Foundations of Modern Macroeconomics Second Edition

Chapter 17: Overlapping generations in discrete time (sections 17.1 – 17.2)

Ben J. Heijdra

Department of Economics & Econometrics
University of Groningen

1 September 2009

Outline

Aims of this chapter (1)

- Study second "work-horse" model of overlapping generations based on discrete time. Motivation for doing this:
  - Key model in modern macroeconomics and public finance theory.
  - Better captures life-cycle behaviour.
  - Chain of bequests easier to study.
  - Natural extension to Computable General Equilibrium (CGE) policy models (e.g. Auerbach & Kotlikoff).

Aims of this chapter (2)

- Apply model to various issues:
  - Funded vs. unfunded pensions.
  - Pension reform.
  - Pensions and induced retirement.
  - Ageing and the macroeconomy.
- Study various extensions.
  - Growth and human capital.
  - Public investment.
  - Endogenous fertility.

Households (1)

- Live two periods: "youth" (superscript Y) and "old age" (superscript O).
- Consume in both periods.
- Work only during youth.
- Unlinked with past or future generations (no bequests).
- Save during youth to finance old-age consumption (life-cycle saving).
- Utility function of young agent at time t:

$$\Lambda_t^Y \equiv U(C_t^Y) + \frac{1}{1+\rho} U(C_{t+1}^O) \tag{S1}$$

Households (2)

- Continued.
  - $U(\cdot)$ is felicity function (Inada-style conditions).
  - $\rho > 0$ captures time preference.
- Budget identities:

$$\begin{aligned} C_t^Y + S_t &= w_t \\ C_{t+1}^O &= (1 + r_{t+1}) S_t \end{aligned}$$

  - S_t is saving.
  - w_t is wage income (exogenous labour supply).
  - r_{t+1} is real interest rate.
- Consolidated (lifetime) budget constraint:

$$w_t = C_t^Y + \frac{C_{t+1}^O}{1 + r_{t+1}} \qquad \text{(S2)}$$

Households (4)

- Utility maximization yields consumption Euler equation:

$$\frac{U'(C^O_{t+1})}{U'(C^Y_t)} = \frac{1+\rho}{1+r_{t+1}} \tag{S3}$$

- Savings function:

$$S_t = S(w_t, r_{t+1}) \tag{S4}$$

  - $0 < S_w < 1$: both goods are normal.
  - S_r ambiguous (offsetting income and substitution effects).
  - If intertemporal substitution elasticity is high ($\sigma > 1$) then $S_r > 0$ (and vice versa).

Firms (1)

- Perfect competition, CRTS technology $Y_t = F(K_t, L_t)$, Inada conditions.
- Hire L_t from young (at wage w_t) and K_t from old (at rental rate $r_t + \delta$):

$$\begin{aligned} w_t &= F_L(K_t, L_t) \\ r_t + \delta &= F_K(K_t, L_t) \end{aligned}$$

- Interest rate facing young depends on future (aggregate) capital-labour ratio: $r_{t+1} + \delta = F_K(K_{t+1}, L_{t+1})$.

Firms (2)

- Intensive-form expressions:

$$y_t = f(k_t) \qquad \text{(S5)}$$

$$w_t = f(k_t) - k_t f'(k_t) \qquad \text{(S6)}$$

$$r_{t+1} = f'(k_{t+1}) - \delta \qquad \text{(S7)}$$

where $y_t \equiv Y_t/L_t$ and $k_t \equiv K_t/L_t$.

Aggregate market equilibrium (1)

- Resource constraint:

$$Y_t + (1-\delta)K_t = K_{t+1} + C_t, \tag{S8}$$

where C_t is *aggregate* consumption:

$$C_t \equiv L_{t-1}C_t^O + L_t C_t^Y$$

- Consumption by the old:

$$L_{t-1}C_t^O = (r_t + \delta)K_t + (1-\delta)K_t$$

- Consumption by the young:

$$L_t C_t^Y = w_t L_t - S_t L_t$$

Aggregate market equilibrium (2)

- Hence, aggregate output is:

$$\begin{aligned} C_t &= (r_t + \delta)K_t + (1 - \delta)K_t + w_t L_t - S_t L_t \\ &= Y_t + (1 - \delta)K_t - S_t L_t \end{aligned} \tag{S9}$$

- Comparing (S8) and (S9) yields:

$$S_t L_t = K_{t+1} \tag{S10}$$

saving by the young determines the future capital stock.

- Population growth:

$$L_t = L_0(1 + n)^t, \quad n > -1$$

- Intensive-form expression:

$$S\left(w_t, r_{t+1}\right) = (1 + n)\, k_{t+1}$$

Fundamental difference equation: General case

- Model can be expressed in single nonlinear difference equation:

$$(1+n)k_{t+1} = S\Big[\underbrace{f(k_t) - k_t f'(k_t)}_{w_t}, \underbrace{f'(k_{t+1}) - \delta}_{r_{t+1}}\Big]$$

- Slope of fundamental difference equation:

$$\frac{dk_{t+1}}{dk_t} = \frac{-S_w k_t f''(k_t)}{1 + n - S_r f''(k_{t+1})}$$

  - Stability condition is $\left|\frac{dk_{t+1}}{dk_t}\right| < 1$.
  - Numerator is positive (because $0 < S_w < 1$ and $f''(\cdot) < 0$).
  - Denominator is ambiguous (because S_r is).

Fundamental difference equation: Unit-elastic case

- For expository purposes focus on *unit-elastic* case:

$$y_t = k_t^{1-\varepsilon_L} \quad \text{so that} \quad w_t = \varepsilon_L k_t^{1-\varepsilon_L}$$

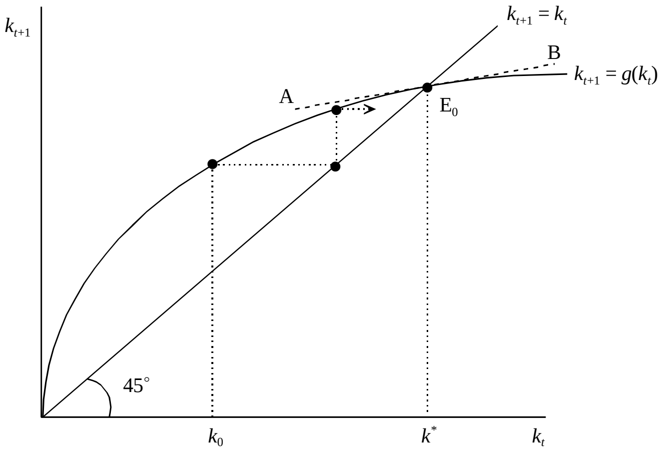
$$U(x) = \ln x \quad \text{so that} \quad S_t = w_t/(2+\rho)$$

- Fundamental difference equation for unit-elastic model:

$$k_{t+1} = g(k_t) \equiv \frac{\varepsilon_L}{(1+n)(2+\rho)} k_t^{1-\varepsilon_L}$$

  - **Figure 17.1** shows the phase diagram.
  - Steady-state equilibrium is unique and stable.

Figure 17.1: The unit-elastic Diamond-Samuelson model



Steady-state efficiency (1)

- Ignoring transitional dynamics, what would an *optimal steady-state* look like?
- Optimal steady-state is such that the lifetime utility of a "representative" young agent is maximized subject to the resource constraint:

$$\max_{\{C^Y,C^O,k\}} \quad \Lambda^Y \quad \equiv \quad U(C^Y) + \frac{1}{1+\rho} U(C^O)$$

$$\text{subject to:} \qquad f(k) - (n+\delta)k = C^Y + \frac{C^O}{1+n}$$

Steady-state efficiency (2)

- The first-order conditions give rise to two types of golden rules:
  - FONC #1, biological-interest-rate *consumption* golden-rule:

$$\frac{U'(C^O)}{U'(C^Y)} = \frac{1+\rho}{1+n}$$

  - FONC #2, *production* golden-rule:

$$f'(k) = n + \delta$$

  - Even if one is violated the other must still hold.
- In decentralized setting, $r = f'(k) - \delta$ so production rule calls for $r = n$. If $r < n$ there is overaccumulation (dynamic inefficiency). This is quite possible in the unit-elastic model.

Some basic applications of the model

- Old-age pensions.
  - Fully-funded versus pay-as-you-go (PAYG) pensions.
  - Reforming the pension system: transitional problems.
- Pensions and induced retirement.
- Ageing of the population.

Old-age pensions (1)

- To study a pension system we must add government taxes and transfers to the model.
- Budget identities:

$$\begin{aligned} C_t^Y + S_t &= w_t - T_t \\ C_{t+1}^O &= (1 + r_{t+1})S_t + Z_{t+1} \end{aligned}$$

  - T_t is tax levied on the young.
  - Z_t is transfer provided to the old.
- Consolidated lifetime budget constraint:

$$w_t - T_t + \frac{Z_{t+1}}{1 + r_{t+1}} = C_t^Y + \frac{C_{t+1}^O}{1 + r_{t+1}}$$

Old-age pensions (2)

- Financing method of the government distinguishes two prototypical systems:
  - Fully-funded system:

    $$Z_{t+1} = (1 + r_{t+1})T_t$$

    Contribution T_t earns market interest rate r_{t+1}.
  - PAYG system:

    $$L_{t-1}Z_t = L_t T_t \quad \Leftrightarrow \quad Z_t = (1 + n)T_t$$

    Contribution T_t earns the right to receive $(1 + n)T_{t+1}$ when old, where n is the biological interest rate.

Fully-funded pensions (1)

- Striking neutrality property.
- Recall that lifetime budget constraint is:

$$w_t - T_t + \frac{Z_{t+1}}{1 + r_{t+1}} = C_t^Y + \frac{C_{t+1}^O}{1 + r_{t+1}}$$

- Recall that under fully-funded system we have:

$$Z_{t+1} = (1 + r_{t+1})T_t$$

- So T_t and Z_{t+1} drop out of the lifetime budget constraint:

$$w_t = C_t^Y + \frac{C_{t+1}^O}{1 + r_{t+1}}$$

Fully-funded pensions (2)

- Economies with or without fully-funded system are identical!
- *Intuition*: household only worries about its total saving $S_t + T_t = S(w_t, r_{t+1})$. Part of this is carried out by the government but it carries the same rate of return.
- Proviso: system should not be "too severe" $(T_t < S(w_t, r_{t+1}))$. Otherwise households are forced to save too much by the pension system.

PAYG pensions (1)

- Features transfer from young to old in each period.
- We look at *defined-contribution* system: $T_t = T$ for all t so that $Z_{t+1} = (1+n)T$.
- Household lifetime budget constraint becomes:

$$\hat{w}_t \equiv w_t - \frac{r_{t+1} - n}{1 + r_{t+1}} T = C_t^Y + \frac{C_{t+1}^O}{1 + r_{t+1}}$$

- Ceteris paribus factor prices, the PAYG system expands (contracts) the household's resources if the market interest rate, r_{t+1}, falls short of (exceeds) the biological interest rate (n).

PAYG pensions (2)

- For logarithmic felicity the savings function becomes:

$$S(w_t, r_{t+1}, T) \equiv \frac{1}{2+\rho} w_t - \left[1 - \frac{1+\rho}{2+\rho} \cdot \frac{r_{t+1} - n}{1 + r_{t+1}}\right] T$$

  with $0 < S_w < 1$, $S_r > 0$, $-1 < S_T < 0$ (if $r_{t+1} > n$), and $S_T < -1$ (if $r_{t+1} < n$).

- Capital accumulation:

$$S(w_t, r_{t+1}, T) = (1+n)\, k_{t+1}$$

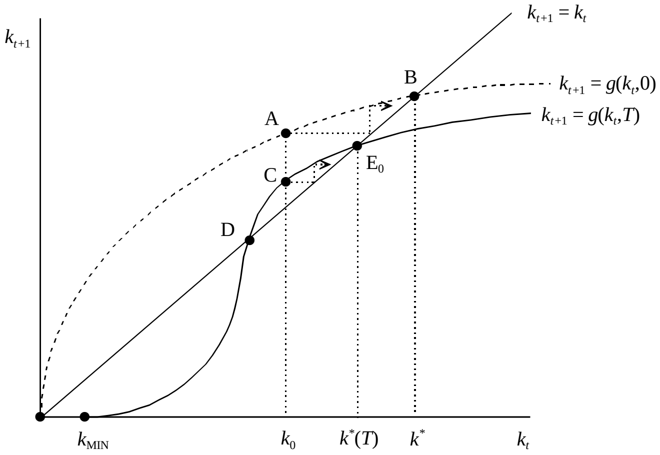
- Factor rewards under Cobb-Douglas technology:

$$\begin{aligned} w_t \equiv w(k_t) &= \varepsilon_L k_t^{1-\varepsilon_L} \\ r_{t+1} \equiv r(k_{t+1}) &= (1-\varepsilon_L) k_{t+1}^{-\varepsilon_L} - \delta \end{aligned}$$

PAYG pensions (3)

- Fundamental difference equation is illustrated in **Figure 17.4**.
  - Two equilibria: unstable on (at D) and stable one (at E_0).
  - Introduction of PAYG system is windfall gain to the then old but leads to crowding out of capital (see path A to C to E_0). In the long run, wages fall and the interest rate rises.

Figure 17.4: PAYG pensions in the unit-elastic model



Digression: Welfare effect of PAYG system (1)

- Ignoring transitional dynamics, what is the effect on welfare if T is changed marginally?
- Two useful tools:
  - Indirect utility function.
  - Factor price frontier.
- Indirect utility function is defined as follows:

$$\bar{\Lambda}^Y(w, r, T) \equiv \max_{\{C^Y, C^O\}} \left\{ \Lambda^Y(C^Y, C^O) \text{ subject to } \hat{w} = C^Y + \frac{C^O}{1+r} \right\}$$

with:

$$\hat{w} = w - \frac{r-n}{1+r}T$$

Digression: Welfare effect of PAYG system (2)

- Key properties of the IUF:

$$\begin{aligned}
\frac{\partial \bar{\Lambda}^Y}{\partial w} &= \frac{\partial \Lambda^Y}{\partial C^Y} > 0 \\
\frac{\partial \bar{\Lambda}^Y}{\partial r} &= \frac{S}{1+r} \cdot \frac{\partial \Lambda^Y}{\partial C^Y} > 0 \\
\frac{\partial \bar{\Lambda}^Y}{\partial T} &= -\frac{r-n}{1+r} \cdot \frac{\partial \Lambda^Y}{\partial C^Y} \gtreqless 0
\end{aligned}$$

- An increase in T has three effects:
  - Wage effect: $w \downarrow$ which is bad for welfare.
  - Interest rate effect: $r \uparrow$ which is good for welfare.
  - Direct effect depending on sign of $r - n$.

Digression: Welfare effect of PAYG system (3)

- Factor price frontier is defined as follows:

$$w_t = \phi(r_t)$$

- Key property of FPF:

$$\frac{dw_t}{dr_t} \equiv \phi'(r_t) = -k_t$$

Digression: Welfare effect of PAYG system (4)

- Welfare effect of marginal change in T:

$$
\begin{aligned}
\frac{d\bar{\Lambda}^Y}{dT} &= \frac{\partial\bar{\Lambda}^Y}{\partial w}\frac{dw}{dT} + \frac{\partial\bar{\Lambda}^Y}{\partial r}\frac{dr}{dT} + \frac{\partial\bar{\Lambda}^Y}{\partial T} \\
&= \frac{\partial\Lambda^Y}{\partial C^Y}\left[\frac{dw}{dT} + \frac{S}{1+r}\cdot\frac{dr}{dT} - \frac{r-n}{1+r}\right] \\
&= -\frac{r-n}{1+r}\cdot\frac{\partial\Lambda^Y}{\partial C^Y}\left[1 + k\frac{dr}{dT}\right] \propto \operatorname{sgn}(n-r)
\end{aligned}
$$

- There is thus an intimate link between the welfare effect and dynamic (in)efficiency:
  - If $r = n$ then $\frac{d\bar{\Lambda}^Y}{dT} = 0$ (no first-order welfare effects despite capital crowding out).
  - If economy is initially dynamically inefficient ($r < n$) then $\frac{d\bar{\Lambda}^Y}{dT} > 0$ (yield on PAYG pension is higher than market interest rate *and* capital crowding out is a good thing).

Pension reform: From PAYG to funded system (1)

- Ignoring transitional dynamics is not a good idea: there may be non-trivial welfare costs due to transition from one to another equilibrium.
- In a dynamically inefficient economy (with $r < n$ initially) an *increase* in T moves the economy in the direction of the golden-rule equilibrium *and* improves welfare for all generations during transition. Optimal to expand and not to abolish the system.

Pension reform: From PAYG to funded system (2)

- In a dynamically efficient economy (with $r > n$ initially) an *decrease* in T moves the economy in the direction of the golden-rule equilibrium *but* during transition it improves welfare for some generations (e.g. those born in the steady-state) and deteriorates it for other generations (e.g. the currently old). How do we evaluate the desirability?
  - Postulate social welfare function, weighting all generations.
  - Adopt the Pareto criterion.
- In a dynamically efficient economy it is impossible to move from a PAYG to a funded system in a Pareto-improving manner: a cut in T makes the old worse off and there is no way to compensate them without making some future generation worse off.

Induced retirement (1)

- Martin Feldstein: PAYG system not only affects the household's savings decision but also its retirement decision.
  - Labour supply is endogenous during youth.
  - The pension contribution rate is potentially distorting (proportional to labour income).
  - *Intragenerational* fairness: pension is proportional to contribution during youth (the lazy get less than the diligent).

Induced retirement (2)

- Preview of some key results:
  - Pension contribution acts like an employment *subsidy* if the so-called *Aaron condition* holds.
  - The general model displays a continuum of perfect foresight equilibria (Cobb-Douglas case has unique perfect foresight equilibrium).
  - If economy is in golden-rule equilibrium ($r = n$) then the contribution rate is non-distorting at the margin.
  - Pareto-improving transition from PAYG to fully-funded system *may* now be possible.

Households (1)

- Retired in old-age but endogenous labour supply during youth (early retirement).
- Utility function of aa young agent:

$$\Lambda_t^Y \equiv \Lambda^Y(C_t^Y, C_{t+1}^O, 1 - N_t)$$

- Budget identities:

$$\begin{aligned} C_t^Y + S_t &= w_t N_t - T_t \\ C_{t+1}^O &= (1 + r_{t+1})S_t + Z_{t+1} \end{aligned}$$

Households (2)

- Pension contribution proportional to wage income:

$$T_t = t_L w_t N_t$$

where t_L is the statutory tax rate ($0 < t_L < 1$).

- Pension received during old age:

$$Z_{t+1} = \left[t_L w_{t+1} \overline{NL}_{t+1} \right] \cdot \frac{N_t}{\overline{NL}_t}$$

  - Term 1: pension contributions of the future young generation (to be disbursed to the then old).
  - Term 2: share of pension revenue received by household (intragenerational fairness).

Households (3)

- Consolidated (lifetime) budget constraint:

$$(1 - t_{Et}) w_t N_t = C_t^Y + \frac{C_{t+1}^O}{1 + r_{t+1}}$$

$$t_{Et} \equiv t_L \cdot \left[1 - \frac{w_{t+1}}{w_t} \frac{\overline{NL}_{t+1}}{\overline{NL}_t} \frac{1}{1 + r_{t+1}} \right]$$

  - Agent has perfect foresight regarding labour supply of the future young.
  - Effective tax rate, t_{Et}, different from the statutory tax rate, t_L.

Households (4)

- Household chooses C_t^Y, C_{t+1}^O, and N_t in order to maximize lifetime utility subject to the lifetime budget constraint. First-order conditions:

$$\frac{\partial \Lambda^Y}{\partial C_{t+1}^O} = \frac{1}{1+r_{t+1}} \cdot \frac{\partial \Lambda^Y}{\partial C_t^Y}$$

$$\left[-\frac{\partial \Lambda^Y}{\partial N_t} = \right] \frac{\partial \Lambda^Y}{\partial (1-N_t)} = (1-t_{Et}) w_t \frac{\partial \Lambda^Y}{\partial C_t^Y}$$

  - MRS between future and present consumption is equated to the relative price of future consumption.
  - MRS between leisure and consumption (during youth) is equated to the after-effective-tax wage rate.
  - It is not t_L but t_{Et} which exerts a potentially distorting effect on labour supply.

Households (5)

- Symmetric solution as all agents are identical. With constant population growth $L_{t+1} = (1+n)L_t$ and t_{Et} simplifies to:

$$t_{Et} \equiv t_L \cdot \left[1 - \frac{w_{t+1}}{w_t}\frac{N_{t+1}}{N_t}\frac{1+n}{1+r_{t+1}}\right] = \frac{t_L}{1+r_{t+1}} \cdot \left[r_{t+1} - \frac{\Delta \overline{WI}_{t+1}}{\overline{WI}_t}\right]$$

- t_{Et} is negative if the *Aaron condition* holds, i.e. if the combined effect of growth in wage income and in the population exceeds the interest rate:

$$t_{Et} < 0 \qquad \Leftrightarrow \qquad \frac{\Delta \overline{WI}_{t+1}}{\overline{WI}_t} > r_{t+1}$$

Households (6)

- Continued.
  - Growth in wage income widens the revenue obtained per young household.
  - Population growth increases the number of young households and thus widens the total revenue.
- Effect of t_L on labour supply is ambiguous for two reasons:
  - Depends on Aaron condition (is t_{Et} negative of positive?).
  - Depends on income versus substitution effect.

The macroeconomy (1)

- Relation between household saving and the capital-labour ratio:

$$S_t = (1+n) N_{t+1} k_{t+1}$$

  where $k_t \equiv K_t / (L_t N_t)$.

- Labour supply and the savings function:

$$\begin{aligned} N_t &= N\left(w_t(1-t_{Et}), r_{t+1}\right) \\ S(\cdot) &\equiv \frac{C^O\left(w_t(1-t_{Et}), r_{t+1}\right) - (1+n) t_L w_{t+1} N_{t+1}}{1+r_{t+1}} \end{aligned}$$

The macro-economy (2)

- Fundamental difference equation:

$$S\left[w_t(1-t_{Et}), r_{t+1}, t_L w_{t+1} N\left(w_{t+1}(1-t_{Et+1}), r_{t+2}\right)\right] = (1+n) N\left(w_{t+1}(1-t_{Et+1}), r_{t+2}\right) k_{t+1}$$

  - (Bad) $w_t = w(k_t)$ and $r_t = r(k_t)$ so expression contains k_t, k_{t+1}, *and* k_{t+2} via the factor prices alone!
  - (Worse) t_{Et+1} depends on N_{t+2} which itself depends on k_{t+2}, k_{t+3}, and t_{Et+2} (infinite regress).
  - (Disaster) FDE depends on the entire sequence of capital stocks $\{k_{t+\tau}\}_{\tau=0}^{\infty}$ so there is a continuum of perfect foresight equilibria.
  - (But) if the utility function is Cobb-Douglas, then labour supply is constant and the perfect foresight equilibrium is unique (case discussed below).

Steady-state welfare effect

- Despite non-uniqueness of transition path, the steady state equilibrium is unique, so we can study its welfare properties.
- The indirect utility function is now:

$$\bar{\Lambda}^Y(w, r, t_L) \equiv \max_{\{C^Y, C^O, N\}} \Lambda^Y(C^Y, C^O, 1-N)$$

$$\text{subject to:} \qquad wN\left[1 - t_L \frac{r-n}{1+r}\right] = C^Y + \frac{C^O}{1+r}$$

- The welfare effect of a marginal change in the statutory tax is:

$$\frac{d\Lambda^Y}{dt_L} = -N\frac{r-n}{1+r} \cdot \frac{\partial \Lambda^Y}{\partial C^Y}\left[w + (1-t_L)k\frac{dr}{dt_L}\right]$$

  - No first-order welfare effect if $r = n$ (golden-rule equilibrium).
  - If $r \neq n$ then welfare effect is ambiguous because $\frac{dr}{dt_L}$ is ambiguous.

Cobb-Douglas preferences (1)

- Assume that the utility function is now:

$$\Lambda_t^Y \equiv \ln C_t^Y + \lambda_C \ln(1 - N_t) + \frac{1}{1+\rho} \ln C_t^O$$

where $\lambda_C \geq 0$ regulates the strength of the labour supply effect.

- Optimal household decision rules:

$$\begin{aligned} C_t^Y &= \frac{1+\rho}{2+\rho+\lambda_C(1+\rho)} w_t^N \\ C_{t+1}^O &= \frac{1+r_{t+1}}{2+\rho+\lambda_C(1+\rho)} w_t^N \\ N_t &= \frac{2+\rho}{2+\rho+\lambda_C(1+\rho)} \end{aligned}$$

Cobb-Douglas preferences (2)

- Continued. With:

$$w_t^N \equiv w_t(1 - t_{Et}) \equiv w_t \left[1 - t_L \left(1 - \frac{w_{t+1}}{w_t} \cdot \frac{1+n}{1+r_{t+1}}\right)\right]$$

  - Labour supply is constant (IE and SE offset each other).
  - Consumption during youth depends on the future interest rate via the effective tax rate.
- Fundamental difference equation is now:

$$(1+n)k_{t+1} = \frac{w(k_t)(1-t_L)}{2+\rho} - \frac{1+\rho}{2+\rho} \cdot \frac{t_L\,(1+n)\,w(k_{t+1})}{1+r(k_{t+1})}$$

  - First-order difference equation in the capital-labour ratio so the transition path is determinate.
  - Assuming stability, there is a unique perfect foresight equilibrium adjustment path.
  - An increase in t_L leads to crowding out of the steady-state capital stock (just as when lump-sum taxes are used).

Cobb-Douglas preferences (3)

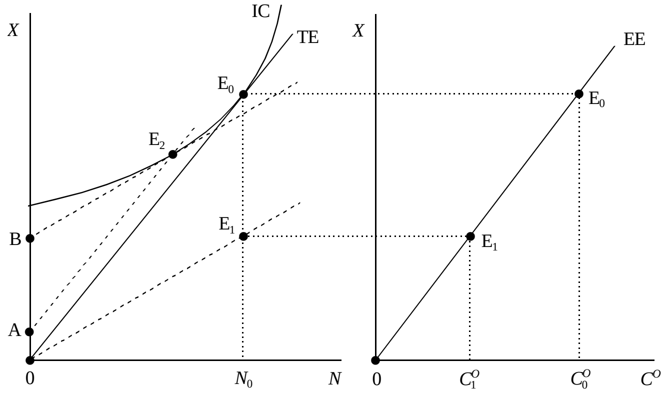
- Unlike the lump-sum case, the increase in t_L causes a distortion in the labour supply decision (provided $r \neq n$).
  - Recall that the deadweight loss of the distorting tax hinges on the elasticity of the *compensated* labour supply curve (which is positive) not of the *uncompensated* labour supply curve (which is zero for CD preferences).
  - (Weak) implication for pension reform: provided lump-sum contributions can be used during transition, a gradual move from PAYG to a funded system is possible.

Digression on deadweight loss of taxation (1)

- Deadweight loss of a distorting tax: the loss in welfare due to the use of a distorting rather than a non-distorting tax.
- In the context of our model, the DWL of the pension tax t_L can be illustrated with **Figure 17.5**.
- Assumptions: (w, r) held constant and $r > n$ (dynamic efficiency).
- Model solved in two steps to develop diagrammatic approach.
- We define lifetime income as:

$$X \equiv wN \left[1 - t_L \frac{r-n}{1+r} \right] \equiv wN(1 - t_E)$$

Figure 17.5: Deadweight loss of taxation



Digression on deadweight loss of taxation (2)

- *Stage 1*: Household chooses C^Y and C^O to maximize:

$$\ln C^Y + \frac{1}{1+\rho} \ln C^O \quad \text{s.t.} \quad C^Y + \frac{C^O}{1+r} = X$$

  This yields:

$$C^Y = \frac{1+\rho}{2+\rho} X, \qquad C^O = \frac{1+r}{2+\rho} X$$

  Second expression plotted in the right-hand panel of **Figure 17.5**.

- By substituting the solutions for C^Y and C^O into the utility function we find:

$$\Lambda^Y \equiv \frac{2+\rho}{1+\rho} \ln X + \lambda_C \ln[1 - N_t] + \text{constant}$$

Digression on deadweight loss of taxation (3)

- *Stage 2*: The household chooses X and N to maximize Λ^Y subject to the constraint $X = wN(1 - t_E)$. The resulting expressions are:

$$N = \frac{2+\rho}{2+\rho+\lambda_C(1+\rho)}$$

$$X = \frac{(2+\rho)\,w(1-t_E)}{2+\rho+\lambda_C(1+\rho)}$$

The maximization problem is shown in the left-hand panel of **Figure 17.5**: IC is the indifference curve and TE is the constraint. Both are downward sloping because $-N$ is measured on the horizontal axis!

Digression on deadweight loss of taxation (4)

- The optimal solution for $t_E = 0$ is given by point E_0 in both panels. Now consider what happens if t_E is increased:
  - Right-hand panel: no effect on EE curve (r is constant).
  - Left-hand panel: TE rotates counterclockwise. New equilibrium at E_1 (directly below E_0).
  - Decomposition of total effect: SE: move from E_0 to E_2; IE move from E_2 to E_1.
- On the vertical axis:
  - 0B is the income one would have to give the household to restore it to its initial indifference curve IC (hypothetical transfer Z_0).
  - AB is the tax revenue collected from the agent (i.e. $t_E w N$).
  - 0B minus AB is the dead-weight loss of the tax.
- If lump-sum tax were used then the slope of TE would not change and the DWL would be zero (hypothetical transfer equal to tax revenue).

Macroeconomic effects of ageing (1)

- The old-age *dependency ratio* is the number of retired people divided by the working-age population.
- In the models studied so far, the old-age dependency ratio is assumed to be constant: $\frac{L_{t-1}}{L_t} = \frac{1}{1+n}$.
- As the data in **Table 17.1** show, this is rather unrealistic:
  - In the OECD and the US the population is ageing: proportion of young falls whilst proportion of old rises.
  - Note: Demographic predictions are notoriously unreliable!

Table 17.1: Age composition of the population

	1950	1990	2025
World			
0-19	44.1	41.7	32.8
20-65	50.8	52.1	57.5
65+	5.1	6.2	9.7
OECD			
0-19	35.0	27.2	24.8
20-64	56.7	59.9	56.6
65+	8.3	12.8	18.6
United States			
0-19	33.9	28.9	26.8
20-65	57.9	58.9	56.0
65+	8.1	12.2	17.2

Macroeconomic effects of ageing (2)

- In the absence of immigration, there are two causes for ageing:
  - Decrease in fertility.
  - Decrease in mortality.
- We can study the first effect with D-S model: focus on interaction with pension system.

Revised model (1)

- Population:

$$L_t = (1 + n_t)\, L_{t-1}$$

with n_t variable.

- Saving-capital link:

$$S(w_t, r_{t+1}, n_{t+1}, T) = (1 + n_{t+1})k_{t+1}$$

  - $S_n < 0$: as n_{t+1} decreases, the future pension decreases ($Z_{t+1} = (1 + n_{t+1})\, T$), and saving increases.
  - LHS: a reduction in n_{t+1} allows for a higher capital-labour ratio for a given level of saving.

Revised model (2)

- A permanent decrease in the fertility rate increases the long-run capital stock. The transition path is shown in **Figure 17.6**. Economy-wide asset ownership rises because the proportion of old increases.
- Qualitatively the same conclusion as Auerbach & Kotlikoff reach on basis of detailed CGE model!

Figure 17.6: The effects of ageing