Foundations of Modern Macroeconomics Second Edition

Chapter 15: Real business cycles

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Outline

1. Extended Ramsey model
   - The basic model
   - Fiscal policy
   - Putting numbers in, getting numbers out

2. A Real Business Cycle model
   - Building a unit-elastic RBC model
   - Simulating the unit-elastic RBC model
   - Puzzles

Aims of this chapter

- To extend the Ramsey-Cass-Koopmans model by endogenizing the labour supply decision of households.
- To show the effects of fiscal policy.
- Demonstrate the critical role of the intertemporal labour supply elasticity.
- To turn the model into an RBC model by assuming stochastic technology shocks.
  - Theory of fluctuations at business cycle frequencies.
  - Impulse response functions.
  - Matching real world data (calibration).
  - Evaluation of the RBC approach.

Household behaviour (1)

- *Key idea*: To get an interesting theory of short-term fluctuations in macroeconomic variables such as output, employment, consumption, investment, wages, and the interest rate we must augment the Ramsey model by adding an endogenous labour supply decision by households.
- Fortunately, endogenizing labour supply is straightforward. In what follows we abstract from population growth ($n_L = 0$ throughout).

Household behaviour (2)

- The household lifetime utility function is:

$$\Lambda(t) \equiv \int_t^{\infty} \underbrace{\left[\varepsilon_C \ln C(\tau) + (1-\varepsilon_C)\ln\left[1-L(\tau)\right]\right]}_{(a)} e^{\rho(t-\tau)} d\tau$$

(a) The felicity function is logarithmic; sub-felicity is Cobb-Douglas: $C^{\varepsilon_C}[1-L]^{1-\varepsilon_C}$.
  - The planning period is t so the time index is in the interval $\tau \in [t, \infty)$.
  - $\rho > 0$ is the pure rate of time preference.
  - **Note**: If we set $\varepsilon_C = 1$ we have the case with exogenous labour supply (the traditional Ramsey formulation).

- The agent's budget *identity* is:

$$\dot{A}(\tau) \equiv r(\tau)A(\tau) + w(\tau)L(\tau) - T(\tau) - C(\tau) \qquad \text{(S1)}$$

where the only thing that has changed is that labour income is now wL (recall that L is a choice variable).

Household behaviour (3)

- Integrating (S1) yields the agent's budget restriction:

$$A(t) = \int_t^{\infty} [C(\tau) - w(\tau)L(\tau) + T(\tau)] \, e^{-R(t,\tau)} d\tau \qquad \text{(S2)}$$

where we have used $R(t,\tau) \equiv \int_t^{\tau} r(s)ds$ and:

$$\lim_{\tau \to \infty} A(\tau)e^{-R(t,\tau)} = 0 \qquad \text{(NPG)}$$

  - Equation (S2) says that the present value of household "primary deficits" (i.e., the excess of spending over non-interest income, $C - wL + T$) equals the value of initial financial wealth in the planning period.
  - NPG stands for "no Ponzi game". (Ponzi was a famous swindler who ran chain letter schemes and ended his life in jail.) This is the solvency condition.
  - $R(t,\tau)$ is the discounting factor. Note that the real interest rate, r, is endogenous (depends on accumulation and on labour supply decisions by aggregate household sector).

Household behaviour (4)

- The household chooses paths for consumption, labour supply, and financial assets, such that lifetime utility is maximized subject to the household budget constraint (S2). In the text we derive the first-order conditions:

$$\frac{\dot{C}(\tau)}{C(\tau)} = r(\tau) - \rho \qquad \text{(S3)}$$

$$\frac{C(\tau)}{1 - L(\tau)} \frac{1 - \varepsilon_C}{\varepsilon_C} = w(\tau) \qquad \text{(S4)}$$

  - Eq. (S3): dynamic part. Consumption Euler equation: the optimal time profile of consumption depends on the gap between the interest rate and the rate of time preference. If $r > \rho$, postpone consumption until later and adopt an upward sloping time profile of consumption.
  - Eq. (S4): static part. The MRS between consumption and leisure should be equated to the real wage.

The unit-elastic extended Ramsey model (1)

- The equations of the extended Ramsey model are given in **Table 15.1**. Show table
- The extended Ramsey model can be analyzed with the aid of its phase diagram in **Figure 15.1**. Show figure
- The diagram reveals that there is a unique equilibrium (at E_0) which is saddle-point stable. The saddle path (SP) is upward sloping. How do we know this?
- The $\dot{K} = 0$ line represents (C, K) combinations for which *net* investment is zero.
  - The golden rule capital stock is K^{GR} (maximum consumption point).
  - For points above (below) the $\dot{K} = 0$ line, consumption is too high (too low), and net investment is negative (positive)–see the horizontal arrows.

The unit-elastic extended Ramsey model (2)

- The $\dot{C} = 0$ line represents (C, K) combinations for which the consumption profile is flat, i.e. for which $r = \rho$.
  - Recall that:

$$\begin{aligned} r &= F_K(K, L) - \delta \\ &= F_K\left(\frac{K}{L}, 1\right) - \delta \end{aligned}$$

    so $r = \rho$ implies a constant K/L ratio. Hence, w and Y/K are constant also along the $\dot{C} = 0$ line. This means that $C/[1 - L]$ is also constant. Combining all these "great ratios" yields the conclusion that the $\dot{C} = 0$ line is linear and downward sloping.
  - For points above (below) the $\dot{C} = 0$ line, consumption is too high (too low), labour supply is too low (too high), and the K/L ratio is too high (too low), i.e. $r < \rho$ $(r > \rho)$ and $\dot{C} < 0$ $(\dot{C} > 0)$. See the vertical arrows.

Figure 15.1: Phase diagram of the unit-elastic model

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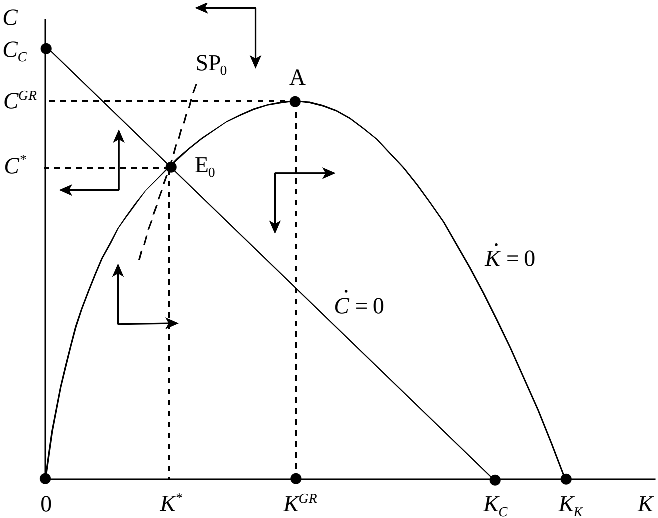


Table 15.1: The unit-elastic model

Go back

$$\dot{K}(t) = I(t) - \delta K(t) \quad \text{(T1.1)}$$

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho \quad \text{(T1.2)}$$

$$G(t) = T(t) \quad \text{(T1.3)}$$

$$w(t) = \varepsilon_L \frac{Y(t)}{L(t)} \quad \text{(T1.4)}$$

$$r(t) + \delta = (1 - \varepsilon_L) \frac{Y(t)}{K(t)} \quad \text{(T1.5)}$$

$$Y(t) = C(t) + I(t) + G(t) \quad \text{(T1.6)}$$

$$w(t) [1 - L(t)] = \frac{1 - \varepsilon_C}{\varepsilon_C} C(t) \quad \text{(T1.7)}$$

$$Y(t) = Z_0 L(t)^{\varepsilon_L} K(t)^{1-\varepsilon_L} \quad \text{(T1.8)}$$

Permanent increase in government consumption (1)

- To get to know the extended Ramsey model, and to prepare for the RBC analysis to come, we first study a simple example of fiscal policy: a permanent increase in government consumption financed by means of a lump-sum tax ($G \uparrow$ and $T \uparrow$).
- The *long-run effects* are obtained with back-of-the-envelope calculations.
- $\dot{K} = 0$ implies $I/K = \delta$ (a constant).
- $\dot{C} = 0$ implies $r = \rho$, $Y/K = (\rho + \delta)/(1 - \varepsilon_L)$ (constants).
- w and $C/[1 - L]$ constant (see above).

Permanent increase in government consumption (2)

- In summary:

$$\frac{dY(\infty)}{Y} = \frac{dK(\infty)}{K} = \frac{dI(\infty)}{I} = \frac{dL(\infty)}{L} = -\omega_{LL}\frac{dC(\infty)}{C} \qquad \text{(S5)}$$

where $\omega_{LL} \equiv [1-L]/L$ represents the intertemporal substitution elasticity of labour supply.

- Using the GME locus (T1.6) yields:

$$\frac{dY(\infty)}{Y} = \omega_C\frac{dC(\infty)}{C} + \omega_I\frac{dI(\infty)}{I} + \omega_G\frac{dG}{G} \qquad \text{(S6)}$$

where $\omega_C \equiv C/Y$, $\omega_I \equiv I/Y$, and $\omega_G \equiv G/Y$ $[\omega_C + \omega_I + \omega_G = 1]$.

Permanent increase in government consumption (3)

- Combining (S5) and (S6) yields the multiplier for output, consumption, and the capital stock:

$$\begin{aligned} \frac{dY(\infty)}{dG} &= \frac{1}{1-\omega_I+\omega_C/\omega_{LL}} > 0 \\ -1 < \frac{dC(\infty)}{dG} &= -\frac{\omega_C/\omega_{LL}}{1-\omega_I+\omega_C/\omega_{LL}} < 0 \\ \frac{dK(\infty)}{dG} &= \frac{1}{\delta}\frac{dI(\infty)}{dG} = \frac{\omega_I/\delta}{1-\omega_I+\omega_C/\omega_{LL}} > 0 \end{aligned}$$

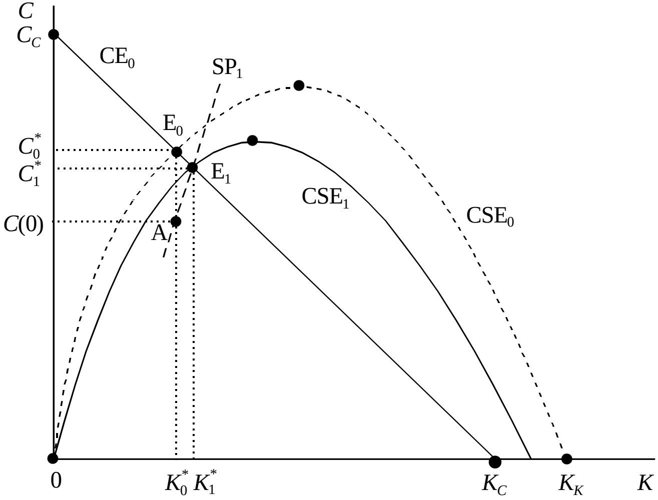
Permanent increase in government consumption (4)

- In the long run output is *crowded in* by additional government consumption, more so the more elastic is labour supply (the higher is ω_{LL})! Consumption is crowded out [though by less than one for one], and the capital stock rises. *Economic intuition:*
  - The household feels poorer because the lump-sum tax goes up permanently ($T \uparrow$).
  - This means that the value of human wealth falls ($H(\infty) \equiv [w - T]/r$), so that on that account consumption and leisure fall ($C \downarrow$ and $[1 - L] \downarrow$ and thus $L \uparrow$).
  - The drop in C makes it possible for the household to save more so that $K \uparrow$ (to restore constant K/L ratio).
  - Note the sharp contrast with the case of exogenous labour supply!

Permanent increase in government consumption (5)

- The *short-run effects* of the fiscal shock are more difficult to compute. We can use **Figure 15.2**.
  - The capital stock equilibrium (CSE) line (for which $\dot{K} = 0$) shifts down because $G \uparrow$. Nothing happens to the consumption equilibrium (CE) line (for which $\dot{C} = 0$) because lump-sum taxes are used.
  - At impact the capital stock is predetermined [at K_0^*] but the economy jumps to the new saddle path [from E_0 to A]. Households immediately adjust their consumption downward and their labour supply upwards due to the higher taxes.
  - Since $C \downarrow$ and $L \uparrow$ (and thus $Y \uparrow$):
    - Households can save more [accumulate capital]: point A lies below CSE_1 so that $\dot{K} > 0$.
    - The K/L ratio falls (capital relatively scarce) so that $r \uparrow$ and thus $\dot{C} > 0$ in point A.
- *During transition* both C and K rise until the new equilibrium in E_1 is reached.

Figure 15.2: Effects of fiscal policy



Quantification of the results (1)

- *Key idea*: Graphical methods are of limited use because they only yield *qualitative results* ("plus" or "minus"). We would like to know more, namely how large are the effects? We want *quantitative results*.
- In the text we show in detail how we can obtain quantitative results (for impact, transitional, and long-run effects) by *log-linearizing the model*. The resulting expressions are found in **Table 15.2**. Advantages of loglinearizing:
  - We can solve the model for all kinds of shocks (not just fiscal).
  - The loglinearized model is expressed in terms of parameters which can be measured by econometric means (adding empirical content to the model), e.g. income shares of various macro variables (ω_C, ω_I, ω_G, ε_L) etcetera.
  - We can simulate realistically calibrated models on the computer and see how well they fit the real world data (the *Lucas program*).

Table 15.2: The loglinearized model

$$\dot{\tilde{K}}(t) = \delta\left[\tilde{I}(t) - \tilde{K}(t)\right] \quad \text{(T2.1)}$$

$$\dot{\tilde{C}}(t) = \rho\tilde{r}(t) \quad \text{(T2.2)}$$

$$\tilde{G}(t) = \tilde{T}(t) \quad \text{(T2.3)}$$

$$\tilde{w}(t) = \tilde{Y}(t) - \tilde{L}(t) \quad \text{(T2.4)}$$

$$\rho\tilde{r}(t) = (\rho+\delta)\left[\tilde{Y}(t) - \tilde{K}(t)\right] \quad \text{(T2.5)}$$

$$\tilde{Y}(t) = \omega_C\tilde{C}(t) + \omega_I\tilde{I}(t) + \omega_G\tilde{G}(t) \quad \text{(T2.6)}$$

$$\tilde{L}(t) = \omega_{LL}\left[\tilde{w}(t) - \tilde{C}(t)\right] \quad \text{(T2.7)}$$

$$\tilde{Y}(t) = \varepsilon_L\tilde{L}(t) + (1-\varepsilon_L)\tilde{K}(t) \quad \text{(T2.8)}$$

Quantification of the results (2)

- In the text we show what a reasonable calibration looks like and derive multipliers and elasticities. See **Table 15.3**.
- In the text we show how the loglinearized model can be used to study more difficult types of shocks. The worked example deals with *temporary fiscal policy* and its effects of the key macroeconomic variables. The degree of persistence of the shock critically influences the adjustment path. (See **Figures 15.4** for details of the derivation. ▸ Show figures)
  - Consumption falls regardless of the degree of shock persistence.
  - Capital rises initially if labour supply is highly elastic and the shock is relatively persistent.
  - Capital falls initially if labour supply is not very elastic and the shock is relatively transitory.

Table 15.3: Government consumption multipliers

Variable	*Impact effect*	*Long-run effect*
$\frac{dY}{dG}$	1.029	1.054
$\frac{dC}{dG}$	-0.539	-0.158
$\frac{dI}{dG}$	0.568	0.212
$\frac{dK}{K^*} / \frac{dG}{G}$	0	0.211
$\frac{dL}{L^*} / \frac{dG}{G}$	0.309	0.211
$\frac{dr}{r^*} / \frac{dG}{G}$	0.518	0
$\frac{dw}{w^*} / \frac{dG}{G}$	-0.103	0

Figure 15.3: Phase diagram of the loglinearized model

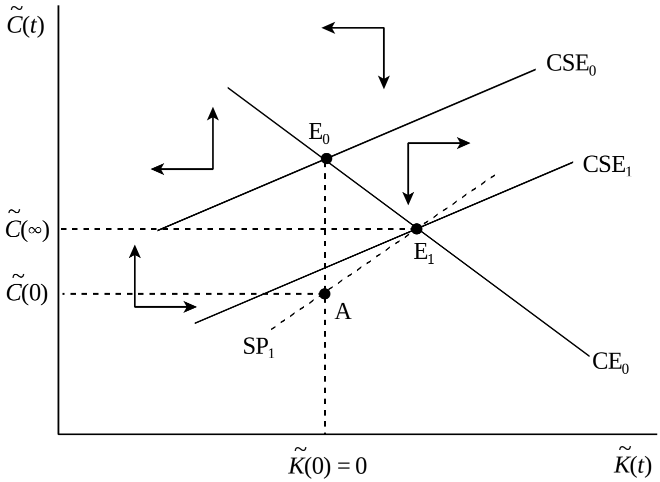


Figure 15.4: Temporary fiscal policy

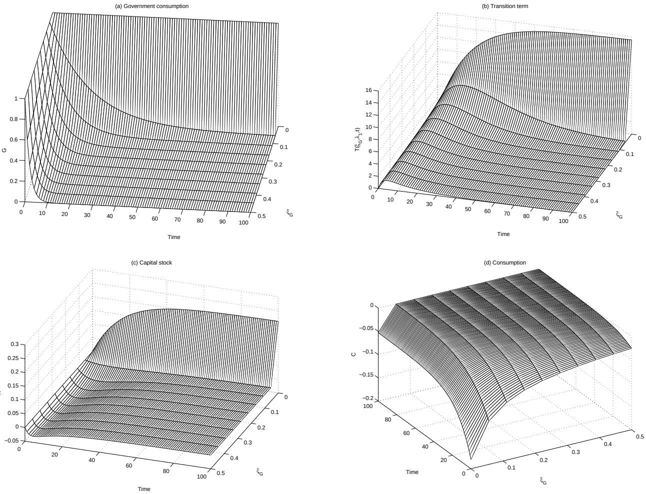


Figure 15.4: Temporary fiscal policy (continued)

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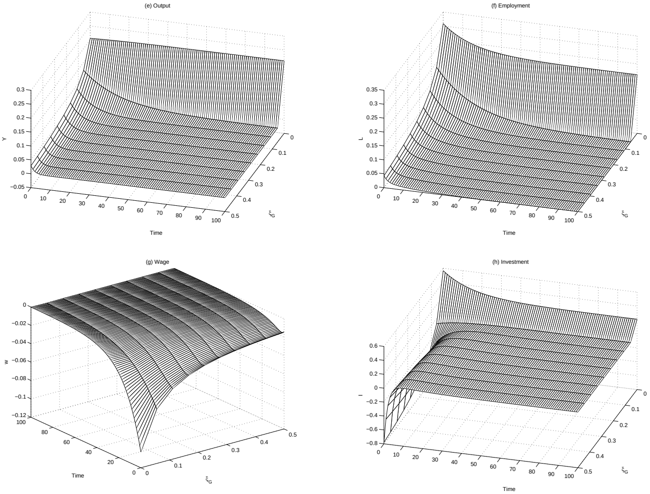
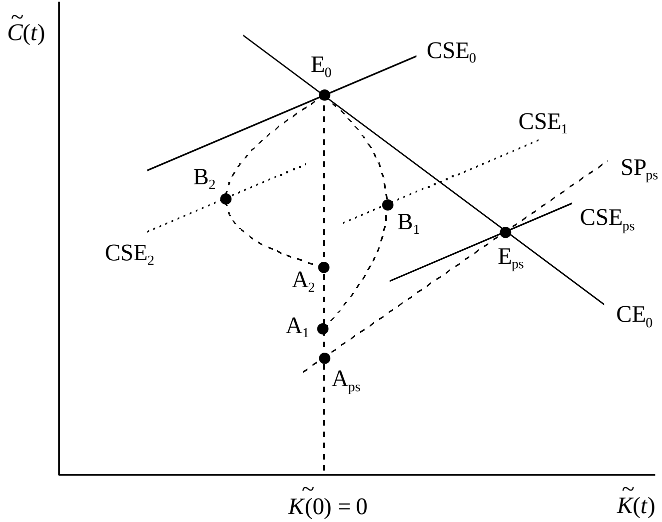


Figure 15.5: Phase diagram for temporary shock



The Lucas research program

- *Key idea*: Macroeconomists should build so-called *structural models*, i.e. models that are based on microeconomic foundations (maximizing households and firms, flexible prices/wages, market clearing, etcetera).
- The Lucas Research Program (LRP) is the logical outcome of the Rational Expectations Revolution of the 1970s.
- Kydland & Prescott accepted the challenge posed by Lucas: they built the first Real Business Cycle (RBC) model.
- We are going to build a simple RBC model right before your very eyes.

Outline of the RBC methodology

- Construct a discrete-time stochastic model of the economy populated by maximizing households and firms.
- Typically the source of the stochastic fluctuations is the level of general productivity (our Z in the production function). Since Z_{t+1} is unknown at time t, agents must form expectations about it. They adopt the REH to do so.
- Calibrate the model in a realistic fashion.
- Find the stochastic equilibrium process for the macroeconomic variables (output, employment, consumption, investment, the capital stock, and factor prices).
- Compute basic statistics (correlations, and standard deviations) for the different variables both for the artificial economy and for the actual economy. Compare how well the model economy matches the actual economy's characteristics.

Building an RBC model: Steps

- We have most of the ingredients already. A few tasks remain.
- Reformulate model in discrete time (rather than continuous time).
- Introduce stochastic productivity shock.
- Rederive firm and household behaviour.
- Solve model
- analyze and visualize solutions (theoretically and quantitatively)

Building an RBC model: Firms

- Technology:

$$Y_\tau = F(Z_\tau, K_\tau, L_\tau) \equiv Z_\tau L_\tau^{\varepsilon_L} K_\tau^{1-\varepsilon_L}, \quad 0 < \varepsilon_L < 1$$

  where Z_τ is the index of general technology.

- Firms rent factors of production from the household sector. The marginal productivity conditions are:

$$\begin{aligned} F_L(Z_\tau, K_\tau, L_\tau) &= w_\tau \\ F_K(Z_\tau, K_\tau, L_\tau) &= R_\tau^K \end{aligned}$$

  where R^K is the rental charge on capital.

Building an RBC model: Households (1)

- Preferences (expected lifetime utility):

$$E_t \Lambda_t \equiv E_t \sum_{\tau=t}^{\infty} \left(\frac{1}{1+\rho} \right)^{\tau - t} \left[\varepsilon_C \ln C_\tau + (1 - \varepsilon_C) \ln[1 - L_\tau] \right]$$

where E_t is the expectations operator (i.e. information dated up to and including period t is used).

- Budget identity:

$$C_\tau + I_\tau = w_\tau L_\tau + R_\tau^K K_\tau - T_\tau$$

- Capital accumulation:

$$K_{\tau+1} = I_\tau + (1 - \delta) K_\tau$$

Building an RBC model: Households (2)

- The first-order conditions (for the planning period t) are:

$$w_t = \frac{(1-\varepsilon_C)/[1-L_t]}{\varepsilon_C/C_t} \quad \text{(S7)}$$

$$\frac{\varepsilon_C}{C_t} = E_t\left[\frac{1+r_{t+1}}{1+\rho}\cdot\frac{\varepsilon_C}{C_{t+1}}\right] \quad \text{(S8)}$$

$$r_{t+1} \equiv R^K_{t+1} - \delta \quad \text{(S9)}$$

  - Eq. (S7): static. The MRS between consumption and leisure should be equated to the wage rate.
  - Eq. (S8): dynamic. The stochastic consumption Euler equation: the MU of consumption in the planning period (C_t) should be equated to the expected weighted MU of consumption one period later (C_{t+1}).
  - Eq. (S9): definition. The real interest rate is the rental rate minus the depreciation rate.
- The full model is given in loglinearized form in Table 15.4.

Table 15.4: The loglinearized stochastic model

$$\tilde{K}_{t+1} - \tilde{K}_t = \delta \left[\tilde{I}_t - \tilde{K}_t \right] \quad \text{(T4.1)}$$

$$E_t \tilde{C}_{t+1} - \tilde{C}_t = \frac{\rho}{1+\rho} E_t \tilde{r}_{t+1} \quad \text{(T4.2)}$$

$$\tilde{G}_t = \tilde{T}_t \quad \text{(T4.3)}$$

$$\tilde{w}_t = \tilde{Y}_t - \tilde{L}_t \quad \text{(T4.4)}$$

$$\rho \tilde{r}_t = (\rho + \delta) \left[\tilde{Y}_t - \tilde{K}_t \right] \quad \text{(T4.5)}$$

$$\tilde{Y}_t = \omega_C \tilde{C}_t + \omega_I \tilde{I}_t + \omega_G \tilde{G}_t \quad \text{(T4.6)}$$

$$\tilde{L}_t = \omega_{LL} \left[\tilde{w}_t - \tilde{C}_t \right] \quad \text{(T4.7)}$$

$$\tilde{Y}_t = \tilde{Z}_t + \varepsilon_L \tilde{L}_t + (1 - \varepsilon_L) \tilde{K}_t \quad \text{(T4.8)}$$

Building an RBC model: Market equilibrium

- Apart from the fact that the model is now in discrete time, it looks virtually identical to the deterministic model of Table 15.2.
- Because general technology is stochastic, so is the future interest rate. For that reason, $E_t \tilde{r}_{t+1}$ appears in the log-linearized Euler equation. Recall:

$$r_{t+1} = F_K(\underbrace{Z_{t+1}}_{(a)}, \underbrace{K_{t+1}}_{(b)}, \underbrace{L_{t+1}}_{(c)}) - \delta$$

(a) Future general technology; unknown in period t (but may be partially forecastable if the shock is persistent – see below).
(b) Future capital stock; known in period t as it depends only on present accumulation decisions.
(c) Future labour supply; unknown in period t as it depends on w_{t+1} and C_{t+1} and thus on Z_{t+1}.

Building an RBC model: The shock

- The specification of the model is completed once the stochastic process for general productivity is specified. A commonly used specification is first-order autoregressive:

$$\begin{aligned} \ln Z_t &= \alpha_Z + \rho_Z \ln Z_{t-1} + \varepsilon_t^Z, \qquad 0 < \rho_Z < 1, \qquad \Longrightarrow \\ \tilde{Z}_t &= \rho_Z \tilde{Z}_{t-1} + \varepsilon_t^Z \end{aligned}$$

where $\tilde{Z}_t \equiv \ln(Z_t/Z)$ and:

  - ρ_Z is the *degree of persistence* of the shock [special cases: $\rho_Z = 0$ purely transitory shock; $\rho_Z = 1$ permanent shock]
  - ε_t^Z is the stochastic *innovation term* (identically and independently distributed with mean zero and variance σ_Z^2).
  - If ρ_Z is nonzero, general productivity in the next period is partially forecastable. Under REH the agents best forecast is:

$$E_t \tilde{Z}_{t+1} = \rho_Z \tilde{Z}_t$$

Building an RBC model: Model solution

- The loglinearized model in Table 15.4 can be solved under the REH. In the text we show two methods. The easiest of these looks directly at so-called *impulse-response functions* for the different variables. *Key idea:*
  - Assume that the system is initially in steady state and trace the effect of a single innovation at time $t = 0$: $\varepsilon_0^Z > 0$ and $\varepsilon_t^Z = 0$ for $t = 1, 2, \ldots$. We call ε_0^Z the *impulse* hitting the economic system.
  - Compute the implied *response* of the different variables to the impulse.
  - In the text we derive the general case for which $0 < \rho_Z < 1$. To understand the general result it pays to look at the special cases.

A purely temporary productivity improvement

- **A purely temporary shock**: $\rho_Z = 0$. The impulse-response functions for this type of shock are given in **Figure 15.7**. Salient features:
  - No long-run effect on general productivity and thus no long-run effect on any variable.
  - Productivity only higher than normal in period $t = 0$.
  - Agents are a little richer and thus $C_0 \uparrow$, and $I_0 \uparrow$ (agents spread gain over present and future consumption).
  - Strong incentive to work when productivity is high: $w_0 \uparrow$, $(1 - L_0) \downarrow$, $L_0 \uparrow$, $Y_0 \uparrow$ (see **Figure 15.6**).
  - For $t = 1, 2, 3, \ldots$ general productivity back to normal. Agent gradually runs down extra savings by consuming more than normal: $C_t \searrow$, $K_t \searrow$, Y_t, L_t, and I_t almost back to normal.
  - Note: Output response looks virtually identical to impulse (lack of internal propagation).

Figure 15.7: Purely transitory productivity shock

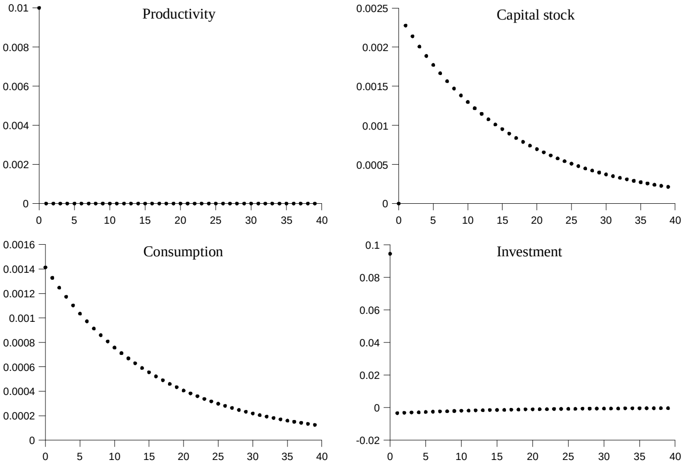


Figure 5.7: Purely transitory productivity shock (continued)

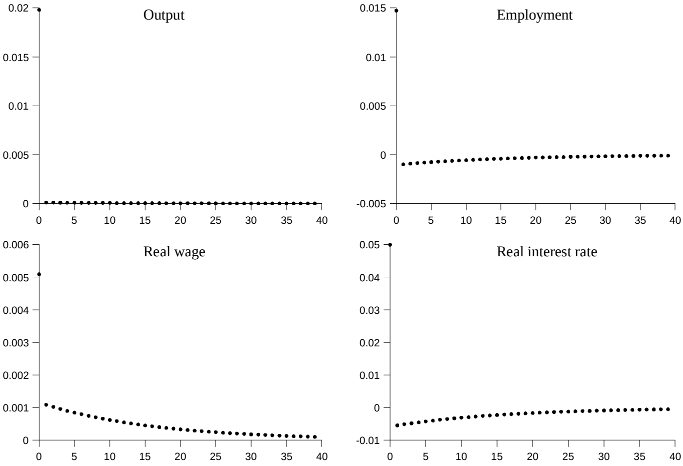
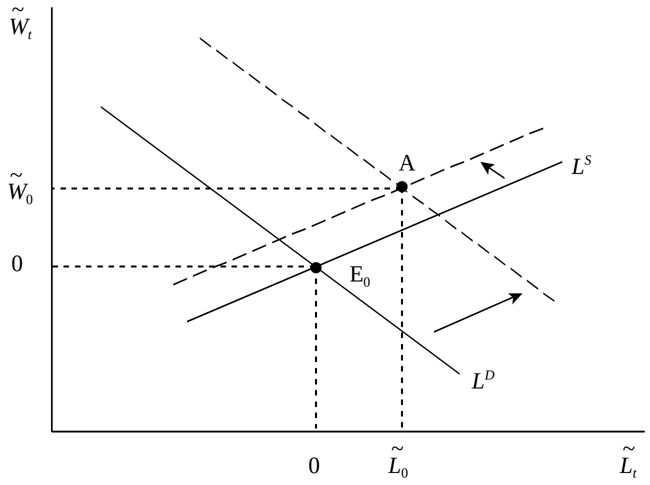


Figure 15.6: A shock to technology and the labour market



A purely permanent productivity improvement

- **A purely permanent shock**: $\rho_Z = 1$. The impulse-response functions for this type of shock are given in **Figure 15.8**. Salient features:
  - There is a long-run effect on productivity and thus on most macro variables: the great ratios explain that $Y_\infty \uparrow$, $C_\infty \uparrow$, $K_\infty \uparrow$, $I_\infty \uparrow$, and $L_\infty \downarrow$ (if $\omega_G > 0$ so that IE effect dominates SE in labour supply).
  - Agents are a lot richer and thus $C_0 \uparrow$, and $I_0 \uparrow$ (agents spread gain over present and future consumption).
  - Even though $w_0 \uparrow$ and SE says $L_0 \uparrow$, there is a smaller upward jump in employment (than for temporary shock) because IE says $L_0 \downarrow$.
  - For $t = 1, 2, 3, \ldots$ general productivity stays high. Agent gradually keep accumulating capital and consumption continues to rise: $C_t \nearrow$, $K_t \nearrow$, $L_t \searrow$, and $I_t \searrow$.
  - Note: Output response again looks virtually identical to impulse (lack of internal propagation).

Figure 15.8: Permanent productivity shock

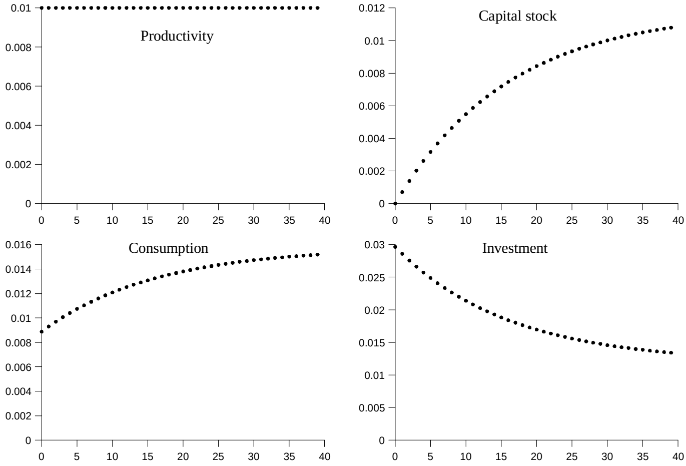
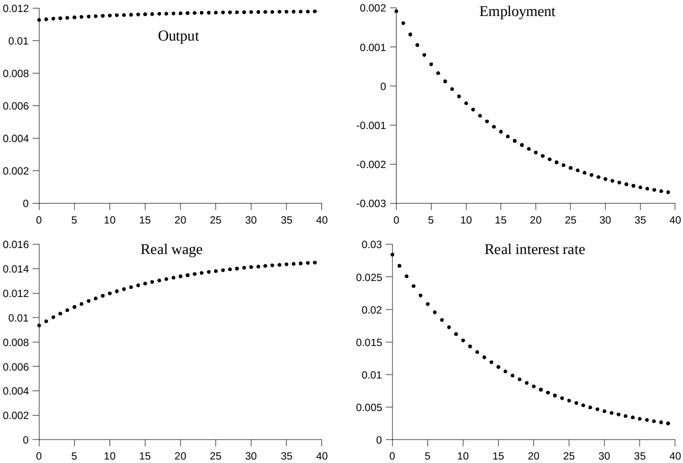


Figure 15.8: Permanent productivity shock (continued)



What is a realistic shock process? (1)

- What would a **realistic shock** look like?
- The seminal work by Solow (1957) has been used to estimate the nature of technological change.
- *Solow residual*: compute how much of output growth can be explained by growth in inputs. The remainder is now called the Solow residual.
- In our model the Solow residual is equal to the general productivity index Z_t:

$$\ln SR_t \equiv \ln Y_t - \varepsilon_L \ln L_t - (1 - \varepsilon_L) \ln K_t = \ln Z_t.$$

What is a realistic shock process? (2)

- We can obtain estimates for α_Z, ρ_Z, and σ_Z^2 [the variance of ε_t^Z] by regressing:

$$\ln SR_t = \alpha_Z + \rho_Z \ln SR_{t-1} + \varepsilon_t^Z$$

- For the US one finds:

$$\hat{\rho}_Z = 0.979$$

  which means that the technology shocks are not permanent but nevertheless display a very high degree of persistence.

- In **Figure 15.9** we show the different impulse-response functions for a whole range of ρ_Z values (including the realistic one). The key thing to note is the highly nonlinear behaviour of the IR functions for values of ρ_Z near unity.

Figure 15.9: Temporary productivity shock

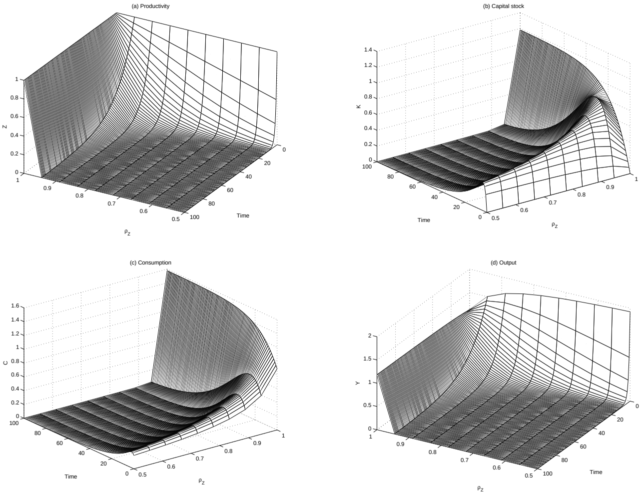
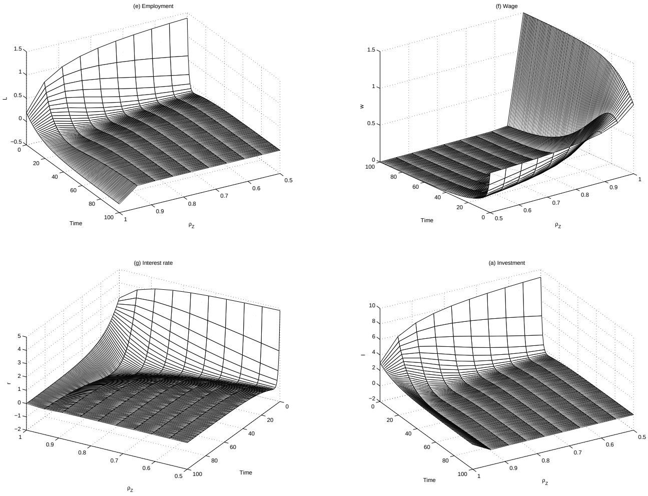


Figure 15.9: Temporary productivity shock (continued)



Correlations and (co)variances

- Although the impulse-response functions display a lot of information about the model, most RBC modellers judge the performance of their model by looking at the match between model-generated and actual statistics. In **Table 15.5** we show an example of this approach. The standard model yields the results in panel (b) whilst actual statistics for the US economy are found in panel (a). Salient features:
  - Model captures that $\sigma(I_t) \gg \sigma(Y_t)$, $\sigma(C_t) < \sigma(Y_t)$.
  - Model more or less matches $\rho(C_t, Y_t)$, $\rho(I_t, Y_t)$, $\rho(K_t, Y_t)$, and $\rho(L_t, Y_t)$, but overstates $\rho(Y_t/L_t, Y_t)$.
  - Given the simple structure of the model, the fit is quite impressive.
  - ....But recall the lack of propagation (explanation is almost entirely exogenous).

Table 15.5: The unit-elastic RBC model

	(a) *US economy*		(b) *Model economy I*		(c) *Model economy II*	
x_t:	$\sigma(x_t)$	$\rho(x_t, Y_t)$	$\sigma(x_t)$	$\rho(x_t, Y_t)$	$\sigma(x_t)$	$\rho(x_t, Y_t)$
Y_t	1.76		1.35		1.76	
C_t	1.29	0.85	0.42	0.89	0.51	0.87
I_t	8.60	0.92	4.24	0.99	5.71	0.99
K_t	0.63	0.04	0.36	0.06	0.47	0.05
L_t	1.66	0.76	0.70	0.98	1.35	0.98
Y_t/L_t	1.18	0.42	0.68	0.98	0.50	0.87

Problematic features of the RBC model

- A number of puzzles remain. Solving these puzzles is at the forefront of current research in the area.
  (A) Employment variability puzzle.
  (B) Pro-cyclical real wage.
  (C) Productivity puzzle.
  (D) Unemployment.
  (E) Monetary aspects.

(A) Employment variability puzzle (1)

- *Key idea*: In reality $\sigma(Y_t)$ close to $\sigma(L_t)$, employment strongly pro-cyclical ($\rho(L_t, Y_t)$ near unity), and wages a-cyclical or mildly pro-cyclical ($\rho(w_t, Y_t)$ near zero). In the model:
  - With productivity shocks: ε_t^Z shifts labour demand, given upward sloping labour supply both w_t and L_t should be pro-cyclical.
  - With low labour supply elasticity (micro-evidence) $\sigma(L_t)$ should be low and $\sigma(w_t)$ should be high.
  - Hence, model under-predicts $\sigma(L_t)$ by quite a margin!
  - See **Figures 15 A and 15.10** to visualize correlations.

Figure 15 A: The good, the bad, and the average

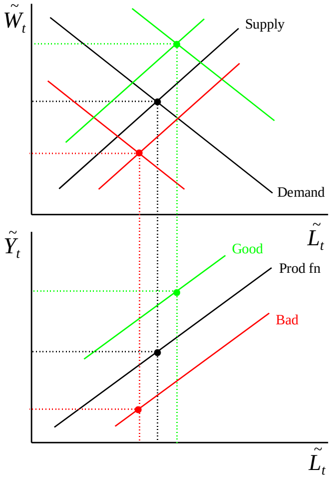
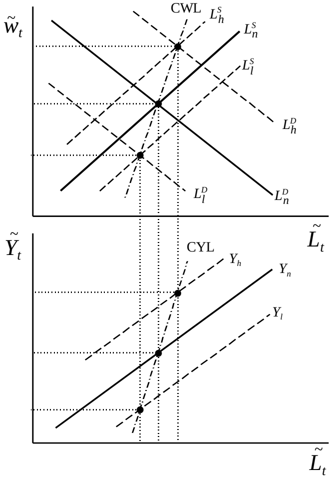


Figure 15.10: Visualizing contemporaneous correlations



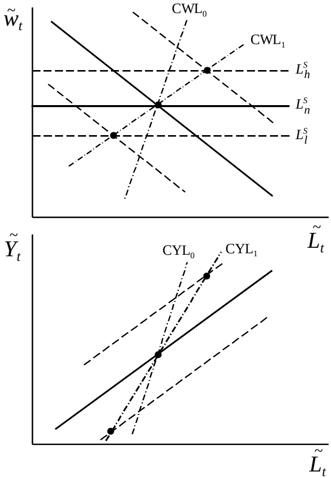
(A) Employment variability puzzle (2)

- *Solution to the puzzle*: We need a high substitution elasticity of labour supply [near horizontal labour supply curve] despite micro-evidence to the contrary. Indivisible labour model:
  - You either work 8 hours per day or 0 hours per day.
  - Lottery determines which is which each period.
  - Firm provides full insurance to the worker, and aggregate outcome is *as if* the representative agent has an infinite intertemporal substitution elasticity of labour supply.
  - See **Figure 15.11**.
  - In Table 15.5 panel (c), we observe that the lottery (or indivisible labour) model does better than the unit elastic model at matching $\sigma(L_t)$.

Table 15.5: The unit-elastic RBC model

	(a) *US economy*		(b) *Model economy I*		(c) *Model economy II*	
x_t:	$\sigma(x_t)$	$\rho(x_t, Y_t)$	$\sigma(x_t)$	$\rho(x_t, Y_t)$	$\sigma(x_t)$	$\rho(x_t, Y_t)$
Y_t	1.76		1.35		1.76	
C_t	1.29	0.85	0.42	0.89	0.51	0.87
I_t	8.60	0.92	4.24	0.99	5.71	0.99
K_t	0.63	0.04	0.36	0.06	0.47	0.05
L_t	1.66	0.76	0.70	0.98	1.35	0.98
Y_t/L_t	1.18	0.42	0.68	0.98	0.50	0.87

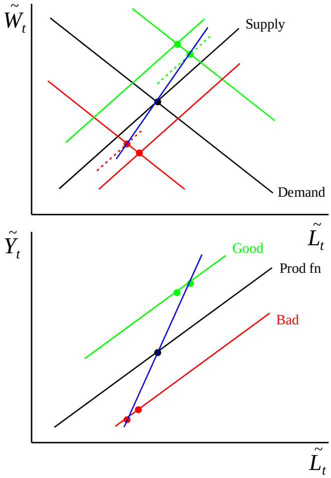
Figure 15.11: Lottery model and contemporaneous correlations



(B) Pro-cyclical wage

- *Key idea*: the unit-elastic model predicts too high a correlation between labour productivity [the wage] and output, $\rho(Y_t/L_t, Y_t) = 0.98$. In Hansen model we have $\rho(Y_t/L_t, Y_t) = 0.78$. In reality this correlation is much lower (0.42 for US).
- *Solution(s) of the puzzle*:
  - introduce shift factors in the labour supply function [both L^D and L^S shift out]
  - See **Figure 15 D**.
  - Use any of the theories explaining real wage rigidity (efficiency wages, union model, etcetera).

Figure 15D: Contemporaneous correlations and shift factors



(C) Productivity puzzle

- *Key idea*: If productivity shocks are predominant then L^D shifts explain high $\rho(Y_t/L_t, L_t)$ and $\rho(Y_t/L_t, Y_t)$. In reality $\rho(Y_t/L_t, L_t) \approx 0$ and $\rho(Y_t/L_t, Y_t)$ is weaker than predicted.
- *Solution(s) of the puzzle*:
  - Introduce shift factors in the labour supply function (both L^D and L^S shift out).
  - Nominal wage contracts and money supply shocks (nominal innovation shifts effective labour supply).
  - Labour hoarding by firms.
  - Non-market sector also subject to technology shocks.
  - Preference shocks affecting labour supply.
  - Shocks to government spending.

(D) Unemployment

- *Key idea*: The standard RBC models assume market clearing in the labour market. Hence all variation in employment is due to adjustment in hours worked. In reality 2/3 is explained by the extensive margin (in/out of employment) and 1/3 by the intensive margin (overtime etcetera).
- *Solution(s) to the puzzle*: Introduce unemployment model in the RBC framework, such as:
  - Search-theoretic approach.
  - Efficiency wage theory, union models.

Evaluation of the RBC approach

- The standard RBC model has a hard time matching data for real economies.
- It is difficult to believe that the productivity shocks explain all fluctuations in the economy: "If they are so important then why don't we read about them in the Wall Street Journal".
- Link between micro-data and calibration values not strong.
- Most important contribution of the approach is a methodological one:
  - Approach is flexible.
  - Micro-foundations for macro are important and can be improved (alternative market structures, heterogeneous households, etcetera).
  - Other shocks can be introduced (government spending shocks, tax shocks, changes in the real exchange rate, etcetera).
- Hence, the RBC^{+} approach is worth pursuing!