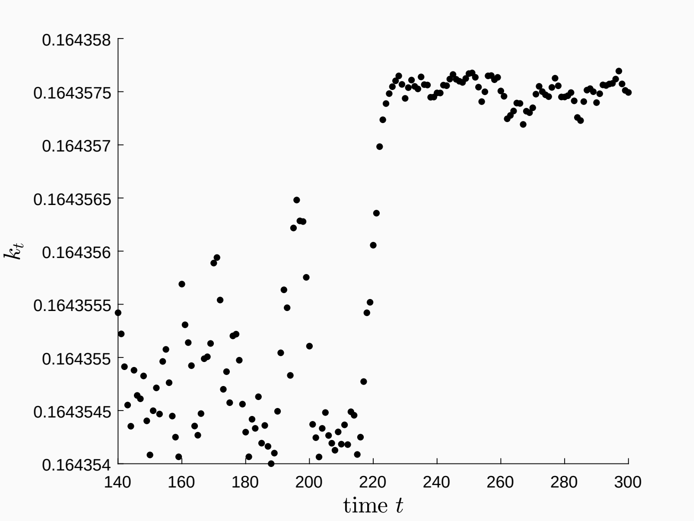


Figure 7(b): Environmental quality Q_t

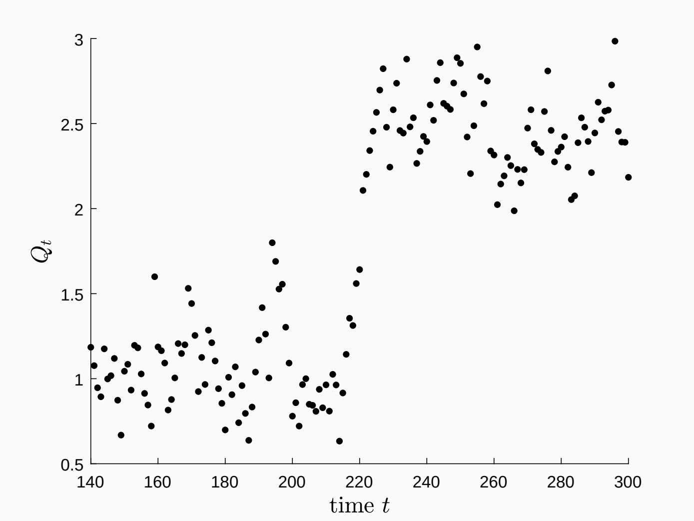


Figure 7(c): Private abatement m_t

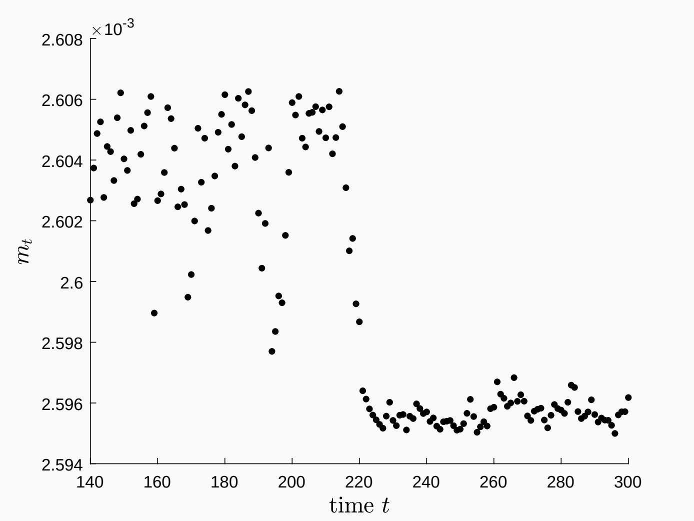


Figure 7(d): Youth consumption c_t^y

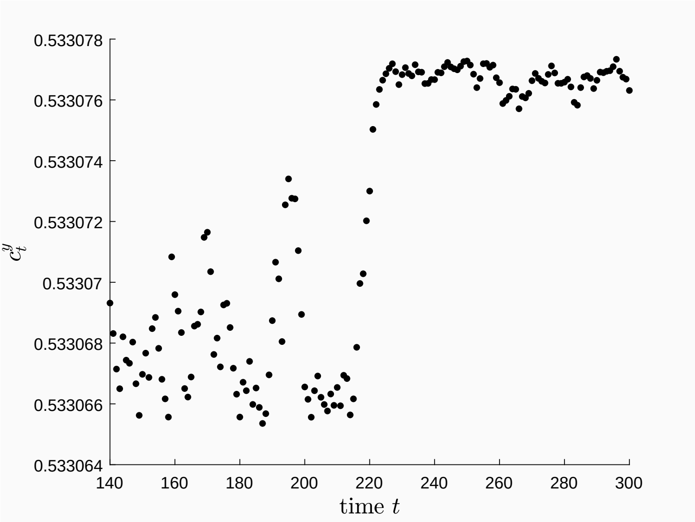


Figure 7(e): Net dirt flow d_t^n

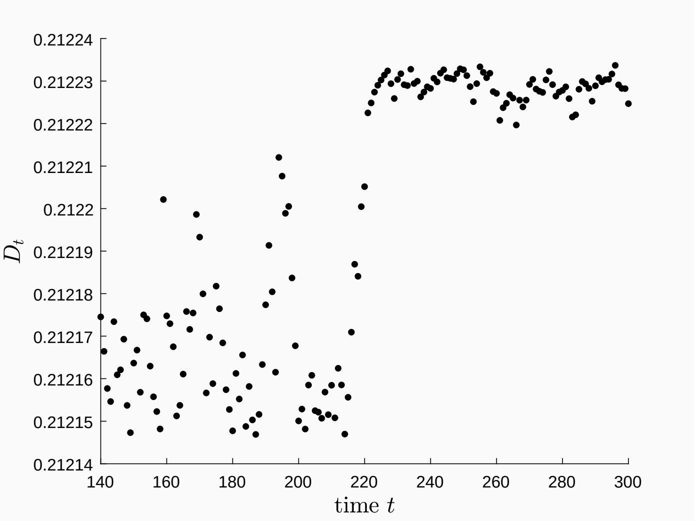
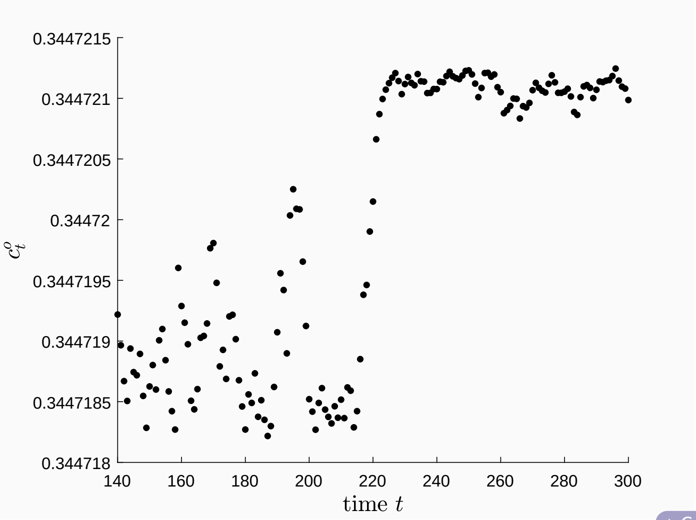


Figure 7(f): Old-age consumption c_t^o



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The optimally managed economy-ecology (1)

- What would happen in a world with a benevolent social planner at the helm?
- A utilitarian approach: social welfare function is the discounted sum of lifetime welfare of current and future generations:

$$\mathbb{E}_t\left[SW_t\right] \equiv \mathbb{E}_t \sum_{\tau=0}^{\infty} \omega^{\tau-1}\left[\ln c^y_{t+\tau-1} + \chi \ln m_{t+\tau-1} + \beta \ln c^o_{t+\tau} + \zeta\beta \ln Q_{t+\tau}\right]$$

$$= \frac{1}{\omega}\left[\ln c^y_{t-1} + \chi \ln m_{t-1}\right] + \mathbb{E}_t \sum_{\tau=0}^{\infty} SF(c^y_{t+\tau}, m_{t+\tau}, c^o_{t+\tau}, Q_{t+\tau})\omega^{\tau}$$

  - ▷ planner's discount rate is ω (should the planner be more patient than households themselves?)
  - ▷ 'reverse discounting' applied to the old in the planning period ensures dynamic consistency of social planning

The optimally managed economy-ecology (2)

- the 'within-period' social felicity function is:

$$SF(\cdot) \equiv \ln c^y_{t+\tau} + \chi \ln m_{t+\tau} + \frac{\beta}{\omega}\Big[\ln c^0_{t+\tau} + \zeta \ln Q_{t+\tau}\Big]$$

- Recursive formulation of the **Stochastic Social Optimum** (SSO):

$$\mathcal{V}(k_t, Q_t) = \max_{\{c^y_t, m_t, c^o_t, g_t\}} SF(c^y_t, m_t, c^o_t, Q_t) + \omega \mathbb{E}_t\left[\mathcal{V}(k_{t+1}, Q_{t+1})\right]$$

$$\begin{aligned} \text{s.t.} \quad & k_{t+1} = f(k_t) + (1-\delta)k_t - c^y_t - c^o_t - m_t - g_t \\ & Q_{t+1} = H(Q_t) - \phi_0 - D_t + \varepsilon_{t+1} \\ & D_t \equiv \xi L f(k_t) e^{-\gamma L m_t - \eta L g_t} \\ & g_t \geq 0 \end{aligned}$$

Model solution

- $\mathcal{V}(k_t, Q_t)$ is the value function
- k_t and Q_t are the state variables
- c_t^y, m_t, c_t^o, and g_t are the control variables
- the policy functions that solve the DP problem are $\mathbf{g}(k_t, Q_t)$, $\mathbf{m}(k_t, Q_t)$, $\mathbf{c}(k_t, Q_t)$, $\mathbf{k}^+(k_t, Q_t)$, $\mathbf{Q}^+(k_t, Q_t)$:

$$\begin{aligned} g_t &= \mathbf{g}(k_t, Q_t) \\ m_t &= \mathbf{m}(k_t, Q_t) \\ c_t^y = c_t^o &= \mathbf{c}(k_t, Q_t) \\ k_{t+1} &= \mathbf{k}^+(k_t, Q_t) \\ Q_{t+1} &= \mathbf{Q}^+(k_t, Q_t) + \varepsilon_{t+1} \end{aligned}$$

▷ Figures 8(a)–(f) depict the policy functions

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Figure 8(a): Public abatement $\mathbf{g}(k_t, Q_t)$

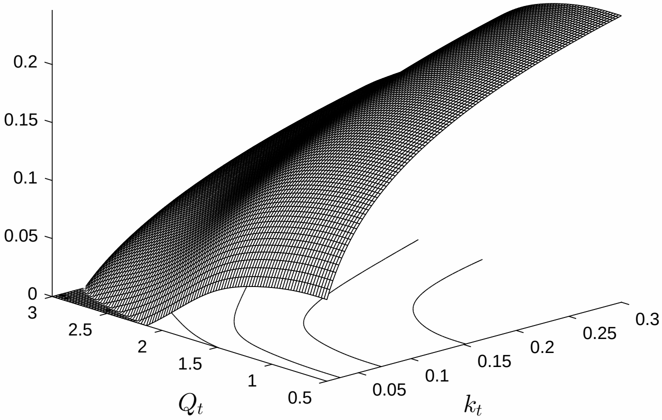


Figure 8(b): Private abatement $\mathbf{m}(k_t, Q_t)$

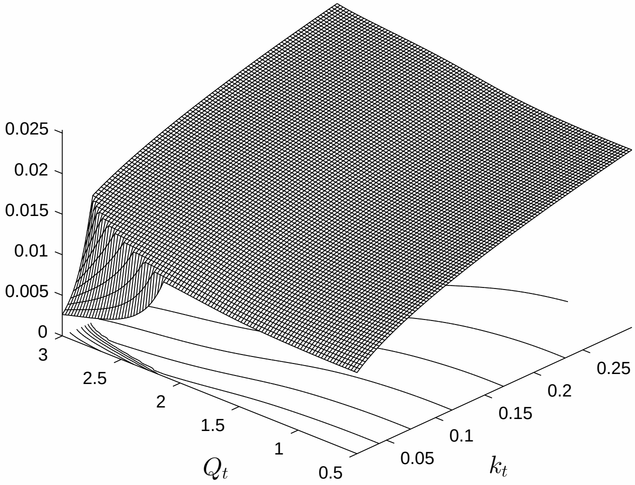


Figure 8(c): Consumption $\mathbf{c}(k_t, Q_t)$

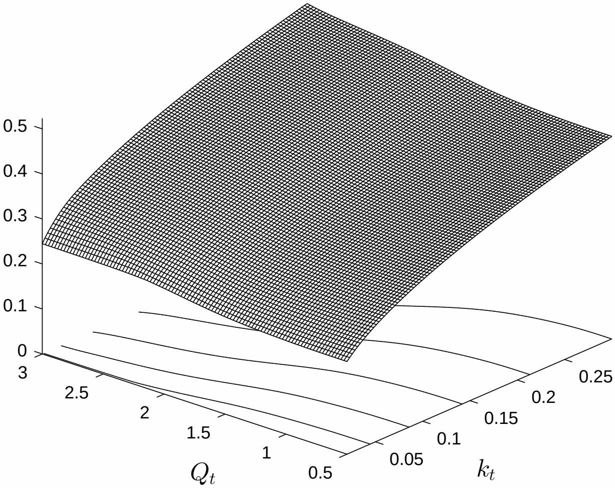


Figure 8(d): Future capital intensity $\mathbf{k}^+(k_t, Q_t)$

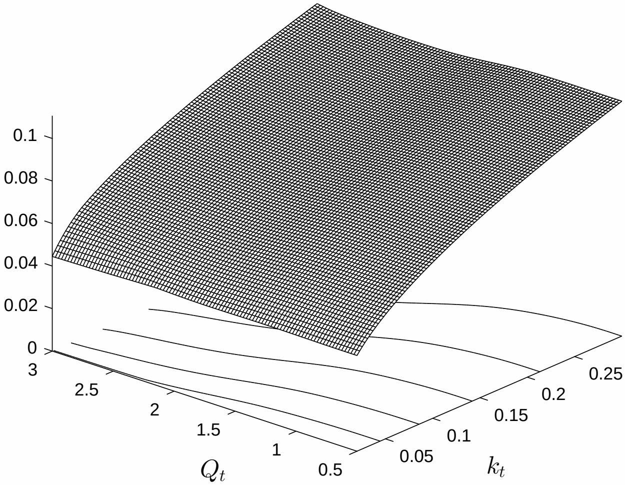
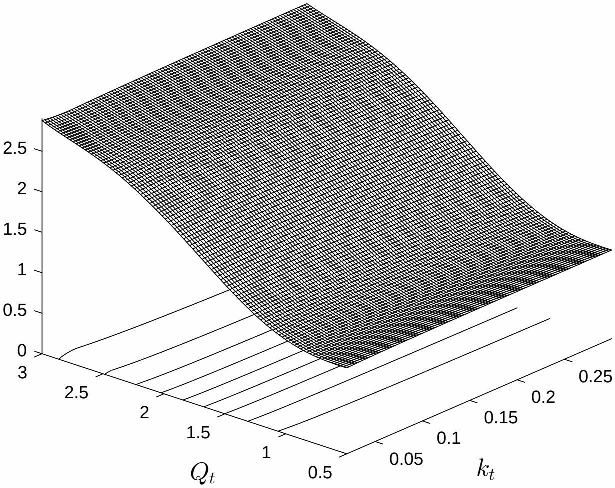


Fig 8(e): Planned future environmental quality $\mathbf{Q}^+(k_t, Q_t)$



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Statistical properties of the SSO

- what are the long-run statistical properties of the economic-ecological system run by a social planner?
- we simulate the model for $T = 10^4$ periods and use a kernel estimation method to compute the resulting probability density functions for the different choice variables
- **Figure 9(a)–(d)** illustrate the PDFs for public abatement g_t, private abatement m_t, the capital intensity k_t, and environmental quality Q_t
  - $g_t > 0$ almost all of the time
  - $m_t > 0$ but low all of the time
  - k_t shows little variability
  - Q_t is single-peaked around the clean steady-state equilibrium

Figure 9(a): Public abatement g_t

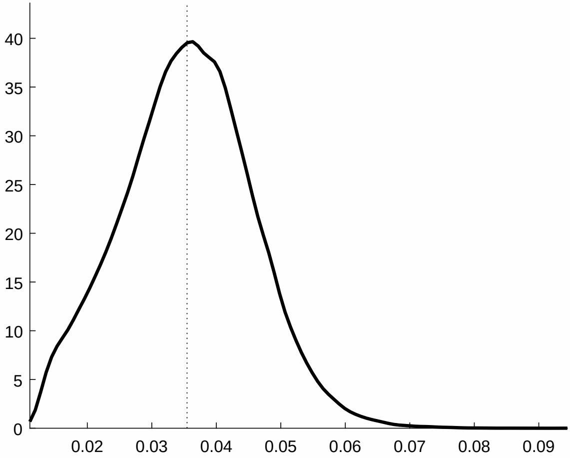


Figure 9(b): Private abatement m_t

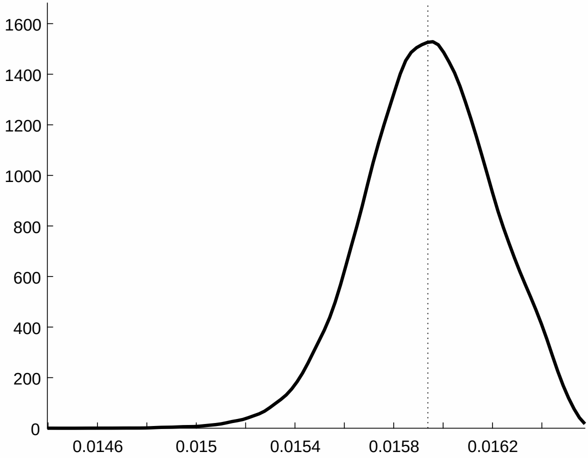


Figure 9(c): Capital k_t

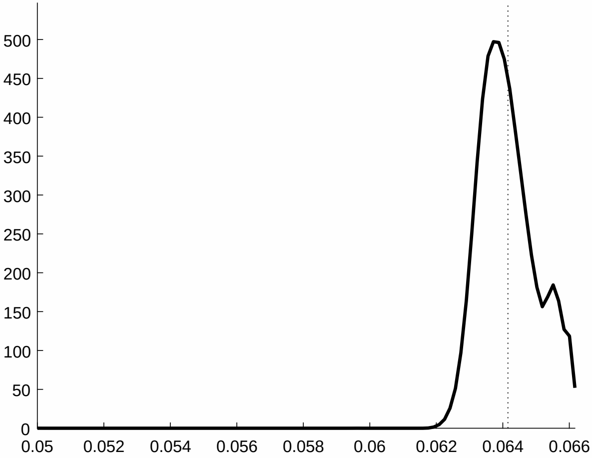
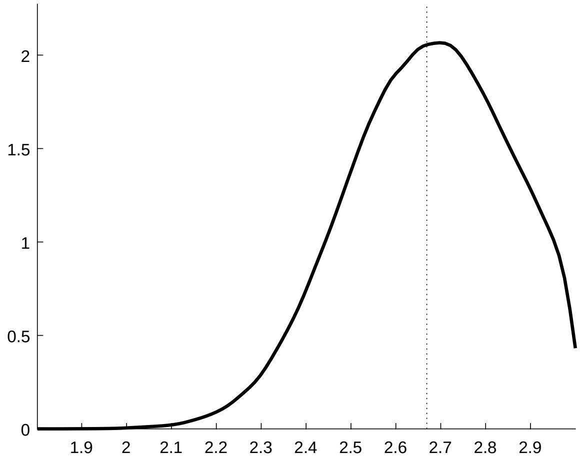


Figure 9(d): Environmental quality Q_t



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Comparison between the SUME and the SSO

- **Figure 10(a)–(b)** illustrate the PDFs for environmental quality Q_t and expected utility at birth $\mathbb{E}_t\left[\Lambda_t^y(k_t, Q_t)\right]$ for the SUME (solid lines) and the SSO (dashed lines)
- Key features:
  - in the SUME both Q_t and $\mathbb{E}_t\left[\Lambda_t^y(k_t, Q_t)\right]$ are multi-modal (epochs again)
  - in the SUME there is a lot of inequality between generations (it matters when you are born)
  - in the SSO both Q_t and $\mathbb{E}_t\left[\Lambda_t^y(k_t, Q_t)\right]$ are single-peaked and feature a tight support
  - in a very small percentage of cases a 'lucky generation' exists which is better off under SUME than under SSO

Figure 10(a): Environmental quality Q_t

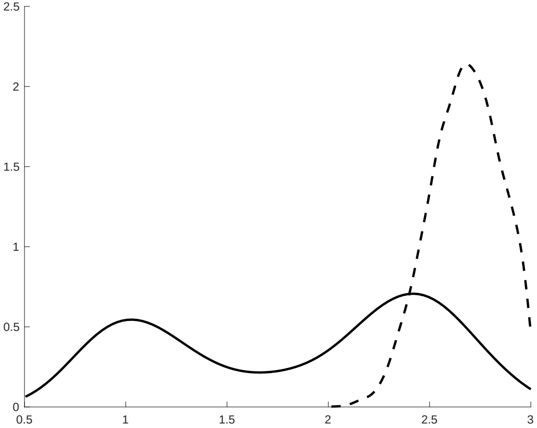
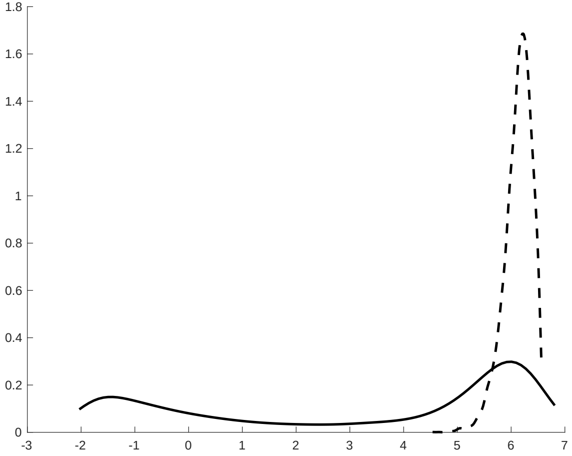


Fig 10(b): Expected lifetime utility at birth $\mathbb{E}_t\left[\Lambda_t^y(k_t, Q_t)\right]$



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An ad hoc policy rule for public abatement?

- In practice decentralization of the SSO is quite complicated
  - g_t must be set optimally
  - m_t must be set optimaly (which instrument is available for encouraging warm-glow spending?)
  - for $\beta = \omega$ (the benchmark) the SSO calls for $c_t^y = c_t^o$ so perfect redistribution instruments must be available
- How well would an ad hoc rule perform?
- Since m_t and k_t show little variation in the SSO we pick:

$$g_t = \pi_0 - \pi_1 Q_t \tag{S11}$$

for $Q_t \in [0.5, \bar{Q}]$ and with $\pi_0 = 0.2601$ and $\pi_1 = -0.0616$

- **Figure 11(c)** illustrates the ad hoc abatement (AHA) rule
- **Figure 11(a)–(b)** give the PDFs for Q_t and $\mathbb{E}_t \left[\Lambda_t^y(k_t, Q_t)\right]$
  - SSO is the dashed PDF
  - AHA is the solid PDF $\longrightarrow$ environmental catastrophes virtually eliminated under this rule

Figure 11(c): Public abatement g_t

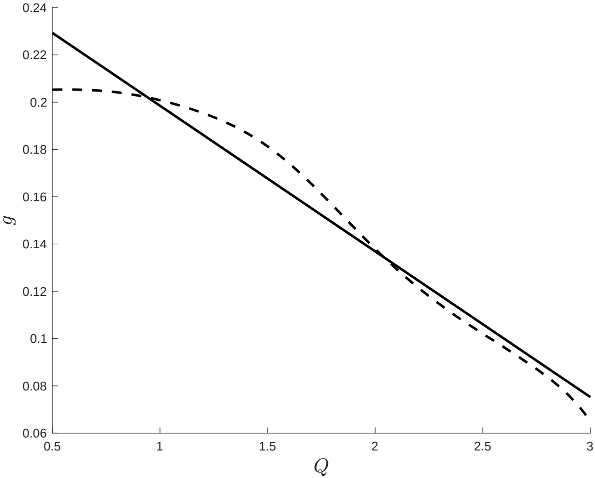


Figure 11(a): Environmental quality Q_t

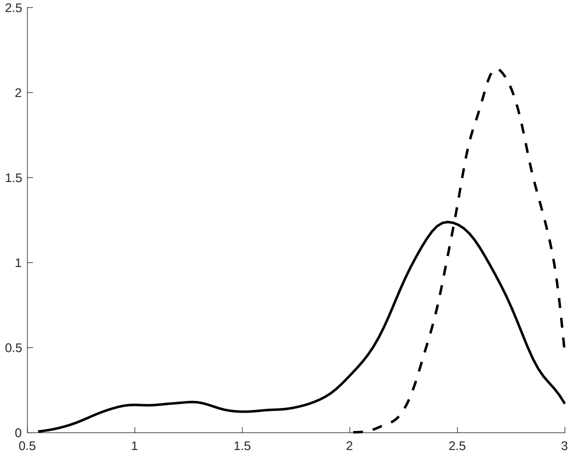
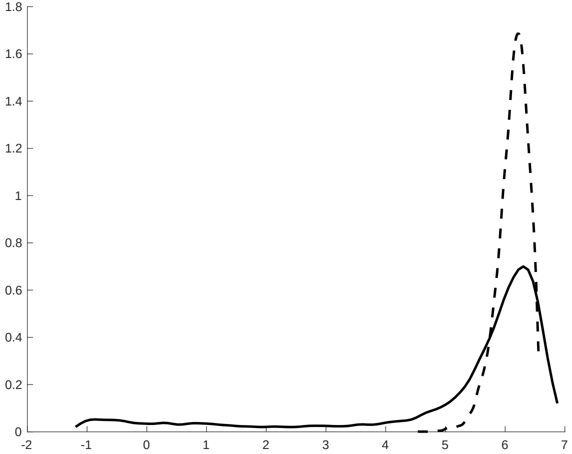


Figure 11(b): Expected lifetime utility at birth $\mathbb{E}_t\left[\Lambda_t^y\right]$



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How robust are our results?

- Key message: the PDF for Q_t is bimodal under SUME but single-peaked in the SSO
- . . . these conclusions are model-specific . . . and depend (critically?) on the values assigned to the structural parameters
- Robustness variations based on alternative parameter choices:
  - **Figure 12(a)**: impatient individuals ($\beta \downarrow$): unimodal PDF with high Q_t also under SUME
  - **Figure 12(b)**: impatient individuals (compensated): unimodal PDF with high Q_t under SUME
  - **Figure 12(c)**: environmentally care-free individuals ($\zeta \downarrow$): unimodal PDF of SUME and SSO with low Q_t
  - **Figure 12(d)**: does free riding matter? Different values for L
  - **Figure 12(e)**: does the planner's impatience matter? Different values for ω

Figure 12(a): Impatient individuals

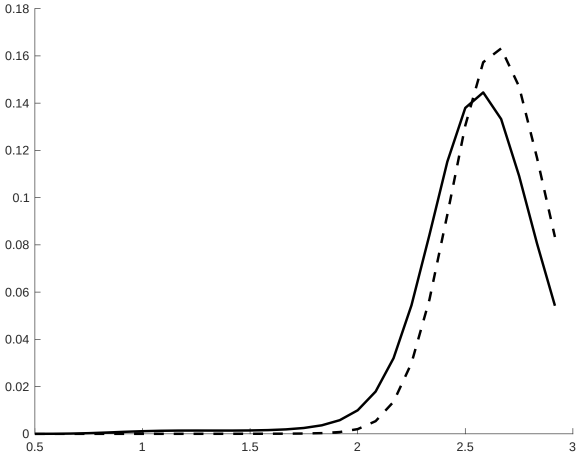

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Figure 12(b): Impatient individuals (compensated)

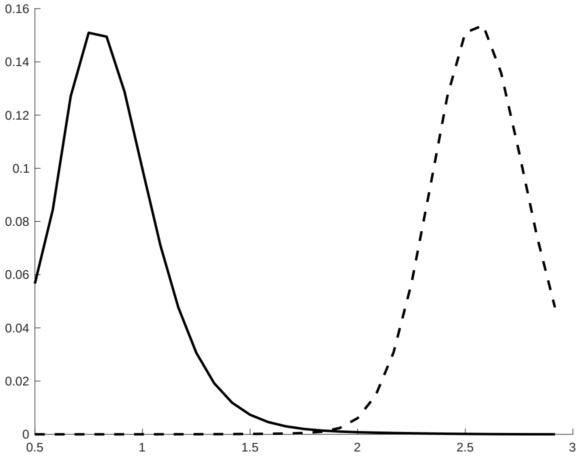

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Figure 12(c): Environmentally care-free individuals

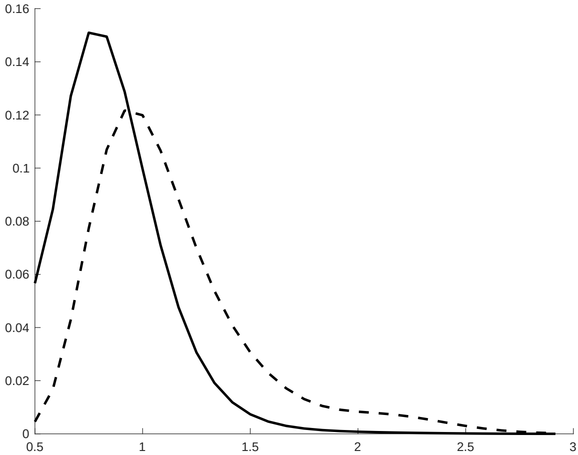

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Figure 12(d): group size L

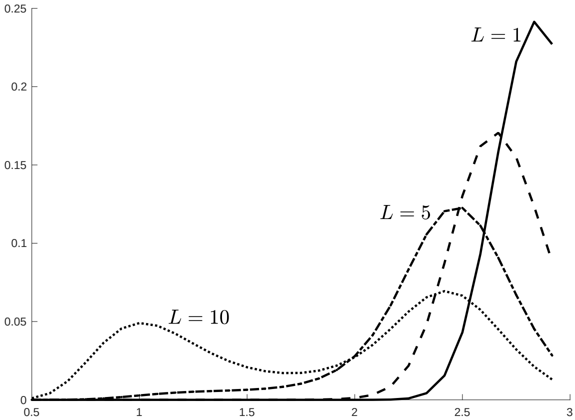

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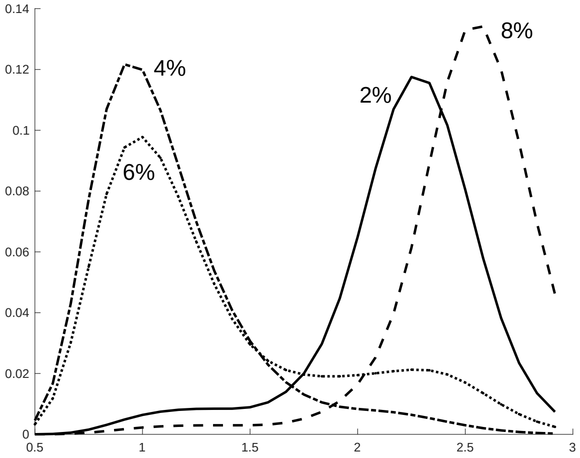
Figure 12(e): social discounting ω



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Conclusions

- With a nonlinear environmental regeneration function, the SUME displays often long-lasting polluted epochs
- Individuals weakly care for the environment they are unable to avoid such low-welfare epochs in an unregulated setting $\longrightarrow$ useful role for government intervention
- A dynamically consistent social planner will ensure that the low-quality trap is eliminated altogether $\longrightarrow$ both public abatement and a pollution tax (on capital) are needed in a decentralized setting
  - $\mathbf{g}(k_t, Q_t)$ is strongly decreasing in Q_t whilst $\mathbf{m}(k_t, Q_t)$ displays the opposite pattern
  - $\mathbf{g}(k_t, Q_t)$ and $\mathbf{m}(k_t, Q_t)$ are increasing in k_t
- An ad hoc linear rule for public abatement, $g_t = \pi_0 + \pi_1 Q_t$, captures most of the benefits attained under the first-best policy $\longrightarrow$ is a constitutional rule useful?

Further work

- Stochastic shocks also in the economic subsystem, e.g. productivity shocks
- Construction and calibration of an N-period overlapping generations model (less severe time aggregation)
- Pollution effects on firms productivity (as in many environmental models)
- Construct a link with the DICE model: richer model of the economy-ecology linkages

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