Foundations of Modern Macroeconomics
Second Edition

Chapter 12: New Keynesian economics

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Outline

Aims of this lecture

- Monopolistic competition as a micro-foundation for the multiplier (is it Keynesian?).
- Monopolistic competition and welfare-theoretic aspects.
  - The marginal costs of public funds and the multiplier.
- Monetary non-neutrality and price adjustment costs.
- Nominal and real rigidity: definitions and interaction.

The "Keynesian multiplier"

- Literature is related to the quantity rationing approach of the 1970s and 1980s.
- Key question: Who sets the prices?
    - Auctioneer? Fictional *deus ex machina*.
    - Price setting firms? Monopolistic competition.
- Develop simple static macro model with monopolistic competition.
- Study two cases:
    - Flexible prices.
    - Sticky prices.

A static model of monopolistic competition

- Households, many small firms, government.
- Horizontal product differentiation.
- Single production factor: labour.

Representative household

- Household utility:

$$U \equiv C^{\alpha}(1-L)^{1-\alpha}, \quad 0 < \alpha < 1$$

  - C is *composite* consumption.
  - L is labour supply ($1-L$ is leisure).
  - U is utility.

Representative household

- Composite consumption: S-D-S preferences:

$$C \equiv N^{\eta} \left[N^{-1} \sum_{j=1}^{N} {C_j}^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)}$$

  - N is number of product varieties.
  - C_j is variety j.
  - $\infty \gg \theta > 1$: close but imperfect substitutes.
  - $\eta \geq 1$: preference for diversity ("taste for variety"). Spread given production over as many varieties as possible if $\eta > 1$.

Representative household

- Household budget constraint:

$$\sum_{j=1}^{N} P_j C_j = WL + \Pi - T$$

  - P_j is price of variety j.
  - W is nominal wage rate.
  - Π is profit income of the household (from MC sector).
  - T is lump-sum taxes.

- Household chooses L, C_j (for $j = 1, 2, \cdots, N$) to maximize U subject to the budget constraint and taking as given W and P_j (for $j = 1, 2, \ldots, N$).

Representative household

- Solutions:

$$
\begin{aligned}
PC &= \alpha\,[W + \Pi - T] \\
W\,(1-L) &= (1-\alpha)\,[W + \Pi - T] \\
\frac{C_j}{C} &= N^{-(\theta+\eta)+\eta\theta}\left(\frac{P_j}{P}\right)^{-\theta} \qquad (j = 1, \cdots, N)
\end{aligned}
$$

  - P is a true price index depending on the P_j's and on N:

$$
P \equiv N^{-\eta}\left[N^{-\theta}\sum_{j=1}^{N} P_j{}^{1-\theta}\right]^{1/(1-\theta)}
$$

  - $W + \Pi - T$ is *full income*.
  - CD preferences imply constant spending shares.
  - Demand for variety j is price elastic (θ is the elasticity).

Representative firm

- Technology:

$$Y_j = \begin{cases} 0 & \text{if } L_j \leq F \\ \frac{L_j - F}{k} & \text{if } L_j \geq F \end{cases}$$

  - Y_j is output of firm j (producing variety j).
  - L_j is labour used by firm j.
  - $1/k$ is the (constant) marginal product of labour.
  - $F > 0$ is fixed costs ("overhead labour"): increasing returns to scale at firm level.

Representative firm

- Profit definition:

$$\Pi_j \equiv P_j Y_j - W\left[kY_j + F\right]$$

  - Π_j is profit of firm j.
  - $P_j Y_j$ is revenue of firm j.
  - $WL_j = W\left[kY_j + F\right]$ is costs of firm j.
- We anticipate that P_j depends on output by firm j (and on competitors' output), $P_j = P_j\left(Y_j\right)$, and adopt the Cournot assumption (firm j takes other firms' output as given).
- The choice problem is:

$$\underset{\{Y_j\}}{\text{Max}}\, \Pi_j = P_j(Y_j)Y_j - W\left[kY_j + F\right]$$

Representative firm

- The optimal decision rule is:

$$\begin{aligned} \frac{d\Pi_j}{dY_j} &= P_j + Y_j \cdot \frac{\partial P_j}{\partial Y_j} - Wk = 0 \Rightarrow \\ P_j &= \mu_j W k \end{aligned} \tag{a}$$

  - (a): price is set equal to a gross markup, μ_j, times marginal (labour) cost, Wk.
  - The gross markup is:

$$\mu_j \equiv \frac{\varepsilon_j}{\varepsilon_j - 1}, \qquad \varepsilon_j \equiv -\frac{\partial Y_j}{\partial P_j}\frac{P_j}{Y_j}$$

  The higher is ε_j, the lower is μ_j (lower market power).

Government

- Levies lump-sum tax, T, on household.
- Employs (useless) civil servants, L_G.
- Consumes a composite good, G, defined analogously to C:

$$G \equiv N^{\eta} \left[N^{-1} \sum_{j=1}^{N} G_j{}^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)}$$

  - η same as in C: price index same.
  - θ same as in C: price elasticity same.

Government

- Assume government is a cost minimizer: chooses G_j (for $j = 1, 2, \cdots, N$) in order to "produce" a given level of G at least cost.
- Derived government demand for variety j is:

$$\frac{G_j}{G} = N^{-(\theta+\eta)+\eta\theta} \left(\frac{P_j}{P} \right)^{-\theta} \qquad (j = 1, \cdots, N)$$

Some loose ends

- Demand facing firm j is:

$$
\begin{aligned}
Y_j &= C_j + G_j \\
&= \underbrace{(C+G)\,N^{-(\theta+\eta)+\eta\theta}}_{(a)} \underbrace{\left(\frac{P_j}{P}\right)^{-\theta}}_{(b)}
\end{aligned}
$$

  - (a): shift factors affecting firm j's demand.
  - (b): relative price factor affecting firm j's demand.

- Demand elasticity is constant (and equal to θ):

$$\mu_j = \frac{\theta}{\theta - 1} = \mu \qquad \text{(for all } j = 1, \cdots, N\text{)}$$

Some loose ends

- Symmetric model: for all $j = 1, \cdots, N$ we have:

$$P_j = \mu W k = \bar{P}, \quad Y_j = \bar{Y}, \quad L_j = \bar{L}$$

- Aggregate quantity index, Y, is:

$$Y \equiv \frac{\sum_{j=1}^{N} P_j Y_j}{P}$$

- Labour market equilibrium (LME):

$$L = L_G + \sum_{j=1}^{N} L_j$$

- Summary of the model is provided in Table 12.1. (Briefly run through table; LME implied by model via Walras Law).
- We can use W as the numeraire (everything measured in wage units).

Table 12.1: A simple macro model with monopolistic competition

$$Y = C + G \tag{T1.1}$$

$$PC = \alpha I_F, \; I_F \equiv [W + \Pi - T] \tag{T1.2}$$

$$\Pi \equiv \sum_{j=1}^{N} \Pi_j = \frac{1}{\theta} PY - WNF \tag{T1.3}$$

$$T = PG + WL_G \tag{T1.4}$$

$$P = N^{1-\eta}\bar{P} = N^{1-\eta}\mu W k \tag{T1.5}$$

$$W(1-L) = (1-\alpha) I_F \tag{T1.6}$$

$$V = \frac{I_F}{P_V}, \; P_V = \left(\frac{P}{\alpha}\right)^{\alpha} \left(\frac{W}{1-\alpha}\right)^{1-\alpha} \tag{T1.7}$$

Balanced-budget multiplier

- Short-run multiplier (N fixed–no entry of new firms).
  - Financed with lump-sum taxes, $T \uparrow$.
  - Financed by firing civil servants, $L_G \downarrow$ (proxy for "bond financing" in a static model).
- Long-run multiplier (N variable–free entry of firms).

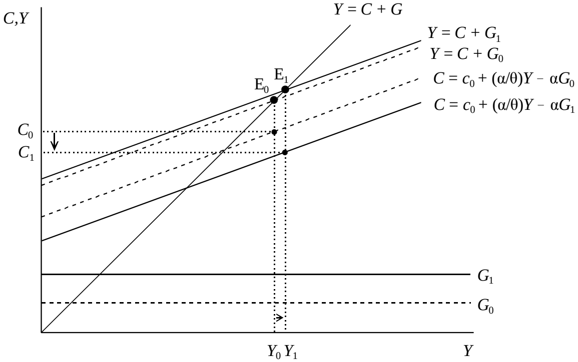
Short-run multiplier

- $N = N_0$ (fixed); GBC: $dG = d\left(T/P\right)$.
- Aggregate *consumption function* is:

$$C = \underbrace{\alpha\left[1 - N_0 F - L_G\right]W}_{c_0} + \frac{\alpha}{\theta}Y - \alpha G$$

  - c_0 is fixed in the short run.
  - W/P is fixed in the short run (in the SE P depends on N only).
  - C depends on Y via the profit channel (as $Y \uparrow \Rightarrow$ aggregate profit income, $\Pi \uparrow \Rightarrow C \uparrow$ (and $(1-L)\uparrow$).
  - G effect is due to taxation ($G \uparrow \Rightarrow T \uparrow$, $C \downarrow$ (and $(1-L)\downarrow$).
- The effect of an increase in G is illustrated in Figure 12.1.

Figure 12.1: Government spending multiplier



Short-run multiplier I

- Effect on output:

$$\left(\frac{dY}{dG}\right)_T^{SR} = \left(\frac{\theta d\Pi}{PdG}\right)_T^{SR}$$
$$= (1-\alpha)\left[1+\sum_{i=1}^{\infty}(\alpha/\theta)^i\right] = \frac{1-\alpha}{1-\alpha/\theta} > 1-\alpha$$

  - Degree of monopoly, $\frac{1}{\theta}$, does magnify the expansionary effect!

- Effect on consumption:

$$-\alpha < \left(\frac{dC}{dG}\right)_T^{SR} = -\frac{\theta-1}{\theta-\alpha}\alpha < 0$$

  - Inconsistent with Haavelmo b-b multiplier (where C is unaffected)!

Short-run multiplier I

- Effect on employment:

$$0 < W \left(\frac{dL}{dG} \right)_T^{SR} = \frac{\theta - 1}{\theta - \alpha}(1 - \alpha) < 1 - \alpha$$

  - Labour supply effect explains output expansion (rather classical mechanism).

Short-run multiplier II

- $N = N_0$ (fixed); GBC: $dG = -W dL_G$.
- Aggregate *consumption function* is:

$$C = \alpha \left[1 - N_0 F\right] W + \frac{\alpha}{\theta} Y - \alpha \frac{T}{P}$$

  - T/P is constant (by assumption).

Short run multiplier II

- Effects of increase in government consumption:

$$
\begin{aligned}
\left(\frac{dY}{dG}\right)_{L_G}^{SR} &= \left(\frac{\theta d\Pi}{PdG}\right)_{L_G}^{SR} = \left[1 + \sum_{i=1}^{\infty}(\alpha/\theta)^i\right] = \frac{1}{1-\alpha/\theta} > 1 \\
\left(\frac{dC}{dG}\right)_{L_G}^{SR} &= \frac{\alpha}{\theta-\alpha} > 0 \\
W\left(\frac{dL}{dG}\right)_{L_G}^{SR} &= -\frac{1-\alpha}{\theta-\alpha} < 0
\end{aligned}
$$

  - $\frac{dY}{dG}$ exceeds unity as consumption rises $\left(\frac{dC}{dG} > 0\right)$!
  - Labour supply falls (wealth effect) but output expansion made possible by release of labour from the unproductive to the productive sector.

Long-run multiplier

- Following a fiscal shock there are excess profits to be gained ($\Pi > 0$).
- In absence of barriers to entry one would expect entry of new firms.
- Ad hoc entry/exit rule:

$$\dot{N} = \gamma_N(\Pi/P) = \gamma_N \left[\theta^{-1}Y - WNF\right], \qquad \gamma_N > 0$$

- What is the long-run multiplier? Assume there are no civil servants ($L_G = 0$).
- Goods market equilibrium (GME) line:

$$\begin{aligned} Y &= \alpha\left[1 - NF\right]W + (\alpha/\theta)Y + (1-\alpha)G \\ &= \left[\frac{\alpha(1-NF)}{\mu k(1-\alpha/\theta)}\right] N^{\eta-1} + \left[\frac{1-\alpha}{1-\alpha/\theta}\right] G \qquad \text{(GME)} \end{aligned}$$

Long-run multiplier

- Continued.
  - We have used the pricing rule:

$$W = \frac{N^{\eta-1}}{\mu k}$$

- The zero-profit (ZP) condition is:

$$Y = \frac{\theta F N^{\eta}}{\mu k} \tag{ZP}$$

- In **Figure 13.2** we illustrate the impact, transitional, and long-run effect of a tax-financed increase in government consumption.
- ZP slopes up.
  - There is entry (exit) of firms to the left (right) of the ZP line (see the horizontal arrows).

Long-run multiplier

- Slope of GME is ambiguous due to interplay of offsetting effects.
  - Diversity effect if $\eta > 1$: renders slope positive.
  - Fixed-cost effect (for $F > 0$): renders the slope negative.
- Two special cases for GME:
  - Standard S-D-S preferences (set $\eta = \mu$): GME slopes up (see Figure 12.2). Long-run multiplier is larger than the short-run multiplier:

$$
\begin{aligned}
\left(\frac{dY}{dG}\right)_T^{LR,\eta=\mu} &= \frac{1-\alpha}{1-\frac{\mu-1}{\mu}\left[\alpha+(1-\alpha)\omega_C\right]} \\
&= \frac{1-\alpha}{1-(1/\theta)\left[\alpha+(1-\alpha)\omega_C\right]} > \frac{1-\alpha}{1-\alpha/\theta} \equiv \left(\frac{dY}{dG}\right.
\end{aligned}
$$

Long-run multiplier

- Continued.
  - No PFD at all (set $\eta = 1$): GME slopes down. Long-run multiplier "vanishes":

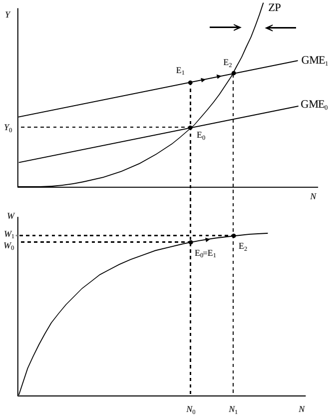
$$0 < \left(\frac{dY}{dG}\right)_T^{LR,\eta=1} = (1-\alpha) < \frac{1-\alpha}{1-\alpha/\theta} \equiv \left(\frac{dY}{dG}\right)_T^{SR}$$

- Diversity effect shows up in the "aggregate production function" for this economy, relating Y to L:

$$Y = \frac{(\theta F)^{1-\eta}}{\mu k} L^{\eta}$$

  - $\eta > 1$ implies IRTS at the aggregate level.
  - Some hard-core Keynesians argue that IRTS are (or should be) the central element of Keynesian economics (PFD is one simple mechanism).

Figure 12.2: Multipliers and firm entry



Welfare effects

- Establish link between the multiplier and welfare.
- Look at short-run multiplier only.
- Handy tool: the indirect utility function (IUF):

$$
\begin{aligned}
V &\equiv \frac{I_F}{P_V} \equiv \frac{W + \Pi/P - T/P}{P_V/P} \\
\frac{P_V}{P} &\equiv \frac{W^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}
\end{aligned}
$$

Tax-financed fiscal policy

- Substitute GBC and profit definition into IUF:

$$V \equiv \frac{[1 - NF - L_G]\, W + (1/\theta)Y - G}{P_V/P}$$

  - Recall: N, W, P, and P_V all fixed in the short run.
  - V rises with Y: output too low from a social point of view.
  - V falls with G: taxation hurts.

- Differentiate V w.r.t. G:

$$\left(\frac{dV}{dG}\right)_T^{SR} = \frac{P}{P_V}\left[\frac{1}{\theta}\left(\frac{dY}{dG}\right)_T^{SR} - 1\right] = -\frac{P}{P_V}\frac{\theta - 1}{\theta - \alpha} < 0$$

Tax-financed fiscal policy

- Fiscal policy is not welfare increasing (contra Keynes claim about empty bottles). Reasons for this un-Keynesian result:
    - Flexible wages and clearing labour market.
    - Every unit of labour is productive.
- Let us reconsider the bond-financed case again.

Bond-financed fiscal policy

- Extra spending financed by firing unproductive civil servants ($dG = -W dL_G$).
- For this case the IUF is:

$$V \equiv \frac{[1 - NF]\, W + (1/\theta) Y - T/P}{P_V/P}$$

  - T/P is fixed.
  - Only output effect remains.
- Differentiate V w.r.t. G:

$$\left(\frac{dV}{dG}\right)^{SR}_{L_G} = \left(\frac{P}{P_V}\right) \frac{1}{\theta} \left(\frac{dY}{dG}\right)^{SR}_{L_G} = \frac{P}{P_V} \frac{1}{\theta - \alpha} > 0$$

Bond-financed fiscal policy

- Fiscal policy increases welfare!
  - Labour shifted from unproductive to productive activities.
  - But: a tax cut would improve welfare even more:

$$\begin{aligned}\left(\frac{dV}{d(T/P)}\right)_{L_G}^{SR} &= \frac{P}{P_V}\left[\frac{1}{\theta}\left(\frac{dY}{d(T/P)}\right)_{L_G}^{SR} - 1\right] \\ &= \frac{P}{P_V}\frac{\theta}{\theta-\alpha} > 0\end{aligned}$$

Monopolistic competition and money

- Turn from real to monetary model.
- Usual short-cut trick: put money in the utility function.
  - Money saves on shoe-leather costs.
  - Shopping costs depend on leisure and money.
  - Money makes shopping easier (saves valuable leisure).
- Household utility function:

$$U \equiv \left[C^{\alpha}(1-L)^{1-\alpha}\right]^{\beta}\left(\frac{M}{P}\right)^{1-\beta}, \quad 0<\alpha,\beta<1,$$

where M is *nominal* money balances.

Monopolistic competition and money

- Household budget constraint:

$$PC + W(1-L) + M = M_0 + W + \Pi - T,$$

where M_0 is *initial* money balances (accumulated in the previous period).

- Household chooses C, L, and M to maximize U subject to the budget constraint. Solutions:

$$\begin{aligned} PC &= \alpha\beta I_F \\ I_F &\equiv M_0 + W + \Pi - T \\ W(1-L) &= \beta(1-\alpha) I_F \\ M &= (1-\beta) I_F \end{aligned}$$

Monopolistic competition and money

- Assume that the policy maker maintains a constant money supply. Money market equilibrium (MME) is then:

$$M = M_0$$

- The monetary monopolistic competition model is summarized in Table 12.2. Some remarks:
  - M_0 features in the indirect utility function (IUF, eqn (T2.8)).
  - Helicopter drop of money, $dM_0 > 0$, has no welfare effects.
  - Money is neutral / classical dichotomy.
  - $dM_0 > 0$ inflates nominal variables but leaves real variables unchanged.
  - In and of itself, monopolistic competition does not cause monetary non-neutrality.

Table 12.2: A simple monetary monopolistic competition model

$$Y = C + G \tag{T2.1}$$

$$C = \alpha\beta \frac{I_F}{P}, \quad \frac{I_F}{P} \equiv \frac{M_0}{P} + \frac{W}{P} + \frac{\Pi}{P} - \frac{T}{P} \tag{T2.2}$$

$$\frac{\Pi}{P} \equiv \frac{1}{\theta} Y - \frac{W}{P} N F \tag{T2.3}$$

$$\frac{T}{P} = G + \frac{W}{P} L_G \tag{T2.4}$$

$$\frac{P}{W} = \mu k N^{1-\eta} \tag{T2.5}$$

$$\frac{W}{P}(1-L) = \beta(1-\alpha)\frac{I_F}{P} \tag{T2.6}$$

$$\frac{M_0}{P} = (1-\beta)\frac{I_F}{P} \tag{T2.7}$$

$$V = \frac{I_F}{P_V}, \qquad P_V = \left(\frac{P}{\alpha\beta}\right)^{\alpha\beta} \left(\frac{W}{\beta(1-\alpha)}\right)^{\beta(1-\alpha)} \left(\frac{P}{1-\beta}\right)^{1-\beta} \tag{T2.8}$$

Properties of the monetary monopolistic competition model

- Model can be reduced to two schedules.
- Focus on the short run: N and W are fixed.
- Goods market equilibrium (GME) locus:

$$Y = \frac{\alpha\left[1 - NF - L_G\right]W + (1-\alpha)G}{1 - \alpha/\theta}$$

- Money market equilibrium (MME) locus:

$$\frac{M_0}{P} = \frac{1-\beta}{\beta}\Big[\left[1 - NF - L_G\right]W + (1/\theta)Y - G\Big]$$

- Classical dichotomy:
  - GME fixes Y independently from M_0.
  - MME then fixes P.

Properties of the monetary monopolistic competition model

- Effects on lump-sum tax financed fiscal policy:

$$
\begin{aligned}
0 &< \left(\frac{dY}{dG}\right)_T^{SR} = \frac{1-\alpha}{1-\alpha/\theta} < 1 \\
\left(\frac{dW}{W}\right)_T^{SR} &= \left(\frac{dP}{P}\right)_T^{SR} = \left(\frac{d\bar{P}}{\bar{P}}\right)_T^{SR} \\
\left(\frac{dM_0/P}{dG}\right)_T^{SR} &= -\frac{M_0}{P^2}\left(\frac{dP}{dG}\right)_T^{SR} = -\frac{(1-\beta)(\theta-1)}{\beta(\theta-\alpha)} < 0
\end{aligned}
$$

- Monetary part of the model is more Classical than Keynesian!

Sticky prices and monetary non-neutrality

- Under which conditions would a price-setting agent change his price or keep it unchanged?
- Key ingredient of the New Keynesian approach: non-trivial price adjustment costs (remember Modigliani (1944)?).
- Two types of price adjustment costs:
  - Menu costs (non-convex): fixed cost per price change (e.g. informing dealers, reprinting price lists or "menu's", etcetera).
  - Convex costs: costs depending on the size of the price change (e.g. adverse reactions by customers to large price changes).

Menu costs

- Develop simplified version of the Blanchard-Kiyotaki model (competitive labour market).
- Focus on the short run: fixed number of firms (N).
- Household utility is additively separable in $(C, M/P)$ and L:

$$\begin{aligned} U(C, M/P, L) &\equiv U^1(C, M/P) - U^2(L) \\ &= C^{\alpha}(M/P)^{1-\alpha} - \gamma_L \frac{L^{1+1/\sigma}}{1+1/\sigma}, \qquad 0 < \alpha < 1 \end{aligned}$$

  - $\sigma > 0$ regulates labour supply elasticity.
  - C is composite differentiated good ($\eta = 1$: no diversity preference).

Menu costs

- Household budget restriction:

$$PC + M = WL + M_0 + \Pi - T \; (\equiv I)$$

- Use two-stage budgeting:
  - *stage 1*: maximizing $U^1(C, M/P)$ subject to $PC + M = I$ yields:

$$\begin{aligned} PC &= \alpha I \\ M &= (1-\alpha)I \\ V^1(I/P) &= \alpha^\alpha (1-\alpha)^{1-\alpha}(I/P) \end{aligned}$$

Menu costs

- Continued.
  - *stage 2*: maximizing $V^1(I/P)$ (the IUF associated with stage 1 problem) subject to $I \equiv WL + M_0 + \Pi - T$ yields:

$$L = \left(\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{\gamma_L}\right)^\sigma \left(\frac{W}{P}\right)^\sigma \qquad \text{(a)}$$

$$\frac{I}{P} = \left(\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{\gamma_L}\right)^\sigma \left(\frac{W}{P}\right)^{1+\sigma} + \frac{M_0 + \Pi - T}{P}$$

- Key feature of the labour supply equation (a): no income effect. Substitution effect parameterized by σ. For future reference:
  - σ large: near horizontal labour supply equation. Small change in W causes large change in L. High degree of *real rigidity* (empirically problematic).
  - σ small: near vertical labour supply equation. Small change in L causes large change in W. Low degree of real rigidity (empirically realistic).

Menu costs

- Firms face demand from private sector and from the government (same elasticity; no diversity effect).

$$Y_j(P_j, P, Y) = \left(\frac{P_j}{P}\right)^{-\theta} \frac{Y}{N}$$

- Aggregate demand is:

$$Y = C + G = \frac{\alpha}{1-\alpha} \cdot \frac{M}{P} + G$$

  - G raises aggregate demand.
  - If P is somehow fixed (e.g. due to menu costs), then M will also raise aggregate demand.

Menu costs

- Technology of the differentiated product firm is slightly more general than before:

$$Y_j = \begin{cases} 0 & \text{if } L_j \leq F \\ \left[\frac{L_j - F}{k}\right]^{\gamma} & \text{if } L_j \geq F \end{cases}$$

  - We had $\gamma = 1$ but now also allow for $0 < \gamma < 1$.
  - γ regulates curvature of the marginal cost curve ($\gamma < 1$, MC falls with output, and AC is *U*-shaped).
- Firm chooses its price, P_j, in order to maximize its profit:

$$\Pi_j(P_j, P, Y) \equiv \underbrace{P_j Y_j(P_j, P, Y)}_{\text{revenue}} - \underbrace{W\left[k\left(Y_j(P_j, P, Y)\right)^{1/\gamma} + F\right]}_{\text{total cost}}$$

  - Bertrand assumption: firm takes prices of close competitors as given (P is an aggregate of these prices).

Menu costs

- The optimal price for firm j satisfies the FONC:

$$\begin{aligned}
\frac{d\Pi_j(P_j, P, Y)}{dP_j} &= [P_j - MC_j]\,\frac{\partial Y_j(P_j, P, Y)}{\partial P_j} + Y_j(P_j, P, Y) \\
&= Y_j(P_j, P, Y)\left[1 + \frac{P_j - MC_j}{P_j}\,\frac{P_j}{Y_j(\cdot)}\,\frac{\partial Y_j(\cdot)}{\partial P_j}\right] \\
&= Y_j(P_j, P, Y)\left[1 - \theta\frac{P_j - MC_j}{P_j}\right] = 0 \qquad \text{(a)}
\end{aligned}$$

Menu costs

- We derive from (a) that the optimal price is a markup times marginal cost (MC_j), i.e. $P_j = \mu MC_j$ or:

$$P_j = \left(\frac{\mu k}{\gamma}\right) W Y_j^{(1-\gamma)/\gamma}, \qquad \mu = \frac{\theta}{\theta - 1} > 1$$

  - Without menu costs optimal pricing rule under Cournot and Bertrand same (not so with menu costs!).
  - Relative price of firm j depends on Y_j and on the real wage, W/P:

$$\frac{P_j}{P} = \frac{\mu k}{\gamma} \frac{W}{P} Y_j^{(1-\gamma)/\gamma}$$

    This is where the aggregate labour market comes into play.

- Model is summarized in Table 12.3.

Table 12.3: A simplified Blanchard-Kiyotaki model (no menu costs)

$$Y = C + G \tag{T3.1}$$

$$C = \frac{\alpha}{1-\alpha}\frac{M_0}{P} = \begin{cases} \alpha\left[\omega^{-\sigma}\left(\frac{W}{P}\right)^{1+\sigma} + \frac{M_0}{P} + \frac{\Pi}{P} - G\right] & \text{(if } \sigma < \infty) \\ \alpha\left[\left(\frac{W}{P}\right)L + \frac{M_0}{P} + \frac{\Pi}{P} - G\right] & \text{(if } \sigma \to \infty) \end{cases} \tag{T3.2}$$

$$\frac{\Pi}{P} \equiv \frac{\mu-\gamma}{\mu}Y - \frac{W}{P}NF \tag{T3.3}$$

$$\frac{P}{W} = (\mu k/\gamma)\left(\frac{Y}{N}\right)^{(1-\gamma)/\gamma} \tag{T3.4}$$

$$\frac{W}{P} = \begin{cases} \omega L^{1/\sigma} & \text{(if } \sigma < \infty) \\ \omega & \text{(if } \sigma \to \infty) \end{cases} \tag{T3.5}$$

Notes: $\omega \equiv \gamma_L[\alpha^\alpha(1-\alpha)^{1-\alpha}]^{-1} > 0$ and $\mu \equiv \theta/(\theta-1)$.

The flex-price version of the B-K model

- Money is neutral: doubling M_0 doubles all nominal variables (P, Π, W) but leaves the real variables $(Y, C, L, M_0/P, \Pi/P, W/P)$ unaffected.
- Fiscal policy completely ineffective. There is no income effect in labour supply, so concomitant tax increase does not affect employment: $\frac{dY}{dG} = \frac{dL}{dG} = \frac{dW}{dG} = 0$ and $\frac{dC}{dG} = -1$ (one-for-one crowding out of private by public consumption)!
- Flex-price B-K model is hyper-classical indeed.

The menu-cost insight

- Small costs of changing one's actions can have large allocational and welfare effects.
- Or: "small deviations from rationality make significant differences to equilibria" (Akerlof & Yellen).
- In macro-context: following a shock to aggregate demand, is it possible that:
  (a) Price stickiness is privately efficient?
  (b) Price stickiness exists in general equilibrium?
  (c) Price stickiness has first-order effect on economic welfare?

Menu-cost insight

- In our version of the B-K model we verify the various parts of the “menu-cost agenda”:
    - Part (a) easy: application of the envelope theorem.
    - Part (b) tricky: intricate general equilibrium effects (interaction nominal and real rigidity).
    - Part (c) follows once (a)-(b) are covered.

Can it be rational not to change one's price?

- Individual firm cares about its profits only.
- Optimal price chosen by firm j satisfies:

$$\frac{P_j^*}{P} = \left[\frac{\mu k}{\gamma} \cdot \frac{W}{P} \cdot \left(\frac{Y}{N}\right)^{(1-\gamma)/\gamma}\right]^{\gamma/[\gamma+\theta(1-\gamma)]}$$

  - We have combined the optimal pricing rule with the firm's demand function.
  - P, Y, and W are all taken as exogenous by the firm (as is N).
  - We can write $P_j^* = P_j^*\,(P, Y, W)$.
- The optimized profit function of firm j can be written as:

$$\Pi_j^*(P,Y,W) \equiv P_j^*(\cdot)Y_j\left(P_j^*(\cdot),P,Y\right) - W\left[k\left[Y_j\left(P_j^*(\cdot),P,Y\right)\right]^{1/\gamma} + \right.$$

Can it be rational not to change one's price?

- By differentiating $\Pi_j^*(\cdot)$ w.r.t. aggregate demand, Y, we find the envelope result:

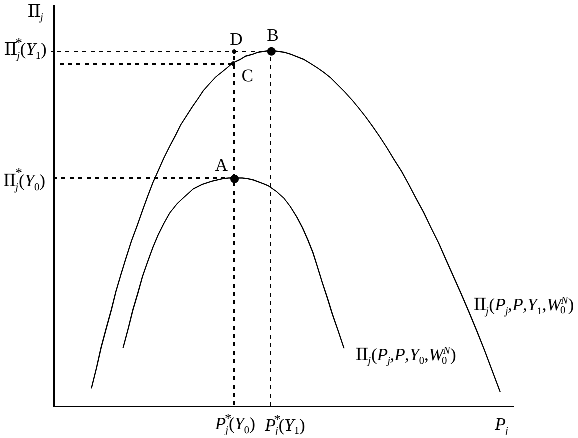
$$\begin{aligned}
\frac{d\Pi_j^*(\cdot)}{dY} &= \left[\left[P_j^*(\cdot) - MC_j^*(\cdot)\right] \left(\frac{\partial Y_j(P_j, P, Y)}{\partial P_j} \right)_{P_j = P_j^*} + Y_j(P_j^*(\cdot), P, Y) \right] \times \\
&\quad \frac{dP_j^*(\cdot)}{dY} + \left[P_j^*(\cdot) - MC_j^*(\cdot)\right] \frac{\partial Y_j(P_j^*(\cdot), P, Y)}{\partial Y} \\
&= \left[\frac{\partial \Pi_j(\cdot)}{\partial P_j} \right]_{P_j = P_j^*} \left(\frac{dP_j^*(\cdot)}{dY} \right) + \left[P_j^*(\cdot) - MC_j^*(\cdot)\right] \frac{\partial Y_j(P_j^*(\cdot), P, Y)}{\partial Y} \\
&= \left[P_j^*(\cdot) - MC_j^*(\cdot)\right] \frac{\partial Y_j(P_j^*(\cdot), P, Y)}{\partial Y} \equiv \frac{\partial \Pi_j(\cdot)}{\partial Y}
\end{aligned}$$

- To a first-order of magnitude, the effect on the profit of firm j of a change in aggregate demand is the same whether or not firm j changes its price optimally following the aggregate demand shock. *Hence, small menu costs will prevent price adjustment by firm j.*

Graphical representation

- The menu-cost result is illustrated in **Figure 12.3**.
- Initially aggregate demand is Y_0 and optimum is at point A.
- Assume Y rises (to Y_1): ceteris paribus (W, P):
  - Profit is higher for all P_j and (provided $\gamma < 1$) and new optimum is at point B (north-east from A–output expansion increases marginal cost).
  - Keeping the old price costs firm j DC in foregone profits. This is small because "objective functions are flat at the top".
  - We have completed $\boxed{\text{part (a)}}$! Next we work on the GE repercussions.

Figure 12.3: Menu costs



General equilibrium effects

- All firms are in the same position as firm j is in, so they all want to expand output following an increase in aggregate demand.
  - Where does the required labour come from?
  - Will there be cost increases because labour is scarce?
- Two cases:
  - σ large (highly elastic labour supply): menu cost equilibrium exists.
  - σ finite/low (moderate labour supply elasticity): general equilibrium effects destroy menu cost equilibrium (simulations in **Tables 12.4-12.5**).

Table 12.4: Menu costs and the markup

	$\mu = 1.10$			$\mu = 1.25$		
$\Delta M = 0.05$, $\sigma_Y = 0.1$	menu costs	welfare gain	ratio	menu costs	welfare gain	ratio
$\sigma = 0.2$	20.44	28.6	1.40	18.10	29.1	1.61
$\sigma = 0.5$	7.85	28.9	3.68	6.96	29.4	4.22
$\sigma = 1$	3.95	29.0	7.35	3.51	29.5	8.40
$\sigma = 2.5$	1.69	29.1	17.18	1.51	29.5	19.49
$\sigma = 5$	0.94	29.1	30.80	0.86	29.6	34.37
$\sigma = 10^6$	0.20	29.1	146.12	0.20	29.6	145.73

Table 12.4: Menu costs and the markup (continued)

	$\mu = 1.50$			$\mu = 2$		
$\sigma = 0.2$	15.23	29.8	1.96	11.53	30.6	2.65
$\sigma = 0.5$	5.87	30.0	5.11	4.55	30.8	6.76
$\sigma = 1$	2.99	30.1	10.06	2.35	30.8	13.12
$\sigma = 2.5$	1.32	30.1	22.80	1.06	30.8	29.12
$\sigma = 5$	0.76	30.1	39.56	0.63	30.9	48.68
$\sigma = 10^6$	0.21	30.1	144.67	0.21	30.9	144.95

Table 12.5: Menu costs and the elasticity of marginal cost

	$\sigma_Y = 0$			$\sigma_Y = 0.05$		
$\Delta M = 0.05$ $\mu = 1.25$	menu costs	welfare gain	ratio	menu costs	welfare gain	ratio
$\sigma = 0.2$	17.44	29.2	1.67	17.72	29.2	1.65
$\sigma = 0.5$	6.61	29.4	4.45	6.76	29.4	4.35
$\sigma = 1$	3.17	29.5	9.31	3.34	29.5	8.84
$\sigma = 2.5$	1.19	29.5	24.73	1.36	29.5	21.69
$\sigma = 5$	0.52	29.6	56.72	0.70	29.6	42.23
$\sigma = 10^6$	$\rightarrow 0$	29.6	$\rightarrow \infty$	0.04	29.6	672.74

Table 12.5: Menu costs and the elasticity of marginal cost (continued)

	$\sigma_Y = 0.1$			$\sigma_Y = 0.2$		
$\sigma = 0.2$	18.10	29.1	1.61	18.54	29.1	1.57
$\sigma = 0.5$	6.96	29.4	4.22	7.34	29.4	4.00
$\sigma = 1$	3.51	29.5	8.40	3.84	29.5	7.67
$\sigma = 2.5$	1.51	29.5	19.49	1.83	29.5	16.16
$\sigma = 5$	0.86	29.6	34.37	1.15	29.5	25.60
$\sigma = 10^6$	0.20	29.6	145.73	0.49	29.6	60.60

The menu-cost equilibrium (MCE)

- Assume $\sigma \to \infty$ so that the real wage is rigid: if P does not change (because all firms keep their old prices) then neither does the nominal wage W.
- Our partial equilibrium story is equivalent to the general equilibrium effects–see Figure 13.3. $\boxed{\text{Part (b)}}$ is confirmed.
- Properties of the menu cost equilibrium:
  - Fiscal policy is highly effective.
  - Monetary policy is highly effective.
  - Both policies have first-order welfare effects. $\boxed{\text{Part (c)}}$ is confirmed.

Fiscal policy in the MCE

- in the MCE, the model can be condensed to:

$$Y = C + G$$
$$C = \frac{\alpha}{1-\alpha}\frac{M_0}{P} = \alpha\left[Y + M_0/P - G\right]$$

where P is fixed (because all firms keep their price unchanged).

Fiscal policy in the MCE

- Fiscal policy: a lump-sum tax financed increase in G is quite effective:

$$
\begin{aligned}
\left(\frac{dY}{dG}\right)_T^{MCE} &= 1 \\
\left(\frac{dC}{dG}\right)_T^{MCE} &= \left(\frac{d(M_0/P)}{dG}\right)_T^{MCE} = 0 \\
\frac{W}{P}\left(\frac{dL}{dG}\right)_T^{MCE} &= \frac{1}{\mu}\left(\frac{dY}{dG}\right)_T^{MCE} = \frac{\theta-1}{\theta} > 0
\end{aligned}
$$

  - $G \uparrow$ causes shift in aggregate demand.
  - $Y_j \uparrow$ (for all firms) but P_j (and thus P) unaffected.
  - Labour supply horizontal, so W unchanged but $L \uparrow$.
  - Household income rise (both wage income and profit income)–multiplier effect.
- Looks exactly like the Haavelmo multiplier (mentioned in Chapter 1).

Monetary policy in the MCE

- Helicopter drop of money balances: $dM_0 > 0$ also increases output, consumption, and employment:

$$P\left(\frac{dY}{dM_0}\right)^{MCE} = P\left(\frac{dC}{dM_0}\right)^{MCE} = \mu W\left(\frac{dL}{dM_0}\right)^{MCE} = \frac{\alpha}{1-\alpha} >$$

  - $M_0 \uparrow$ causes increase in C (wealthier households).
  - $Y_j \uparrow$ (for all firms) but P_j (and thus P) unaffected.
  - Labour supply horizontal, so W unchanged, but $L \uparrow$.
  - Household income rise (both wage income and profit income)–multiplier effect.

Welfare effects of policy in the MCE

- In the menu-cost equilibrium, the hyper-classical model becomes hyper-Keynesian (strong effects of policy).
- But: what are the welfare effects of fiscal and monetary policy?

Welfare effects of policy in the MCE

- Indirect utility function (IUF):

$$
\begin{aligned}
V &= \alpha^{\alpha}(1-\alpha)^{1-\alpha}\left[Y + \frac{M_0}{P} - G\right] - \gamma_L L \\
&= \alpha^{\alpha}(1-\alpha)^{1-\alpha}\left[\frac{M_0+\Pi}{P} - G\right] + \left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}\left(\frac{W}{P}\right) - \gamma_L\right] \\
&= \alpha^{\alpha}(1-\alpha)^{1-\alpha}\left[\frac{M_0+\Pi}{P} - G\right] \qquad \text{(a)}
\end{aligned}
$$

  - Used $\Pi \equiv PY - WL$ in the first step.
  - Used labour supply, $\gamma_L = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\left(\frac{W}{P}\right)$, in second step: labour supply set optimally so variation in L causes no first-order welfare effect.

- From (a) we conclude that both policies cause first-order welfare effect.

Welfare effects of policy in the MCE

- Fiscal policy:

$$
\begin{aligned}
\left(\frac{dV}{dG}\right)_T^{MCE} &= \alpha^\alpha(1-\alpha)^{1-\alpha}\left[\left(\frac{dY}{dG}\right)_T^{MCE} - 1\right] - \gamma_L\left(\frac{dL}{dG}\right)_T^{MC} \\
&= -\frac{\gamma_L}{\mu}\left(\frac{P}{W}\right) \\
&= -\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{\mu} < 0
\end{aligned}
$$

  - $\frac{dY}{dG} = 1$ does not come for free as the household must supply more hours of labour.
  - Net effect on welfare is negative.

Welfare effects of policy in the MCE

- Monetary policy:

$$\begin{aligned}\left(\frac{dV}{dM_0}\right)^{MCE} &= \alpha^{\alpha}(1-\alpha)^{1-\alpha}\left[\frac{1}{P}+\left(\frac{d(\Pi/P)}{dM_0}\right)^{MCE}\right] \\ &= \frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{P}\left[1+P\left(\frac{dY}{dM_0}\right)^{MCE}-W\left(\frac{dL}{dM_0}\right)^{MCE}\right] \\ &= \underbrace{\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{P}}_{(a)}\underbrace{\left[1+\frac{1}{\theta}\frac{\alpha}{1-\alpha}\right]}_{(b)}>0\end{aligned}$$

  - (a): marginal utility of nominal income (positive).
  - (b), first term: *liquidity effect* (also exists in competitive model).
  - (b), second term: *profit effect* (specific to B-K model).
  - Total effect on welfare is positive, more so the larger is the degree of monopoly ($\frac{1}{\theta}$).
- Note: The liquidity effect holds because the competitive monetary equilibrium is sub-optimal: one should not

The general MCE equilibrium

- Now we assume that labour supply features a finite elasticity: $0 < \sigma \ll \infty$.
- Analytical results no longer possible: GE effects too complicated.
- Calibrate the model and simulate robustness of menu-cost result to variations in:
  - The value of σ.
  - The value of μ (the gross monopoly markup).
  - The value of $\sigma_Y \equiv \frac{1-\gamma}{\gamma}$ (the elasticity of the marginal cost function).
- The calibration exercise allows the evaluation of big shocks (inframarginal).
- Assume that the menu costs take the form of labour input Z (e.g. shop assistants changing price tags).

The general MCE equilibrium

- Following a monetary shock there are two scenario's:
  - (FA) full adjustment: all firms change their price and incur the menu costs.

$$\Pi^{FA} = \frac{\mu - \gamma}{\mu} PY - WN\left(F + Z\right)$$

  - (NA) non-adjustment: all firms keep their price unchanged.

$$\Pi^{NA} = P_0 Y - W\left[kY^{1/\gamma} N^{1-1/\gamma} + NF\right]$$

The general MCE equilibrium

- In the simulations we find minimum value of Z (labeled Z_{MIN}) for which non-adjustment is an equilibrium (for which $\Pi^{NA} > \Pi^{FA}$). See **Tables 12.4-12.5**.
  - The entry "menu costs" is defined as follows:

$$\text{menu costs} = 100 \times \left[\frac{N_0 \left(W\right)^{NA} Z_{MIN}}{P_0 Y^{NA}} \right]$$

  - The entry "welfare gain" is defined as follows:

$$\text{welfare gain} = 100 \times \left[\frac{V^{NA} - V_0}{U_C Y^{NA}} \right]$$

  - The entry "ratio" is defined as $\frac{\text{welfare cost}}{\text{menu cost}}$.
  - example from Table 13.4: $\mu = 1.1$, $\sigma_Y = 0.1$, and $\sigma = 10^6$. Menu costs amounting to no more than 0.20% of revenue (tiny) will make non-adjustment of prices an equilibrium in the sense that $\Pi^{NA} > \Pi^{FA}$! The welfare effect is 29.1% of output (huge). Small menu costs have large welfare effects.

The general MCE equilibrium

- Other key features of the simulation results:
  - Welfare measure relatively constant.
  - The markup does not affect menu costs and ratio very much.
  - The labour supply elasticity exerts a very strong effect on menu costs and the ratio. Intuition: if σ is low, then output expansion drives up wages (production costs) which makes non-adjustment less likely to be optimal.
- Table 13.5 has basically very similar results: the key role is played by the labour supply elasticity.

Evaluation of the menu-cost idea

- Runs into same trouble as the RBC literature does: we simply do not observe a high σ.
- Ball & Romer: both *nominal rigidity* (menu cost) and some kind of *real rigidity* (e.g. high σ, customer market, or efficiency wage labour market) are needed to get the menu-cost equilibrium.
- Rotemberg mentions some further problems:
  - MC equilibrium may not be unique.
  - May equally well apply to quantities instead of prices (makes price adjustment more likely).
  - MC insight does not generalize easily to dynamic setting (our next theories do not have that problem).

Quadratic price adjustment costs

- Convex adjustment costs: quadratic in price change.
- Derive approximate pricing rule in two steps:
  - Determine path of equilibrium prices $\{P^*_{j,\tau}\}_{\tau=0}^{\infty}$ which the firm would set in the absence of price-adjustment costs (PACs). This is the desired "target" the firm will aim for.
  - Next determine the quadratic approximation of the profit function around this target price path and incorporate PACs.

Quadratic price adjustment costs

- The objective function of the firm is then:

$$\Omega_0 = \sum_{\tau=0}^{\infty} \left(\frac{1}{1+\rho} \right)^{\tau} \left[\underbrace{\left(p_{j,\tau} - p^*_{j,\tau} \right)^2}_{(a)} + \underbrace{c \left(p_{j,\tau} - p_{j,\tau-1} \right)^2}_{(b)} \right]$$

  - Stay as close as possible to target path: Ω_0 should be minimized.
  - $p_{j,\tau} \equiv \log P_{j,\tau}$ (actual price); $p^*_{j,\tau} \equiv \log P^*_{j,\tau}$ (target price).
  - ρ is the firm's discount factor.
  - (a): intratemporal cost of setting the "wrong" price.
  - (b): intertemporal costs associated with changing the price (annoyed customers, etcetera).

Quadratic price adjustment costs

- The firm minimizes Ω_0 by choosing the appropriate sequence of prices, $\{p_{j,\tau}\}_{\tau=0}^{\infty}$. The FONC is:

$$\begin{aligned}\frac{\partial \Omega_0}{\partial p_{j,\tau}} &= \left(\frac{1}{1+\rho}\right)^{\tau}\left[2\left(p_{j,\tau}-p_{j,\tau}^{*}\right)+2c\left(p_{j,\tau}-p_{j,\tau-1}\right)\right] \\ &- \left(\frac{1}{1+\rho}\right)^{\tau+1}\left[2c\left(p_{j,\tau+1}-p_{j,\tau}\right)\right]=0\end{aligned}$$

or:

$$p_{j,\tau+1}-\left[1+(1+\rho)\frac{1+c}{c}\right]p_{j,\tau}+(1+\rho)p_{j,\tau-1}=-\frac{1+\rho}{c}p_{j,\tau}^{*} \tag{a}$$

- Equation (a) is a second-order difference equation is $p_{j,\tau}$. We need two boundary conditions:
  - Initial condition: $p_{j,-1}$ is pre-determined (set in the past).
  - Terminal condition.

Quadratic price adjustment costs

- The pricing rule in the planning period ($p_{j,0}$) is then:

$$p_{j,0} = \lambda_1 p_{j,-1} + (1-\lambda_1)\left[\frac{\lambda_2 - 1}{\lambda_2}\sum_{\tau=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^{\tau} p^*_{j,\tau}\right] \qquad \text{(b)}$$

  - $0 < \lambda_1 < 1$ is the stable characteristic root of (a).
  - $\lambda_2 > 1$ is the unstable characteristic root of (a).
  - Actual price weighted average of the past price and a long-run target price.
  - Note that (b) contains both backward-looking and forward-looking elements. Anticipated changes in $p^*_{j,\tau}$ will immediately have an effect on the current price.

Slaggered price setting

- Guillermo Calvo (and co-workers) have devised an alternative approach to price stickiness (red light-green light model).
- Price contracts are staggered (old idea of Phelps and Taylor).
- No separate price-adjustment costs.
- Duration of price contract is stochastic via a Poisson process: each period "nature" draws a signal to each firm:
  - "green light" with probability π: go ahead and adjust your contract price (optimally).
  - "red light" with probability $1-\pi$: continue to charge your present contract price.

Slaggered price setting

- Objective function of a firm which has just received a green light:

$$\begin{aligned}\Omega_0 &= \left(p_{j,0}-p^*_{j,0}\right)^2+\frac{1}{1+\rho}\left[\pi\left(p_{j,1}-p^*_{j,1}\right)^2+(1-\pi)\left(p_{j,0}-p^*_{j,1}\right)^2\right]\\ &+\left(\frac{1}{1+\rho}\right)^2\left[\pi^2\left(p_{j,2}-p^*_{j,2}\right)^2+\pi(1-\pi)\left(p_{j,1}-p^*_{j,2}\right)^2\right.\\ &\left.+(1-\pi)^2\left(p_{j,0}-p^*_{j,2}\right)^2\right]+\text{higher-order terms.}\end{aligned}$$

  - Period $\tau=0$: you can set your price at $p_{j,0}$ (take intratemporal costs into account).
  - Period $\tau=1$: you may get a green or a red light. In latter case, you keep old price, $p_{j,0}$. In the former case, you can re-optimize and determine $p_{j,1}$.
  - Period $\tau=2$: three possibilities....

Slaggered price setting

- Collecting terms involving $p_{j,0}$ we get:

$$\begin{aligned}\Omega_0 &= \left(p_{j,0} - p^*_{j,0}\right)^2 + \frac{1-\pi}{1+\rho}\left(p_{j,0} - p^*_{j,1}\right)^2 + \left(\frac{1-\pi}{1+\rho}\right)^2\left(p_{j,0} - p^*_{j,2}\right)^2 + \\ &= \sum_{\tau=0}^{\infty}\left(\frac{1-\pi}{1+\rho}\right)^{\tau}\left(p_{j,0} - p^*_{j,\tau}\right)^2 + \text{uninteresting terms}\end{aligned}$$

  - Pricing friction shows up as heavier discounting: if $\pi \approx 1$ you have almost perfect price flexibility. If $\pi \approx 0$ you attach higher weight to future deviation costs.

- The firm chooses $p_{j,0}$ in order to minimize Ω_0. The FONC is:

$$p_{j,0}\sum_{\tau=0}^{\infty}\left(\frac{1-\pi}{1+\rho}\right)^{\tau} = \sum_{\tau=0}^{\infty}\left(\frac{1-\pi}{1+\rho}\right)^{\tau} p^*_{j,\tau}$$

Slaggered price setting

- We get:

$$p_0^n = \frac{\pi + \rho}{1 + \rho} \sum_{\tau=0}^{\infty} \left(\frac{1 - \pi}{1 + \rho} \right)^{\tau} p_{\tau}^{*} \qquad \text{(new price)}$$

- Firms facing a red light maintain their old prices:

$$p_{-s}^n = \frac{\pi + \rho}{1 + \rho} \sum_{\tau=0}^{\infty} \left(\frac{1 - \pi}{1 + \rho} \right)^{\tau} p_{\tau - s}^{*} \qquad \text{(price set } s \text{ period ago)}$$

- Given the Poisson process and the assumption of a large number of firms we know that $\pi(1-\pi)^s$ is the fraction of firms which last set its price s periods ago. We can aggregate all prices to derive an expression for the aggregate price level:

Slaggered price setting

- Continued.

$$\begin{aligned} p_0 &= \pi p_0^n + \pi(1-\pi)p_{-1}^n + \pi(1-\pi)^2 p_{-2}^n + \pi(1-\pi)^3 p_{-3}^n + \ldots \\ &= \pi \sum_{s=0}^{\infty} (1-\pi)^s p_{-s}^n \\ &= \pi p_0^n + (1-\pi)p_{-1} \end{aligned}$$

or:

$$p_0 = (1-\pi)p_{-1} + \pi \left[\frac{\pi+\rho}{1+\rho} \sum_{\tau=0}^{\infty} \left(\frac{1-\pi}{1+\rho} \right)^{\tau} p_{\tau}^{*} \right]$$

- Actual price is weighed average of new price and past price.
- Rotemberg and Calvo approaches "observationally equivalent" (yield same macro pricing equation). Rotemberg estimates that in the US 8% of all prices are adjusted each quarter (mean time between price adjustments in three years).

Punchlines

- General equilibrium monopolistic competition (MC) model provides micro-foundations for multiplier.
- Intimate link between multplier and welfare effects (pre-existing distortion).
- The existence of MC does **not** render money neutral! We need price-adjustment costs.
- The menu cost insight: small deviations from rationality can have large macroeconomic and welfare effects (need both nominal and real rigidity).
- Practical models: convex adjustment costs (macroeconomic price level becomes backward-looking state variable).