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Education Choices in a Changing Economic, Demographic and Social Environment

Theses in Economics and Business

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Publisher: University of Groningen, Groningen, The Netherlands

Printer: Ipskamp Drukkers B.V.

ISBN: 978-90-367-7624-0 / 978-90-367-7623-3 (eBook)

This document was prepared using LaTeX.

Education Choices in a Changing Economic, Demographic and Social Environment

PhD thesis

to obtain the degree of PhD at the
University of Groningen
on the authority of the
Rector Magnificus Prof. E. Sterken
and in accordance with
the decision by the College of Deans.

This thesis will be defended in public on

Thursday 10 September 2015 at 16:15 hours

by

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Acknowledgements

First of all I would like to thank my supervisor Ben Heijdra. We met in 2006 when I became his Honours Student for the bachelor Economics. What started out as sporadic meetings during which he would question me on my grades and study progress evolved into a fruitful cooperation when I started working as his teaching and research assistant. He showed me that I could achieve more in my studies than I had ever thought possible, which led to a second degree in Econometrics and a master at the London School of Economics. Afterwards he convinced me to return to Groningen for my PhD, a decision which I have never regretted making. We may not always agree, but I have learned a lot from our discussions and I am proud of what we have achieved together.

Secondly my thanks goes out to the assessment committee for taking the time to read my thesis. Hippolyte d'Albis kindly invited me to spend a few days in Paris in February 2014 and gave me the opportunity to present a paper at a workshop there. As editor of the new Journal of Demographic Economics, David de la Croix was responsible for organizing its inaugural conference in Iowa City which I enjoyed attending very much. Rob Alessie has always been a helpful colleague and an inexhaustible source of empirical knowledge.

I am grateful to Fabian Kindermann for teaching me to be a better programmer. I have found that I greatly enjoy this aspect of research and this is in large part due to his help. He has been a very patient teacher, always willing to answer my endless questions through Skype or during one of his many research visits to Groningen. I would also like to thank Pim Heijnen and Petros Milionis for their feedback on my work, both of them have taken the time to read it when asked and to provide helpful comments after presentations.

Vera Heininga is the best friend in the world. I am glad we have managed to keep up our weekly dinners throughout the years (and hopefully will do so in the future) and am proud to have her by my side as my paranymph during the defense. My second paranymph, Paula Gobbi, I would like to thank for introducing me to the fascinating topic of Family Economics and for being very helpful in finding interesting conferences to attend (preferably together).

Peter Dijkstra has been a wonderful office partner over the years. I know that I am not the easiest person to share an office with, but together we have managed to create a pleasant and productive atmosphere in DUI 745.

Finally, I am grateful to my family for all their love and support and their help in putting my work in the right perspective. In particular I want to thank Joost Veenstra for being the sweetest, funniest and most caring husband I could have wished for (and so handsome as well, what a bargain). Together with my two guinea girls Pearl and Ruby he has made our home the best place to be.

Laurie Reijnders

February 2015

Contents

CHAPTER 1

Introduction

> "Human capital analysis starts with the assumption that individuals decide on their education, training, medical care, and other additions to knowledge and health by weighing the benefits and costs. Benefits include cultural and other nonmonetary gains along with improvement in earnings and occupations, while costs usually depend mainly on the foregone value of the time spent on these investments."
>
> *Gary S. Becker* (Nobel lecture, 1992)

Economists assume that when individuals decide whether or not to pursue education, they rationally weigh the corresponding benefits against the costs. As stressed by Gary Becker in his Nobel lecture 'The Economic Way of Looking at Behavior' (see quote above), these are not restricted to monetary gains and losses. Indeed, postulating that individuals maximize their welfare is not the same as assuming that they are necessarily selfish or only driven by material concerns: it all depends on the specification of the welfare function. For example, the benefits of an additional year of education might include a higher wage and more job security in the future, but also a greater probability of marriage and a longer expected life span. And, in addition to tuition fees and foregone earnings, there is a psychic cost of studying which depends on the amount of cognitive and non-cognitive skills that an individual possesses.

In this thesis we study the effect of changes in the economic, demographic and social environment on an individual's decision to invest in tertiary education. In doing so we consider how this decision interacts with various other choices that a person makes over the course of his or her life, including labour market participation, fertility and child care. The models used are necessarily dynamic in nature and assume that people are forward-looking. In most chapters we adopt a general equilibrium perspective, taking into account that individual choices jointly determine outcomes at the aggregate level and how these in turn affect the trade-offs faced by households and firms.

This thesis consists of two parts. In Part I we look at education in the broader context of human capital accumulation. An individual's stock of human capital consists of all skills and knowledge he or she possesses that can be put to productive use. At the start of life this might be limited to some 'innate ability' but over the life cycle it expands through investment in education and by learning on the job. At the same time, some skills will deteriorate and part of the knowledge is forgotten. If the rate at which the process of human capital depreciation proceeds goes up with age then this might induce elderly people to withdraw from the labour market and to settle into retirement.

In Chapter 2 we study the role of longevity in shaping the incentives for an individual to invest in education. There is a horizon effect: if people expect to live longer then they can potentially reap more benefits from their investment and this will stimulate them to extend their schooling period. However, it does not necessarily imply that they also work longer. We show that an improvement in survival probabilities has an ambiguous effect on the optimal retirement age in general. In our numerical simulation such a biological longevity boost prompts individuals to work a little longer. Nevertheless, they expect to spend more years in retirement and add to their stock of savings accordingly. This will lead to a rise in the capital intensity of production and thereby a decrease in the interest rate and an increase in the return to labour. As a consequence, retirees that have all their wealth in the form of savings are worse off while young generations of workers gain. Since the number of working individuals per retiree decreases it becomes harder to maintain an unfunded pension system. Significant adjustments are necessary, in the form of an increase in labour taxes, a decrease in pension benefits or a rise in the statutory retirement age.

We also investigate an alternative scenario, one in which the health improvements that bring about an increase in life expectancy also affect the durability of the human capital stock. Under such a comprehensive longevity boost the rate of human capital depreciation goes down at every age so that individuals lose their skills at a lower pace. This increases both the opportunity cost of time and the amount of lifetime wealth, which have opposite effects on the choice between supplying labour and consuming leisure. It is again not clear from a theoretical perspective whether the optimal retirement age goes up or down, but in the simulations it increases significantly. Human capital becomes relatively abundant in production and factor prices move in exactly the opposite direction as under a biological longevity boost. Because individuals work longer the pressure on the pension system is eased so that only minor adjustments in the tax rate, benefits or statutory retirement age are required to sustain it.

In Chapter 3 we add two important components to this framework. The first is labour market risk. We assume that before they start to work, individuals do not possess full information about their ability to learn on the job. In addition, they face idiosyncratic productivity shocks in every year. This includes the possibility to become unemployed, in which case it is not possible to earn wage income. The second extension is to introduce an explicit system for educational loans. The way in which these are designed affects how risky it is to invest in tertiary education. For example, our benchmark case is a system of subsidized mortgage loans that requires each individual to pay back his or her own study debt. This implies that at each moment in time a fixed redemption payment has to be made, regardless of the amount of wage income. During spells of low labour productivity or unemployment this puts pressure on the funds left to finance consumption. This may deter some individuals from obtaining a college degree.

We then study two possible reform scenarios. The first is the introduction of a graduate labour tax. Instead of building up an explicit study debt each student gets an allowance from the government to cover tuition and living expenses, which is financed out of tax revenue. The corresponding educational tax is levied on the labour earnings of all college graduates. It follows that individuals who receive a low wage income automatically pay less than those in more affluent circumstances. This kind of risk sharing among educated individuals primarily affects the intensive margin of education, in the sense that the number of college graduates hardly changes but each stays in school for a longer period. In addition there is a positive effect on aggregate welfare, provided that generations who gain as a result of the policy reform compensate the losers.

The second policy reform we consider is a system of comprehensive labour taxes. Workers without a college degree then also have to carry the additional tax burden, which prompts a large response at the extensive margin of education: individuals that would not have pursued tertiary education before decide to do so now. In this case there is not only redistribution from lucky individuals (those with a high labour productivity draw) to those who are unlucky (with a lower productivity level), but also from the uneducated to the educated. As a consequence there is an aggregate welfare loss from this policy reform.

In Part II of the thesis we take into account the social environment in which an individual lives and how it influences education choices. First, we recognize that a household usually consists of more than one member and that interactions between these members shape household decisions. For example, couples will jointly determine how many children to have and in which way to allocate the time required for care. Second, at the moment an individual chooses whether to go to college or not he or she is usually still single, but expectations about the likelihood of marriage and the characteristics of a future spouse play an important role in this decision.

In Chapter 4 we study the phenomenon of the 'college gender gap reversal': the fact that in recent years women have surpassed men in terms of college enrollment and graduation rates in most developed countries. At first sight this seems hard to explain, given that women tend to earn less than equally qualified men (the gender wage gap) and usually work fewer hours as a result of childbearing and child rearing. In order to gain insight into the differential incentives for men and women to invest in education we decompose its return into two distinct components. The first is a labour market benefit, which is the payoff to education for an individual who stays single for certain his or her entire life (as in the models of Chapter 2 and 3). Under some realistic assumptions about the degree of curvature in the utility function we prove that if the college wage premium is the same for both sexes but there is a gender wage gap then the labour market benefit of education is greater for women. With strongly diminishing marginal utility of wealth they have more to gain by raising their lifetime earnings through additional schooling.

The remainder of the benefit of education reflects the role of expectations about marriage. The way these distort the education decision depends on the extent to which having a college degree increases the probability of marrying an educated spouse and how the education levels of the spouses affect the division of resources and time within a household. It is likely that the marriage market distortion lowers the benefit of education for women relative to men. In the presence of a gender wage gap a woman can expect to marry a more wealthy spouse, which provides her with fewer incentives to increase her own earnings. In addition, her opportunity cost of time is more important in the decision how many children to raise, as the time cost of child birth cannot be borne by her husband.

We then show numerically which changes in the economic and social environment can lead to a reversal in college graduation rates. For example, a drop in the probability to get married as observed for the United States in recent decades would be sufficient. In the new equilibrium risk-averse women invest more in education than men because being single is more costly for them.

Chapter 5 extends the set-up of Chapter 4 to a general equilibrium framework in order to study family policy. In particular we are interested in how subsidies for child care affect fertility choices and investment in education. Making professional child care more affordable lowers the cost of bringing up a child and allows parents to spend more time in the labour market. Everything else equal this increases the desired number of children and the returns to education. However, in order to finance such a subsidization program the government has to levy taxes. These taxes affect the opportunity cost of time of parents and thus alter the trade-off between parental and formal child care. In addition, a higher demand for professional carers draws unskilled workers from production into the service sector. This leads to a decrease in the college wage premium which has a negative effect on the incentive to invest in education.

In our numerical simulation the benchmark equilibrium without taxes and subsidies is such that fertility is highest for a couple consisting of an uneducated wife and an educated husband and lowest for families in which both parents are educated. There is a negative relationship between the education level of the father and the number of offspring, consistent with empirical stylized facts. Introducing an ad valorem subsidy on child care financed by a proportional labour tax raises the desired number of children in all types of households. The resulting reallocation of labour between sectors reduces the college wage premium so that in the new equilibrium college graduation rates drop for both men and women. We find that if the policy aims to stimulate fertility then this can be more effectively done by providing a fixed subsidy for each child. However, in that case all child care is performed by the parents so that labour supply goes down, especially for uneducated married women as they carry the greatest care burden.

Table 1.1 gives an overview of all the chapters in this thesis. It highlights the economic, demographic and social aspects that potentially have a bearing on the education decision. All in all, this thesis covers a wide range of topics and should therefore provide a comprehensive picture of the trade-offs involved in the choice whether or not to invest in education.

Table 1.1: Overview of the chapters

	Economic	Demographic	Social
Part I			
Chapter 2	pension system	mortality risk	
Chapter 3	educational loans labour market risk		
Part II			
Chapter 4	gender wage gap	fertility choice	marriage expectations
Chapter 5	child care subsidies	fertility choice	marriage expectations

Part I
Individual incentives to invest in education

CHAPTER 2

Longevity shocks with age-dependent productivity growth*

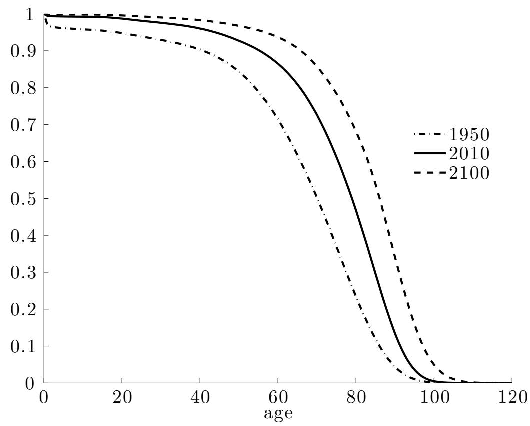
2.1 Introduction

The last decades have witnessed a remarkable increase in the average length of human life. For males, life expectancy at birth in the United States went up from almost 66 years in 1950 to more than 75 years in 2010. This is the result of an increased probability of survival at every age. Figure 2.1 shows the fraction of individuals that are still alive at a given age for both years. Whereas in 1950 child mortality was high and only 20% of the population lived past 80 years, in 2010 young individuals are much more likely to survive to middle age and 40% of them will become older than 80. This demographic trend is expected to continue in the near future as evidenced by the forecasted survival function for 2100. Life expectancy for males will go up with almost 8 more years to about 83.

The aim of this chapter is to study the long-run economic effects of such a predicted longevity increase. In particular we are interested in how it affects individual decisions about education and retirement, taking macroeconomic repercussions through endogenous factor prices and the pension system into account. To that end we construct a model of a closed economy inhabited by overlapping generations of finitely-lived individuals. Over the life cycle their stock of human capital increases with education and the build-up of labour market experience and decreases because of depreciation. As people get older their knowledge and skills deteriorate at an increasing rate so that productivity eventually declines with age. This induces individuals to spend the last years of their life in retirement.

*This chapter is based on Heijdra and Reijnders (2012).

Figure 2.1: Survival function for the United States, males



Source: Bell and Miller (2005).

In this context we present analytical results and a simple quantitative exercise regarding the steady-state effects of two stylized shocks. The first is a *biological longevity boost*, which consists of an outward shift of the survival function in the manner described above. We find that individuals work a little longer but spend most of the additional years in retirement. They substantially increase their savings, which raises the capital intensity of production and lowers the interest rate. In order to maintain an unfunded Pay-As-You-Go pension system there has to be either a substantial increase in the corresponding tax rate, a decrease in benefits or a rise in the statutory retirement age. In the second scenario we consider, the increase in the expected length of life is accompanied by a reduction in the rate of human capital depreciation at any given age. Under this *comprehensive longevity boost* it is possible that human capital becomes relatively abundant in production, resulting in a lower unit cost of effective labour and an increase in the interest rate. As individuals are more productive and work longer hardly any adjustments are required in the pension system.

We make two contributions to the literature on the macroeconomics of ageing. First, we show that it is important to distinguish between the length of 'biological life' (how long a person is expected to live) and the length of 'economic life' (how long a person is able and willing to participate in the labour market) as this matters greatly for the affordability of an unfunded pension system in an ageing society. It has been argued by d'Albis et al. (2012) that in the absence of distorting tax incentives the optimal retirement age may increase or decrease following a rise in life expectancy, depending on the age profile of mortality decline. We show that the optimal length of the retirement period also depends crucially on the extent to which an individual can be productive during the additional years of life. If the improvement in health that brings about an increase in the expected length of life also reduces the rate of human capital depreciation then the pressure on the pension system is significantly alleviated compared to the case that the age-productivity profile remains unchanged.

Second, we show that factor prices could move in a direction opposite to the one accepted as conventional wisdom following an increase in longevity. The usual story is that an increase in the expected length of life raises the stock of physical capital relative to human capital as individuals save more for retirement, see for example Kalemli-Ozcan et al. (2000) and Ludwig et al. (2012). As a consequence the interest rate decreases and wages go up. These relative factor price movements matter, as they affect the intergenerational distribution of welfare and wealth during the transition from one demographic steady state to the next. Recently retired individuals will not benefit from increases in the wage rate but will receive a lower return on their pension savings if the interest rate goes down. We show that if an increase in longevity is accompanied by an improvement in productivity, then human capital might become relatively abundant which would instead raise the return to capital.

The remainder of this chapter is organized as follows. In Section 2.2 we outline the model, followed by a discussion of the implied optimal retirement age in Section 2.3. We parameterize the model in Section 2.4 in order to perform a simple quantitative exercise, the results of which are described in Section 2.5. The final section concludes. The chapter contains three appendices with technical derivations.

2.2 Model

In this section we develop a dynamic micro-founded macro model of a closed economy. First we describe the behaviour of firms (Section 2.2.1) and individuals (Section 2.2.2). After discussing accidental bequests (Section 2.2.3) and the details of the pension system (Section 2.2.4) we characterize the macroeconomic equilibrium (Section 2.2.5).

2.2.1 Firms

There exists a representative firm that produces aggregate output $Y(t)$ which can be used for consumption and investment. The production technology takes the following form:

$$Y(t) = \Phi K(t)^{\phi}[Z(t)N(t)]^{1-\phi}, \quad \Phi > 0, \quad 0 < \phi < 1, \tag{2.1}$$

where $K(t)$ is the stock of physical capital and $N(t)$ is a labour composite:

$$N(t) = \left[\beta N^u(t)^{1-1/\psi} + (1-\beta)N^s(t)^{1-1/\psi}\right]^{\frac{1}{1-1/\psi}}, \quad \psi > 0. \tag{2.2}$$

Following Katz and Murphy (1992) and Heckman et al. (1998), unskilled labour $N^u(t)$ and skilled labour $N^s(t)$ are taken to be imperfect substitutes with a constant substitution elasticity equal to ψ. The index of labour-augmenting technological change $Z(t)$ is assumed to grow at an exogenous rate n_Z.[1] The stock of capital evolves over time according to $\dot{K}(t) = I(t) - \delta_K K(t)$ with $\dot{K}(t) \equiv dK(t)/dt$ the rate of change, $I(t)$ the level of investment and δ_K the depreciation rate. The profit flow of the firm at time t is then given by $\Pi(t) = Y(t) - (r(t) + \delta)K(t) - w(t)N(t)$ where $r(t)$ is the return to capital or interest rate and $w(t)$ is the (minimum) unit cost of effective labour. Profit maximization gives rise to the usual marginal productivity conditions:

$$r(t) + \delta_K = \phi\Phi\left[\frac{K(t)}{Z(t)N(t)}\right]^{\phi-1}, \tag{2.3}$$

$$\frac{w(t)}{Z(t)} = (1-\phi)\Phi\left[\frac{K(t)}{Z(t)N(t)}\right]^{\phi}. \tag{2.4}$$

It follows that a higher capital intensity $K(t)/[Z(t)N(t)]$ is associated with a lower return to capital and a higher return to effective labour. The corresponding rental rates of unskilled labour $w^u(t)$ and skilled labour $w^s(t)$ have to satisfy:

$$\frac{w^u(t)}{Z(t)} = \frac{w(t)}{Z(t)}\beta\left[\frac{N^u(t)}{N(t)}\right]^{-1/\psi}, \tag{2.5}$$

$$\frac{w^s(t)}{Z(t)} = \frac{w(t)}{Z(t)}(1-\beta)\left[\frac{N^s(t)}{N(t)}\right]^{-1/\psi}. \tag{2.6}$$

The more scarce a specific skill type is in production, the greater is its return. Profits are equal to zero as a result of the linear homogeneity of the production function.

[1] Alternatively we could have chosen an endogenous growth specification, for example as in Boucekkine et al. (2002). However, this requires a knife-edge condition on the intergenerational spillover of human capital.

2.2.2 Individuals

The economy is inhabited by overlapping generations of finitely-lived individuals with perfect foresight. During the initial years of life no relevant decisions are made.[2] After reaching the age of majority M the adult individual learns his or her utility cost of schooling θ. He or she then decides whether to obtain a college degree in order to become a skilled worker. We introduce a dummy variable d_s^j that equals 1 if $j = s$ ('skilled') and zero if $j = u$ ('unskilled'). Expected lifetime utility for an individual of skill type j born at time v whose cost of education is θ is given by:

$$\Lambda^j(v|\theta) = \int_{v+M}^{v+\bar{D}} \left[\ln c^j(v,t) + \chi \frac{\ell^j(v,t)^{1-\sigma} - 1}{1-\sigma} \right] e^{-\rho[t-v-M]} S(M, t-v)\, dt - \theta d_s^j, \quad (2.7)$$

where $c^j(v,t)$ is consumption at time t and $\ell^j(v,t)$ is leisure. The parameter ρ is the pure rate of time preference and σ determines the curvature of the felicity from leisure. The function $S(u_1, u_2)$ captures the probability of surviving from age u_1 to $u_2 > u_1$. We assume that everyone dies for certain at or before the maximum age $\bar{D}$.

A college education takes $\bar{E}$ years, so that the age at labour market entry for an individual of skill type j is $E^j = M + \bar{E} d_s^j$. Assuming that the time endowment equals one, leisure is given by:

$$\ell^j(v,t) = \begin{cases} 1-\bar{e} & \text{for } M \le t-v < E^j \\ 1-\bar{l} & \text{for } E^j \le t-v < R^j(v) \\ 1 & \text{for } R^j(v) \le t-v \le \bar{D} \end{cases} \quad (2.8)$$

During the education period the time required for study is $0 < \bar{e} < 1$ and it is not possible to work. We assume that labour supply is indivisible in the sense that an individual works a fixed number of $\bar{l}$ hours (full time) from labour market entry until retirement at a chosen age $R^j(v)$. As in Heijdra and Romp (2009), Kalemli-Ozcan and Weil (2010) and d'Albis et al. (2012) the retirement decision is taken to be irreversible: once an individual has left the labour force he or she cannot return.

The stock of human capital at labour market entry is given by:

$$h^j(v, v+E^j) = 1 + \zeta d_s^j, \quad \zeta > 0, \quad (2.9)$$

[2] In most macroeconomic models the childhood years are ignored altogether and an individual enters the economy at an 'economic age' of 0. However, as this chapter focuses on demographic issues we should not ignore this part of the population.

where ζ captures the direct return to a college education. Over the life cycle human capital evolves as follows:

$$\frac{\dot{h}^j(v,t)}{h^j(v,t)} = \gamma^j(t-v)l^j(v,t) - \delta_h^j(t-v), \tag{2.10}$$

where $\dot{h}^j(v,t) \equiv \partial h^j(v,t)/\partial t$ and $l^j(v,t) = \bar{l}$ when the individual is working and 0 otherwise. A person of age $u \equiv t-v$ and skill type j accumulates human capital in the form of learning-by-doing or experience at rate $\gamma^j(u)$. However, at the same time his or her existing stock of knowledge depreciates at rate $\delta_h^j(u)$. Solving (2.10) given the initial condition (2.9) yields for $t \geq v + E^j$:

$$h^j(v,t) = \left[1 + \zeta d_s^j\right] e^{\int_{v+E^j}^{t} \left[\gamma^j(\tau-v)l^j(v,\tau) - \delta_h^j(\tau-v)\right] d\tau}. \tag{2.11}$$

Hence, both the level of human capital and its rate of growth depend on the individual's age.

There is no clear consensus regarding the empirical relationship between age and labour productivity. This is partly a result of the fact that we cannot directly measure productivity and that the best proxy available, the hourly wage rate, is not observed for individuals who are already retired. The census data that we use to parameterize the model show a hump-shaped wage profile at working ages (see below), which implies that either the rate of experience accumulation should decline with age or the depreciation rate should go up. Recent empirical evidence from Jeong et al. (2014) suggests that there are no decreasing returns to accumulating experience. We interpret this to mean that $\gamma^j(u) = \gamma_0^j$ does not depend on age while the deprecation rate does and parameterize our model accordingly (Section 2.4.2). However, this assumption is not crucial to our findings: what matters is that the overall productivity growth rate $\gamma^j(u)\bar{l} - \delta^h(u)$ depends negatively on u.[3]

Individuals enter adulthood without any assets such that $a^j(v, v+M) = 0$. The accumulation of savings over time proceeds according to:

$$\dot{a}^j(v,t) = r(t)a^j(v,t) + I^j(v,t) + q(v,t) + p(v,t) - c^j(v,t), \tag{2.12}$$

where $\dot{a}^j(v,t) \equiv \partial a^j(v,t)/\partial t$ and $I^j(v,t) \equiv (1-\tau(t))w^j(t)h^j(v,t)l^j(v,t)$ is after-tax wage income earned at time t. There is a proportional labour tax $\tau(t)$ which is used to finance pension benefits $p(v,t)$ for eligible individuals. We assume that there are no annuities or life-insured loans available so that the return on financial assets is the real

[3]With divisible labour the distinction between experience accumulation and human capital depreciation becomes more crucial. See Heijdra and Reijnders (2012) for a discussion of this case.

rate of interest.[4] The assets left behind by individuals who pass away are redistributed to those who are still alive in the form of accidental bequests $q(v,t)$. If there is uncertainty about whether a person might die and there is no life insurance available then individuals cannot borrow money for fear that they will default on their loan. In order to allow people to borrow funds to finance their education we assume that survival is certain up to age $F > M$.[5] For the remainder of life there is a borrowing constraint such that $a^j(v,t) \geq 0$.

An individual of a given skill type has to determine the level of consumption at each moment in time $c^j(v,t)$ and the age at retirement $R^j(v)$ so as to maximize expected lifetime utility (2.7) given the process of human capital accumulation (2.10) and the budget identity (2.12). Assuming that the borrowing constraint does not bind, the first-order condition for consumption can be written as:

$$\frac{1}{c^j(v,t)} e^{-\rho[t-v-M]} S(M, t-v) = \lambda^j(v) e^{-\int_{v+M}^{t} r(\tau) d\tau}. \tag{2.13}$$

At any moment in time, the marginal utility of consumption (left-hand side) should equal the corresponding marginal cost in terms of reduced lifetime wealth (right-hand side) with $\lambda^j(v)$ its shadow price.

The first-order condition for the retirement age is given by:

$$\begin{aligned} &-\chi \frac{(1-\bar{l})^{1-\sigma} - 1}{1-\sigma} e^{-\rho[R^j(v)-M]} S(M, R^j(v)) \\ &= \lambda^j(v) I^j(v, v+R^j(v)) e^{-\int_{v+M}^{v+R^j(v)} r(\tau)\, d\tau}. \end{aligned} \tag{2.14}$$

The left-hand side is the increased felicity from leisure while the right-hand side captures the utility cost of foregone earnings. We discuss the retirement decision in more detail in Section 2.3 below.

Finally, each individual has to decide whether or not to become skilled. In doing so he or she weighs the costs against the benefits. The costs of an education are threefold. First, leisure during schooling years is reduced by the time required for studying. Second, the individual has to postpone entry into the labour market and therefore loses potential wage income. Third, there is a 'psychic' or effort cost of studying equal to θ. The benefit of an education is that it increases human capital and thereby the payoff to each hour of labour. As the cost is increasing in θ while the benefit is independent of it, the optimal education choice is governed by a threshold rule. See the upper panel of

[4] In reality these kind of financial products do exist, but are not used to a great extent. See for example Cannon and Tonks (2008).

[5] Since the survival profile is very flat initially this is not a strong assumption.

Figure 2.2. For a cohort born at time v there is a value $\bar{\theta}(v)$ such that all individuals for whom $\theta \leq \bar{\theta}(v)$ will decide to obtain a college degree while all individuals with $\theta > \bar{\theta}(v)$ remain uneducated. It follows that the fraction of skilled individuals in this cohort equals $\pi(v) = F_\theta(\bar{\theta}(v))$ where F_θ is the cumulative distribution function of the utility cost of education, see the lower panel of Figure 2.2.

Demography and aggregation

At a given time t, the size of the cohort of vintage v is denoted by $P(v,t)$. Over time cohort members pass away so that:

$$P(v,t) = \begin{cases} P(v,v)S(0,t-v) & \text{for } 0 \leq t-v \leq \bar{D} \\ 0 & \text{for } t-v > \bar{D} \end{cases} \tag{2.15}$$

The size of the total population $P(t) = \int_{t-\bar{D}}^{t} P(v,t)\,dv$ is found by summing over all living cohorts. We assume that the economy is in a demographic steady state in which the crude birth rate $b = P(t,t)/P(t)$ and the population growth rate $n_P = \dot{P}(t)/P(t)$ are constant. This gives rise to the following equilibrium condition:[6]

$$b = \frac{1}{\Delta(0,\bar{D},n_P)}, \tag{2.16}$$

where Δ is the 'demographic function':

$$\Delta(u_1,u_2,\xi) = \int_{u_1}^{u_2} e^{-\xi[u-u_1]} S(u_1,u_2)\,du. \tag{2.17}$$

In Appendix 2.B we show that the demographic function is strictly positive, decreasing in ξ and u_1 and increasing in u_2.

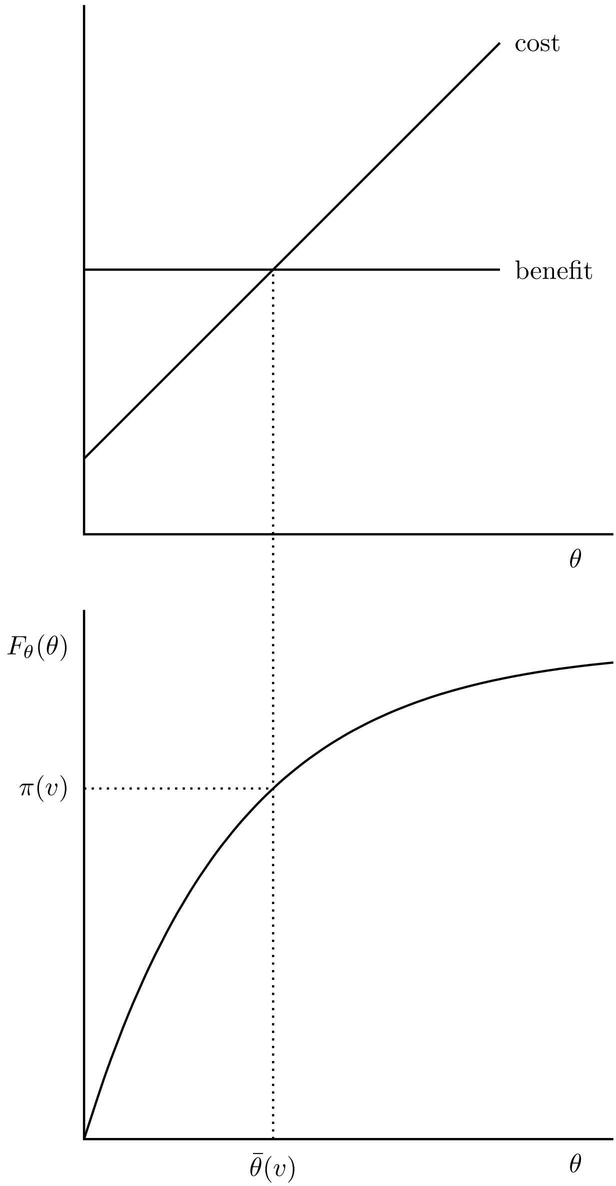
Given the demographic structure of the population we can calculate aggregate values of effective labour, consumption and financial assets by skill type:

$$C^j(t) = \int_{t-\bar{D}}^{t-M} c^j(v,t)P^j(v,t)\,dv,$$
$$L^j(t) = \int_{t-\bar{D}}^{t-M} h^j(v,t)l^j(v,t)P^j(v,t)\,dv,$$

[6]By definition of the total population, the birth rate and the population growth rate:

$$P(t) = \int_{t-\bar{D}}^{t} P(v,t)\,dv = \int_{t-\bar{D}}^{t} bP(v)S(0,t-v)\,dv = bP(t)\int_0^{\bar{D}} e^{-n_P u}S(0,u)\,du = bP(t)\Delta(0,\bar{D},n_P).$$

Figure 2.2: Optimal choice of education



$$A^j(t) = \int_{t-\bar{D}}^{t-M} a^j(v,t)P^j(v,t)\,dv,$$

where $P^s(v,t) = \pi(v)P(v,t)$ is the fraction of skilled individuals in a given cohort and $P^u(v,t) = [1-\pi(v)]P(v,t)$ is the fraction of unskilled. It follows that total consumption and financial assets are given by $C(t) = C^u(t) + C^s(t)$ and $A(t) = A^u(t) + A^s(t)$, respectively.

2.2.3 Accidental bequests

In the absence of life insurance individuals will pass away with a positive stock of financial wealth. The way in which these accidental bequests are distributed among survivors has nontrivial general equilibrium repercussions, see Heijdra et al. (2014). We take a conservative stance and assume that every adult receives the same amount so that $q(v,t) = \overline{q}(t)$. Provided that nothing is wasted this implies:

$$\int_{t-\bar{D}}^{t-M} \mu(t-v)\big[a^u(v,t)P^u(v,t) + a^s(v,t)P^s(v,t)\big]\,dv = \overline{q}(t)\int_{t-\bar{D}}^{t-M} P(v,t)\,dv, \quad (2.18)$$

where $\mu(u)$ is the mortality rate at age u:

$$\mu(u) \equiv -\frac{\partial S(u_1,u)/\partial u}{S(u_1,u)}. \quad (2.19)$$

Total assets left behind (left-hand side) should equal total bequests (right-hand side).

2.2.4 Pensions

We introduce a stylized Pay-As-You-Go (PAYG) pension system that provides a benefit to every person over the age of $\bar{R}$ (the statutory retirement age) so that $p(v,t) = \overline{p}(t)$ for $t - v \geq \bar{R}$ and zero otherwise. The system is unfunded in the sense that benefits are not paid out of accumulated assets but from current contributions by workers:

$$\tau(t)\big[w^u(t)L^u(t) + w^s(t)L^s(t)\big] = \overline{p}(t)\int_{t-\bar{D}}^{t-\bar{R}} P(v,t)\,dv. \quad (2.20)$$

Note that we assume that every elderly individual receives the pension benefit regardless of whether he or she is still working. In this way we prevent large distortions of the retirement decision. In contrast, real-life pension system might provide strong incentives for retirement at or close to the statutory age (see for example Heijdra and Romp (2009)).

2.2.5 Macroeconomic equilibrium

We restrict attention to the long-run equilibrium of the model. A macroeconomic steady state or balanced growth path is a sequence of prices and allocations such that:

(i) Individuals maximize expected lifetime utility taking prices and transfers as given.

(ii) Firms maximize profits taking prices as given.

(iii) The budget of the pension system is balanced.

(iv) Accidental bequests are redistributed to survivors.

(v) All markets clear.

- Capital market:

$$K(t) = A(t)$$

- Goods market:

$$Y(t) = C(t) + I(t)$$

- Labour market:

$$N^u(t) = L^u(t), \quad N^s(t) = L^s(t)$$

(iv) All variables grow at a constant rate, possibly zero.

Our choice of the utility function ensures that the balanced growth path exists, see King et al. (2002). In the steady state the share of skilled workers is the same across cohorts and so is the optimal retirement age for each skill type. Total output, consumption and savings grow at rate $n_Z + n_P$, effective labour grows at rate n_P, wages, pensions and bequests grow at rate n_Z and the interest rate is constant over time.

2.3 The optimal retirement age

When studying the general equilibrium effects of a longevity shock below, changes in the retirement age play an important role. Therefore we discuss in some more detail how the optimal (steady-state) retirement age is determined in the model.

By using (2.13) in (2.14) we find that the optimal retirement age R^* has to satisfy:[7]

$$-\frac{\chi\dfrac{(1-\bar{l})^{1-\sigma}-1}{1-\sigma}}{1/c^j(v,v+R^*)}=I^j(v,v+R^*). \tag{2.21}$$

Recall that there is only a labour supply decision at the extensive margin: an individual works either 0 or $\bar{l}$ hours. Under this assumption, the left-hand side of (2.21) can be seen as the 'marginal rate of substitution' (MRS) between leisure and consumption at age R^*. It is not really 'at the margin' because of the indivisibility of labour, but it captures a similar notion. The numerator is the discrete change in felicity when labour supply changes from $\bar{l}$ to 0 while the denominator equals the marginal utility of consumption.[8] The right-hand side of (2.21) represents the 'opportunity cost of time' (OCT) in terms of foregone labour earnings. At the optimal retirement age R^* the individual is exactly indifferent between working and not working.

In order to derive analytical results we focus on the steady state with a constant interest rate r and growth rate of wages n_Z. We assume that there are no pensions and no accidental bequests and that the borrowing constraint never binds. The lifetime budget constraint can then be written as:

$$\int_{v+M}^{v+\bar{D}} c^j(v,t)e^{-r[t-v-M]}\,dt=\int_{v+M}^{v+\bar{D}} I^j(v,t)e^{-r[t-v-M]}\,dt. \tag{2.22}$$

The discounted value of all consumption expenditures during life (left-hand side) has to be covered by total wage income (right-hand side). For any possible retirement age R we define:

$$MRS^j(R)=-\chi\frac{(1-\bar{l})^{1-\sigma}-1}{1-\sigma}\frac{e^{(r-\rho)[R-M]}S(M,R)}{\Delta(M,\bar{D},\rho)}\int_{E^j}^{R}\hat{I}^j(u)e^{-r[u-M]}du, \tag{2.23}$$

[7] Alternatively we can write:

$$\frac{1}{c^j(v,v+R^*)}I^j(v,v+R^*)=-\chi\frac{(1-\bar{l})^{1-\sigma}-1}{1-\sigma},$$

such that the marginal utility of earning a wage should equal the cost of supplying labour. This is similar to equation (11) in d'Albis et al. (2012) or equation (2) in Prettner and Canning (2014).

[8] Note that the felicity of leisure equals 0 when leisure is equal to 1.

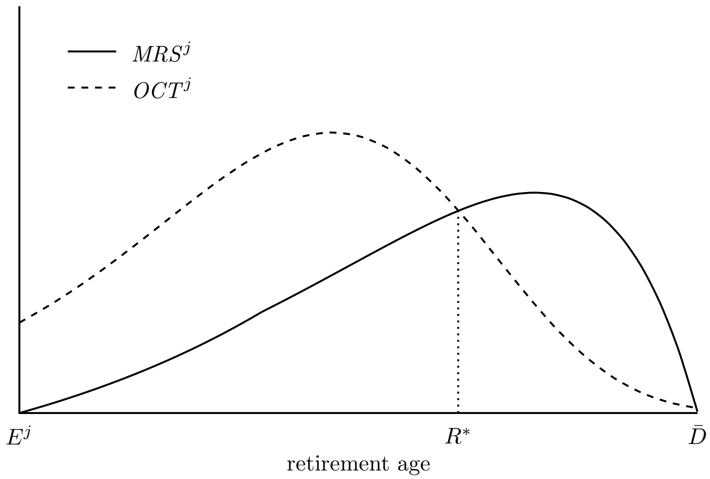
$$OCT^j(R) = \hat{I}^j(R), \tag{2.24}$$

where $\hat{I}^j(u)$ is wage income earned at age u relative to wage income at labour market entry:

$$\hat{I}^j(u) \equiv \frac{I^j(v, v+u)}{I^j(v, v+E^j)} = \begin{cases} e^{\int_{E^j}^{u} \left[n_Z + \gamma_0^j \bar{l} - \delta_h^j(s)\right] ds} & \text{for } E^j \leq u \leq R \\ 0 & \text{otherwise} \end{cases} \tag{2.25}$$

The optimal retirement age satisfies $MRS^j(R^*) = OCT^j(R^*)$. This follows from (2.21) after dividing both sides by $I^j(v, v+E^j)$ and substituting for the optimal level of consumption at retirement given the budget constraint (2.22). Note that equations (2.23) and (2.24) do not depend on the year of birth v, so that in the steady state the optimal retirement age will be the same for all cohorts (as was asserted above).

Figure 2.3: Optimal retirement age



In Figure 2.3 we visualize the two profiles. With a constant felicity of leisure during the working career, MRS^j essentially follows the dynamics of the level of consumption at retirement. According to (2.23) consumption is increasing in lifetime income and the probability of survival. It equals zero when $R = E^j$ (as there is no income) and when $R = \bar{D}$ (as death is certain). The first derivative satisfies:

$$\frac{\partial MRS^j(R)}{\partial R} = \left[r - \rho + \frac{\hat{I}^j(R) e^{-r[R-M]}}{\int_{E^j}^{R} \hat{I}^j(u) e^{-r[u-M]}\, du} - \mu(R) \right] MRS^j(R). \tag{2.26}$$

In the absence of uninsured mortality risk ($\mu(R) = 0$) consumption would increase with the age of retirement because individuals are patient (provided $r > \rho$) and labour earnings go up when the work career is extended. With uncertain survival the increased risk of dying will eventually dominate as the individual gets older so that the expression in (2.26) becomes negative. It follows that the MRS^j profile is strictly concave.

The OCT^j profile mimics the hump-shaped pattern of wages over the life cycle. It satisfies:

$$\frac{\partial OCT^j(R)}{\partial R} = \left[n_Z + \gamma_0^j \bar{l} - \delta_h^j(R)\right] OCT^j(R), \tag{2.27}$$

where $\delta_h^j(R)$ is increasing in R. The opportunity cost of time is normalized to unity when $R = E^j$ and is non-negative for $R = \bar{D}$.

As long as the opportunity cost of time exceeds the marginal rate of substitution between leisure and consumption the individual keeps working. The point of intersection between the two profiles determines the optimal retirement age. The following proposition describes how the retirement age is affected by a change in longevity, human capital depreciation or factor prices.

Proposition 2.1. *Suppose that there are no pensions and bequests and that the borrowing constraint never binds. Assume that there is an interior solution for the optimal retirement age in the steady state. Keeping everything else constant we have that for both skill types:*

(i) An increase in survival rates has an ambiguous effect on the retirement age.

(ii) A decrease in the depreciation rate has an ambiguous effect on the retirement age.

(iii) An increase in the interest rate leads to a decrease in the retirement age.

(iv) An increase in the rental rate of labour does not affect the retirement age.

Proof. See Appendix 2.A. □

Note that in case of an improvement in survival probabilities only the MRS^j profile is affected and not the OCT^j curve. For any possible retirement age there is a positive effect on the level of consumption at that age due to the increased chance of being alive, but there is also a negative effect as financial resources have to be spread over a longer (expected) life time. Hence, in general we cannot say whether individuals are going to retire earlier or later. In the special case that mortality is unchanged at working ages but drops for elderly individuals, only the latter effect is present so that consumption decreases and retirement is postponed.[9] For example, suppose that $S(0,u) = 1$ for $u \leq \bar{D}$ and $S(0,u) = 0$ for $u > \bar{D}$ so that there is no mortality risk but a certain length of life. An increase in $\bar{D}$ would then result in an increase in the retirement age.

In contrast to a longevity boost, a decrease in human capital depreciation affects both profiles. During the working career human capital is higher at any age so that there is an increase in the level of wealth (and thereby consumption) as well as the opportunity cost of time. As a result the effect on the retirement age is again ambiguous. Note, however, that a change in the depreciation rate at a certain age affects the level of human capital in the future but not the past. Hence, if improvements in productivity are limited to elderly individuals then the retirement age remains unaltered.

A change in the interest rate influences the price of consumption and leisure at different points in time and thereby has both an income and substitution effect on the optimal length of the retirement period (which can be seen as the purchase of leisure). In addition it determines the extent to which future income is discounted in lifetime wealth. The overall effect is such that a higher interest rate leads to earlier retirement.

The fact that changes in the rental rate of labour do not influence the retirement decision is a consequence of the fact that the utility function satisfies the King-Plosser-Rebelo conditions (see King et al. (2002)). These ensure that the income and substitution effect of a proportional wage change on labour supply exactly cancel out. This is necessary for a steady state with positive wage growth and a constant retirement age to exist.

2.4 Parameterization

In the next section we will study the long-run effect of a longevity shock on individual choices and macroeconomic outcomes. As it is not possible to solve for the equilibrium of the model in closed form we will complement the analytical insights from the previous section with a simple quantitative exercise. To that end we choose plausible values for the demographic and economic parameters in line with the United States in the year 2010.

[9] A similar result is proved by d'Albis et al. (2012) for the case that there is an annuity market (either perfect or imperfect). Cervelatti and Sunde (2011) show that the age profile of mortality decline also matters for the optimal schooling decision.

2.4.1 Demographic parameters

We set the age of majority equal to $M = 18$. For the survival function we use the functional form suggested by Boucekkine et al. (2002) but extend it to the case that there is certain survival up to age F:

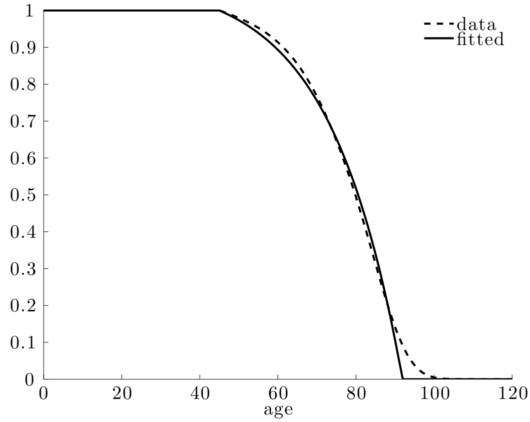
$$S(u_1, u_2) = \begin{cases} 1 & \text{for } u_2 < F \\ \dfrac{\eta_0 - e^{\eta_1 \max\{u_2 - F, 0\}}}{\eta_0 - e^{\eta_1 \max\{u_1 - F, 0\}}} & \text{for } F \leq u_2 < \bar{D} \qquad \eta_0 > 1, \quad \eta_1 > 0, \\ 0 & \text{for } u_2 \geq \bar{D} \end{cases} \tag{2.28}$$

where $u_1 < u_2$ and $\bar{D} \equiv \ln \eta_0 / \eta_1$. The corresponding life expectancy at birth is given by:

$$\mathbb{E}[D] = \int_0^{\bar{D}} S(0, u)\, du = F + \frac{1}{\eta_1}\left[\frac{\eta_0 \ln \eta_0}{\eta_0 - 1} - 1\right]. \tag{2.29}$$

The data on survival probabilities comes from the Office of the Chief Actuary of the Social Security Administration (SSA) and is described in Bell and Miller (2005). We use the period life table for males for 2010. Given that the survival function is very flat and close to 1 up to middle age (see Figure 2.1 in the introduction) we set $F = 45$. We divide the number of individuals who are alive at a given age by the corresponding number at age 45 in order to obtain the data profile in Figure 2.4. Values for the parameters η_0 and η_1 are obtained using nonlinear least squares, see Table 2.1. The corresponding maximum age is $\bar{D} = 91.906$ while the expected length of life is 77.489 years.

Figure 2.4: Fitted survival function for 2010, conditional on survival up to age 45



According to the World Bank the crude birth rate for the United States in 2010 is 14 births per $1,000$ individuals in the population. The demographic equilibrium condition (2.16) then implies that the population growth rate is 0.209%.

Table 2.1: Demographic parameters

Parameter		Value	Source or target
Age at majority	M	18.000	
Age of certain survival	F	45.000	
Level parameter survival function	η_0	12.829	SSA for 2010
Growth parameter survival function	η_1	0.054	SSA for 2010
Crude birth rate	b	0.014	WB for 2010
Population growth rate	n_P	0.002	Demographic equilibrium

Sources: SSA is the Social Security Administration of the United States. WB is the World Bank.

2.4.2 Economic parameters

Even though we do not attempt a full-blown calibration exercise we nevertheless wish to choose the economic parameters of the model in such a way that they are in line with empirical evidence.

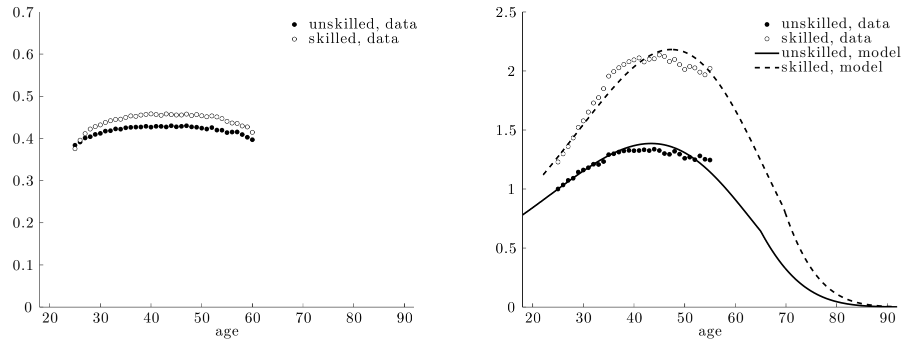
In order to obtain life-cycle profiles for hours worked and the hourly wage earned we follow an approach similar to Wallenius (2011). We use data from the Current Population Survey (CPS) for the United States in the years 1976 up to and including 2012. The sample is confined to males that work a positive number of hours, have at least a high school diploma and are between the age of 25 and 55. The reason for restricting attention to this age range is to avoid sample selection issues as a consequence of schooling and early retirement. For each individual we have data on the birth year, weeks worked last year, usual hours worked per week, wage and salary income and educational attainment. We construct pseudo panel data or synthetic year-of-birth cohorts by following the different representative samples of individuals who are born in the same year over time. A distinction is made between two skill types: those with at least 4 years of college (the ‘skilled’) and those with less (the ‘unskilled’).

We normalize hours worked to a unit time endowment under the assumption that individuals have 14 hours available for work per day or 98 per week. For a given cohort we find the average number of hours worked at each age by averaging over the corresponding observations using the sampling weights. We then take the average over cohorts by age and skill type, see Figure 2.5(a). The hours profile is nearly flat between ages 25 and 55 for both skill types, which fits well with our assumption of a labour supply decision at the extensive margin only. We set the time requirement of a full time job equal to the average value $\bar{l} = 0.440$.

We adjust the hourly wages by the consumer price index so that they are measured in 1999 US dollars and comparable across years. For each cohort we find the average hourly wage at each age by skill type. We then normalize the resulting cohort profiles by the wage at age 25 of the unskilled. After averaging over cohorts we obtain the life-cycle profile depicted in Figure 2.5(b). There is a clear hump-shaped pattern for both skill types.

Figure 2.5: Data profiles and model fit

(a) Hours worked (b) Hourly wage



Source: Current Population Survey for the United States, 1976-2012.

The parameterization then proceeds as follows. We fix the interest rate at 3.5% per year and assume that unskilled and skilled individuals earn the same return per unit of effective labour which is normalized to unity. The long-run economic growth rate is 2%. We set the rate of time preference equal to $\rho = 0.010$ and choose a value for the curvature parameter for the felicity of leisure $\sigma = 2$ such that the Frisch labour supply elasticity is about 0.6.[10] The statutory retirement age is 65 and the tax rate on wage income used to finance the pension system is equal to 10.6%, which corresponds to the combined contributions of employers and employees for the US Old Age and Survivors Insurance from 2000 onwards.

[10]See Keane (2011) for an overview of the empirical estimates of this elasticity.

We parameterize the experience accumulation and human capital depreciation functions in the following way:

$$\gamma^j(u) = \gamma_0^j, \qquad \gamma_0^j > 0,$$
$$\delta_h^j(u) = \delta_0 e^{\delta_1 \max\{u-X,0\}}, \quad \delta_0 > 0, \quad \delta_1 \geq 0, \quad X \geq M.$$

This is similar to Wallenius (2011) but with an age effect in depreciation rather than in experience for reasons alluded to above. By assuming that the depreciation parameters are independent of skill type we have chosen parsimony over degrees of freedom in our data fitting (as described below). Note that if $X > M$ then the depreciation rate is constant at δ_0 for young individuals.

For a given set of human capital technology parameters $\{\gamma_0^u, \gamma_0^s, \delta_0, \delta_1, X, \zeta\}$ we iterate over the life-cycle profiles of both skill types until the pension benefit and accidental bequest satisfy their respective balanced budget conditions. In every round we update the preference parameter χ in such a way that the optimal retirement age for an unskilled individual is equal to 65. We then calculate the squared relative deviation of the simulated wage profiles from the empirical values at ages 25, 35, 45 and 55. We choose the set of parameter values that minimizes this distance.

The resulting profiles are depicted in Figure 2.5(b) and match the data quite closely. The parameter estimates in Table 2.2 show that the return to labour market experience is somewhat higher for skilled individuals and that having a college education increases start-up human capital by about 32%. On average a skilled person between ages 25 and 60 earns 53% more per hour than an unskilled individual, which is somewhat lower than usually reported. Heathcote et al. (2010), for example, calculate a skill premium of 90% for males in 2005. This discrepancy arises because (i) we have excluded individuals with less than a high school diploma from our sample and (ii) the wage profiles are averages over a time period of 37 years during which the premium has risen.

Next we calculate the steady-state education threshold and choose the location parameter of the lognormal utility cost distribution in such a way that the fraction of educated individuals is 38% (in line with the CPS data) under the assumption that the scale parameter equals 1.[11]

Finally we set the technology parameters for the firms. The income share of capital ϕ is one third and the elasticity of substitution between skilled and unskilled labour is

[11] We have two parameters available (μ_θ and σ_θ) to match only one target (the fraction of skilled individuals). We have tried different values of σ_θ but this does not qualitatively change our results, see also Section 2.5.1.

1.41 as estimated by Katz and Murphy (1992). The remaining parameters are chosen so that the factor prices are indeed equal to their postulated values.

2.4.3 Visualization of the benchmark

Some key indicators of the parameterized benchmark equilibrium (BM hereafter) are reported in first column of Table 2.4 below. The ratio of consumption to output is 0.702, while the capital-output ratio is 2.435 (not shown). Both are plausible values.

The steady-state life-cycle profiles for consumption and savings are given in Figure 2.6. These are scaled by the level of technology at the age of majority $Z(v + M)$ to ensure that they are the same for all cohorts. As long as the borrowing constraint does not bind, consumption grows at an exponential rate $r - \rho - \mu(u)$. This rate is initially positive (when mortality is low) but becomes negative later in life (when the risk of dying increases). As a consequence the consumption profile is hump-shaped and reaches a peak around age 70 for both skill types. As skilled individuals cannot work during their education period they have to borrow money at the start of life. These loans are fully repaid by the age of 35, well before survival becomes uncertain.

Ideally individuals would like to let their consumption decrease to zero as they get close to the maximum age and their chances of survival dwindle. As they still receive income in the form of pension benefits and accidental bequests this would imply that it is optimal to borrow money towards the end of life and repay it in the last few years (conditional on survival). Given that this is not allowed, the borrowing constraint will bind and individuals consume exactly their transfer income in each year (which grows at a rate n_Z).[12] This explains the upward sloping part of the consumption profile at the end of life for both skill types. The age at which the constraint starts to bind is such that there is no jump in consumption.

In Table 2.4 we see that skilled individuals retire from the labour force just before reaching age 70, which is almost 5 years later than the unskilled. The second bump in their asset profile (see the dashed line in Figure 2.6(b)) is a consequence of the fact that they start to receive their pension benefits while they are still working.

[12]This result is in line with Leung (1994) who shows that if individuals have no bequest motive and annuity markets do not exist, then savings must be depleted some time before the maximum lifetime.

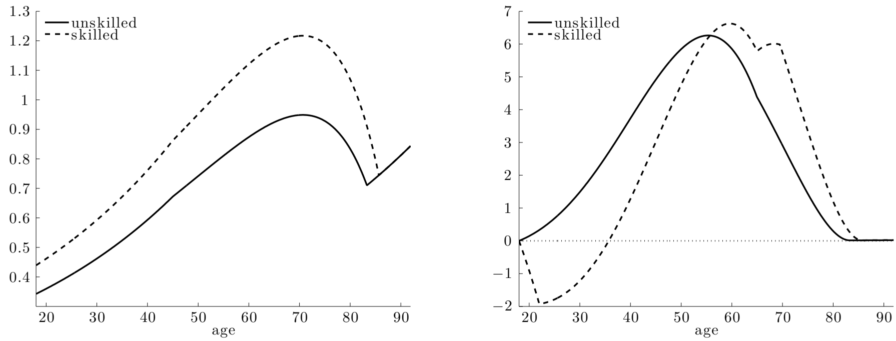
Table 2.2: Economic parameters

Parameter		Value	Source or target
Preferences			
Pure rate of time preference	ρ	0.010	
Curvature parameter for leisure	σ	2.000	
Preference parameter for leisure	χ	0.446	Retirement age unskilled
Production			
Income share of capital	ϕ	0.330	
Constant in production function	Φ	1.549	Rental rates of labour
Depreciation of physical capital	δ_K	0.101	Interest rate
Economic growth rate	n_Z	0.020	
Substitution elasticity between skill types	ψ	1.410	Katz and Murphy (1992)
Time requirement of a full-time job	$\bar{l}$	0.440	Average hours CPS
Weight of unskilled labour in composite	β	0.529	Equal marginal products
Government			
Pension tax rate	τ	0.106	SSA for 2010
Statutory retirement age	$\bar{R}$	65.000	
Human capital			
Experience parameter for unskilled	γ_0^u	0.094	Wage profiles CPS
Experience parameter for skilled	γ_0^s	0.117	Wage profiles CPS
Level parameter depreciation profile	δ_0	0.022	Wage profiles CPS
Growth parameter depreciation profile	δ_1	0.040	Wage profiles CPS
Age after which depreciation increases	X	18.000	Wage profiles CPS
Education			
Return to education	ζ	0.321	Wage profiles CPS
Location parameter talent distribution	μ_θ	2.641	Fraction educated CPS
Scale parameter talent distribution	σ_θ	1.000	
Time requirement of a college education	$\bar{e}$	0.400	

Sources: SSA is the Social Security Administration of the United States. CPS is the Current Population Survey of the United States.

Figure 2.6: Steady-state life-cycle profiles in the benchmark

(a) Consumption (b) Financial assets



2.5 The long-run effects of increased longevity

In this section we show the long-run effects predicted by the model of two stylized longevity shocks. The first is a biological longevity boost (BLB), which consists of an outward shift of the survival function. Secondly we consider what happens if this increase in the expected length of life is accompanied by an improvement in labour productivity at all ages, this is referred to as the comprehensive longevity boost (CLB).

2.5.1 Biological longevity boost

If the survival function shifts outward in the way forecasted by the SSA for 2100 (see Figure 2.1 in the introduction), then the demographic equilibrium changes. We estimate a new set of parameters to fit the data profile for 2100 conditional on survival up to age 45. The maximum age increases to $\bar{D} = 96.968$ and life expectancy at birth goes up by more than 6 years to 83.638, see Table 2.3. As the population growth rate is unaffected (under the assumption that nothing has happened to fertility) the crude birth rate will have to fall. In panel (a) of Figure 2.7 we see that the inverse of the demographic function shifts down which for a given n_P leads to a lower b.

The resulting changes in the age composition of the population can be visualized by means of relative cohort sizes. These are defined as:

$$\frac{P(v,t)}{P(t)} = \begin{cases} be^{-n_P[t-v]}S(0,t-v) & \text{for } 0 \leq t-v \leq \bar{D} \\ 0 & \text{for } t-v > \bar{D} \end{cases} \tag{2.30}$$

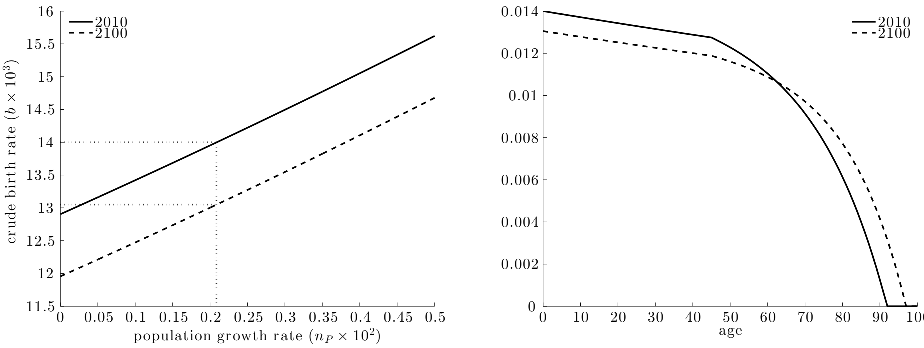
Table 2.3: Demographic steady states

		2010	2100
Maximum age	$\bar{D}$	91.906	96.968
Life expectancy	$\mathbb{E}[D]$	77.489	83.638
Crude birth rate	b	0.014	0.013
Population growth rate	n	0.002	0.002

The size of a cohort relative to the population decreases with its age because cohort members die (reflected in a decreasing probability of survival) while the total number of individuals alive increases (given a positive growth rate). Panel (b) of Figure 2.7 is similar to one half of the population pyramid (since we make no distinction between sexes here) tilted on its side. The total area underneath the line equals 1 by definition. An outward shift of the survival function and a corresponding decrease in the crude birth rate imply that 'mass' is redistributed from the young to the elderly, resulting in an ageing of the population.

Figure 2.7: Demographic changes

(a) Demographic equilibrium (b) Relative cohort sizes



The quantitative long-run consequences of a biological longevity boost are summarized in Table 2.4. Initially we assume that the statutory retirement age and the pension benefit remain fixed and that the tax rate adjusts to balance the budget of the pension system as given in (2.20). This is known as a Defined Benefit (DB) pension. Keeping factor prices constant at their values in the benchmark, the first column under the BLB heading reports the partial equilibrium effects of the longevity shock. The retirement age decreases a little for both skill types, which means that individuals expect to spend a significantly longer part of their life in retirement. As a consequence the pension tax rate has to increase from 10.6% to 14.5%. The fraction of educated individuals goes up by almost 2 percentage points as the increased probability of survival during working ages raises the expected payoff of a college degree.

We wish to make two remarks regarding these partial equilibrium results. First, it would be misleading to interpret the findings as pertaining to a small open economy. For such an economy the factor prices are determined in the rest of the world, but as most countries experience very similar demographic changes these prices cannot be expected to remain constant. Second, the extent to which the fraction of skilled individuals changes depends crucially on the dispersion of educational talent in the population. For a given shift in the education threshold, a lower (higher) value of the scale parameter σ_θ of the utility cost distribution would have increased (decreased) the proportion of educated individuals relative to that reported in Table 2.4.[13] However, qualitatively the results remains the same: it is more attractive to get a college degree.

The next column gives the general equilibrium outcomes under the DB system. Individuals have to save more in order to finance their extended retirement period, which leads to an increase in the capital intensity of production. This results in a drop in the return to capital and a rise in the unit cost of effective labour. The latter has no effect on the retirement decision but the lower interest rate induces an increase in the retirement age, see Proposition 2.1. The change in the skill distribution lowers the rental rate on skilled relative to unskilled effective labour. This reduces the incentive to obtain an education and therefore the general equilibrium effect on the fraction of skilled individuals is smaller than the partial equilibrium effect (although still positive).

In the final two columns under the BLB heading we explore alternative assumptions regarding the closure rule for the pension system. The first is a Defined Contribution (DC) system whereby the tax rate on wage income remains constant while the pension benefit adjusts to balance the budget. Compared to the DB case individuals work about a year longer and save more for old age which results in a further increase in the capital intensity and reduction of the interest rate. The second possibility is to keep both the tax rate and benefit constant and instead change the Statutory Age (SA) for retirement. In terms of macroeconomic outcomes this scenario is in between the previous two. The age at which individuals become eligible for pension benefits goes up by 5.301 years, about 1 year less than the increase in the expected life span. Unskilled individuals choose to retire from the labour force almost 4 years before the pension payments start.

[13]The variance of the log-normal distribution is $(e^{\sigma_\theta^2} - 1)e^{2\mu_\theta + \sigma_\theta^2}$ which is increasing in σ_θ.

Table 2.4: Quantitative results

	BM	BLB				CLB			
		PE	DB	DC	SA	PE	DB	DC	SA
Individuals									
Fraction skilled (in %p)	38.000	39.685	38.619	39.170	39.138	46.907	38.707	38.834	38.882
Retirement age unskilled	65.000	64.593	65.573	66.700	66.588	70.578	70.349	70.632	70.571
Retirement age skilled	69.468	68.893	69.696	70.674	70.534	75.226	74.942	75.192	75.130
Firms									
Capital intensity	7.251		7.559	7.845	7.781		7.183	7.238	7.218
Skilled to unskilled labour	0.849		0.874	0.893	0.892		0.918	0.922	0.922
Factor prices									
Interest rate (in %p)	3.500		3.127	2.803	2.874		3.586	3.517	3.542
Unit cost effective labour	1.995		2.023	2.048	2.042		1.989	1.994	1.992
Rental rate unskilled	1.000		1.024	1.043	1.040		1.023	1.027	1.026
Rental rate skilled	1.000		1.003	1.007	1.004		0.968	0.968	0.967
Pension system									
Statutory retirement age	65.000	65.000	65.000	65.000	70.301	65.000	65.000	65.000	66.535
Pension tax rate (in %p)	10.600	14.536	14.312	10.600	10.600	11.067	11.474	10.600	10.600
Pension payment	0.180	0.180	0.180	0.136	0.180	0.180	0.180	0.167	0.180
Welfare									
Equivalent variation (in %)				5.324	6.744			3.224	2.264

We can compare the three different pension systems in terms of their effect on steady-state welfare. In particular, we calculate the percentage by which consumption should change at each moment in time under the DB system in order to make an individual as well off as under one of the alternative pension schemes (an equivalent variation exercise). For each level of the utility cost of education θ we find $\omega(v|\theta)$ as the solution to:

$$\begin{aligned} &\max\left\{\Lambda_{DB}^{u}(v|\theta),\Lambda_{DB}^{s}(v|\theta)\right\}+\Delta(M,\bar{D},\rho)\ln(1+\omega(v|\theta)) \\ &=\max\left\{\Lambda_{i}^{u}(v|\theta),\Lambda_{i}^{s}(v|\theta)\right\}, \end{aligned} \tag{2.31}$$

where the subscript $i \in \{DB, DC, SA\}$ indicates the type of pension scheme. Note that each individual chooses to be skilled or unskilled depending on whichever option gives the highest expected utility. We can then calculate the average over all different educational ability types to obtain:

$$\bar{\omega}(v)=\int_{0}^{\infty}\omega(v|\theta)\,dF_{\theta}(\theta). \tag{2.32}$$

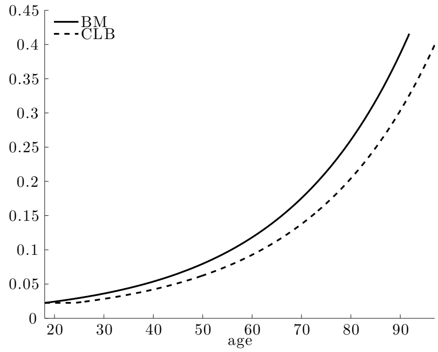
In the steady state this number does not depend on the date of birth v. The last row of Table 2.4 reports the average value multiplied by 100%. For example, if there is a biological longevity boost then on average individuals would require 5.324% more consumption under the DB regime to be as well off as under a Defined Contribution pension scheme and 6.744% to be indifferent with respect to a system that changes the statutory retirement age. It follows that the latter is to be preferred in welfare terms under the BLB.

2.5.2 Comprehensive longevity boost

In case of a comprehensive longevity boost individuals not only expect to live longer but are also more productive during their working career. Unfortunately we do not have any data on forecasted productivity changes. Instead we use a parametric approach and model a productivity improvement as a rightward shift of the human capital depreciation profile through an increase in the parameter X. Figure 2.8 shows the original and new depreciation rates under the assumption that the change in X equals that in life expectancy (about 6 years). This implies that a person of age 50 now loses skills at the rate that someone of age 44 did previously, etcetera.

As before, the first column in Table 2.4 under the CLB heading gives the partial equilibrium effect in case of a Defined Benefit pension system. The retirement age

Figure 2.8: Comprehensive longevity boost



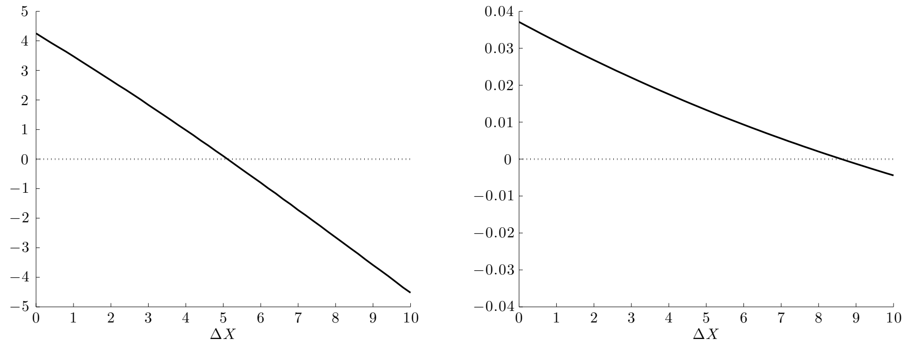
increases with more than 5 years for both skill types and the fraction of skilled individuals rises by about 7 percentage points. Since people are on average more productive and work longer, the pension tax rate need hardly increase. The general equilibrium repercussions through factor price changes again dampen the incentive to obtain an education, as evidenced by the next column. The capital intensity decreases so that the interest rate increases while the unit cost of effective labour goes down.

Note that the change in the interest rate under a CLB is in opposite direction to that under a BLB. Whether it goes up or down depends crucially on the relative scarcity of effective labour (or human capital) versus physical capital. This in turn is affected by the increase in labour productivity relative to the improvement in survival probabilities. In Figure 2.9 we show the general equilibrium outcomes relative to the benchmark for a whole range of possible changes in X. The BLB corresponds to $\Delta X = 0$ while for the CLB we have $\Delta X = 6.149$. In Panel (a) we observe that the capital intensity increases compared to the benchmark for small changes in X (so that the interest rate goes down) but decreases for larger shifts of the depreciation profile (so that the interest rate goes up). The switching point is around $\Delta X = 5$. The required change in the pension tax rate plotted in Panel (b) is also decreasing in ΔX. In the extreme case that individual's productivity improves much faster than their expected life span the tax rate would even go down.

The result that the interest rate might move in a different direction under a CLB compared to the BLB is robust to different closure rules for the pension system. In the last two columns of Table 2.4 we report the long-run equilibrium with a Defined Contribution system or a change in the Statutory Age of retirement. In both cases the interest rate increases relative to the benchmark. The required adjustments in the

Figure 2.9: General equilibrium outcomes for different values of X

(a) Change in capital intensity (in %) (b) Change in pension tax (in %p)



pension system are much smaller when individuals not only live longer but are also more productive. Interestingly, the welfare ranking of the different policy options also changes. It is no longer optimal to adjust the statutory retirement age, instead it is better to keep the contributions fixed.

2.6 Conclusion

In this chapter we have studied the long-run effects of a longevity increase on individual decisions about education and retirement, taking macroeconomic repercussions through endogenous factor prices and the pension system into account. We have constructed a model of a closed economy inhabited by overlapping generations of finitely-lived individuals whose labour productivity depends on their age through the build-up of labour market experience and the depreciation of human capital. In this context we have presented analytical results and a simple quantitative exercise regarding the steady-state effects of two stylized shocks. The first is a biological longevity boost, which consists of an outward shift of the survival function. In this scenario individuals work a little longer but spend most of the additional years in retirement. This prompts an increase in savings, which raises the capital intensity of production and lowers the interest rate. In order to maintain an unfunded Pay-As-You-Go pension system there has to be either a substantial increase in the corresponding tax rate, a decrease in benefits or a rise in the statutory retirement age. In contrast, if the increase in life expectancy is accompanied by an improvement in labour productivity through a decrease in human capital depreciation then the retirement age increases significantly.

Under this comprehensive longevity boost it is possible that human capital becomes relatively abundant in production, resulting in a lower unit cost of effective labour and an increase in the interest rate. As individuals are more productive and work longer hardly any adjustments are required in the pension system.

Appendix

2.A Economic proofs

Proposition 2.1. *Suppose that there are no pensions and bequests and that the borrowing constraint never binds. Assume that there is an interior solution for the optimal retirement age in the steady state. Keeping everything else constant we have that for both skill types:*

(i) An increase in survival rates has an ambiguous effect on the retirement age.

(ii) A decrease in the depreciation rate has an ambiguous effect on the retirement age.

(iii) An increase in the interest rate leads to a decrease in the retirement age.

(iv) An increase in the rental rate of labour does not affect the retirement age.

Proof. The optimal retirement age is at the intersection of the following two profiles:

$$MRS^j(R) = MU_z \frac{e^{-(r-\rho)[R-M]} S(M,R)}{\Delta(M,\bar{D},\rho)} \int_{E^j}^{R} \hat{I}^j(u) e^{-r[u-M]}\, du,$$
$$OCT^j(R) = e^{\int_{E^j}^{R} \left[nz+\gamma^j(s)\bar{l}-\delta_h^j(s)\right] ds},$$

where $MU_z \equiv -\chi[(1-\bar{l})^{1-\sigma}-1]/(1-\sigma)$ is a positive constant. Both profiles are defined on the interval $[E^j, \bar{D}]$. We note the following properties:

(1) $OCT(E^j) = 1 > MRS(E^j) = 0$.

(2) $OCT(\bar{D}) \geq 0 = MRS(\bar{D})$.

(3) MRS^j is initially increasing in R and then decreasing:

$$\frac{\partial MRS^j(R)}{\partial R} = \left[r - \rho + \frac{\hat{I}^j(R) e^{-r[R-M]}}{\int_{E^j}^{R} \hat{I}^j(u) e^{-r[u-M]}\, du} - \mu(R) \right] MRS^j(R),$$

since $r > \rho$ and $\mu(R) = 0$ for $R < F$ but $\mu(R) \to \infty$ as $R \to \bar{D}$.

(4) OCT^j is initially increasing in R and then decreasing:

$$\frac{\partial OCT^j(R)}{\partial R} = \left[n_Z + \gamma_0^j \bar{l} - \delta_h^j(R) \right] OCT^j(R),$$

since $\delta_h^j(R)$ is increasing in R.

Let R_0^* denote the optimal retirement age in the initial steady-state equilibrium. We assume that this is an interior solution so that $OCT^j(R_0^*) = MRS^j(R_0^*)$. The new optimal retirement age is higher than the initial one if $OCT^j(R_0^*)$ increases relative to $MRS^j(R_0^*)$ and lower otherwise.

(i) Suppose that $S(M, u)$ weakly increases for any given u.

- There is no change in $OCT^j(R_0^*)$.
- The change in $MRS^j(R_0^*)$ is ambiguous as $\Delta(M, \bar{D}, \rho)$ increases as well.

It follows that the effect on the retirement age is ambiguous.

(ii) Suppose that $\delta_h^j(u)$ weakly decreases for any given u.

- There is an increase in $OCT^j(R_0^*)$.
- There is an increase in $MRS^j(R_0^*)$.

It follows that the effect on the retirement age is ambiguous.

(iii) Suppose that r increases.

- There is no change in $OCT^j(R_0^*)$.
- There is an increase in $MRS^j(R_0^*)$:

$$\frac{\partial MRS^j(R_0^*)}{\partial r} = MU_z \frac{e^{-\rho[R_0^* - M]} S(M, R_0^*)}{\Delta(M, \bar{D}, \rho)} \int_{E^j}^{R_0^*} [R_0^* - u] \hat{I}^j(u) e^{r[R_0^* - u]} \, du > 0.$$

It follows that the retirement age decreases.

(iv) Suppose that $w^j(t)$ increases.

- There is no change in $OCT^j(R_0^*)$.
- There is no change in $MRS^j(R_0^*)$.

It follows that the retirement age remains unchanged.

□

2.B Demographic proofs

Definition 2.1. *For* $|\xi| \ll \infty$ *and* $0 \le u_1 < u_2 \le \bar{D}$ *the demographic function is defined as:*

$$\Delta(u_1, u_2, \xi) = \int_{u_1}^{u_2} e^{-\xi[u-u_1]} S(u_1, u)\, du.$$

Note that by integrating (2.19) the survival function can be written as:

$$S(u_1, u) = e^{-\int_{u_1}^{u} \mu(s)\, ds},$$

where $\mu(s)$ is the mortality rate at age s. We assume that $\mu(s) = 0$ for $0 \le s < F$ and $\mu(s) > 0$ with $\mu'(s) > 0$ and $\mu''(s) > 0$ for $F \le s \le \bar{D}$.

Lemma 2.1. *The demographic function has the following upper bound:*

$$\Delta(u_1, u_2, \xi) \le \frac{1}{\xi + \mu(u_1)}.$$

Proof. Since $\mu(s) = 0$ for $0 \le s < F$ we can write the demographic function as:

$$\Delta(u_1, u_2, \xi) = \int_{u_1}^{\hat{u}} e^{-\xi[u-u_1]}\, du + \int_{\hat{u}}^{u_2} e^{-\int_{\hat{u}}^{u_1} [\xi + \mu(s)]\, ds}\, du.$$

where $\hat{u} = \max\{u_1, \min\{F, u_2\}\}$. We consider three different possibilities.

(1) $u_1 < u_2 \le F$ so that $\hat{u} = u_2$ and $\mu(u_1) = 0$

The demographic function satisfies:

$$\Delta(u_1, u_2, \xi) = \int_{u_1}^{u_2} e^{-\xi[u-u_1]}\, du = \begin{cases} \dfrac{1 - e^{-\xi[u_2-u_1]}}{\xi} & \text{if } \xi \ne 0 \\ u_2 - u_1 & \text{if } \xi = 0 \end{cases}$$

In either case the result is less than $1/\xi$.

(2) $F < u_1 < u_2$ so that $\hat{u} = u_1$ and $\mu(u_1) > 0$

The function $MU(u_1, u) = \int_{u_1}^{u} \mu(s)\, ds$ for $u \ge u_1$ is a non-negative, increasing and convex function of u:

$$MU(u_1, u_1) = 0, \qquad \frac{\partial MU(u_1, u)}{\partial u} = \mu(u) \ge 0, \qquad \frac{\partial MU^2(u_1, u)}{\partial u^2} = \mu'(u) \ge 0.$$

It follows that:

$$MU(u_1,u) \geq MU(u_1,u_1) + \frac{\partial MU(u_1,u_1)}{\partial u}[u-u_1] = \mu(u_1)[u-u_1].$$

Hence the demographic function satisfies:

$$\begin{aligned}\Delta(u_1,u_2,\xi) &\leq \int_{u_1}^{u_2} e^{-[\xi+\mu(u_1)][u-u_1]}\,du \\ &= \begin{cases} \dfrac{1-e^{-[\xi+\mu(u_1)][u_2-u_1]}}{\xi+\mu(u_1)} & \text{if } \xi+\mu(u_1)\neq 0 \\ u_2-u_1 & \text{if } \xi+\mu(u_1)=0 \end{cases}\end{aligned}$$

In either case the result is less than $1/[\xi+\mu(u_1)]$.

(3) $u_1 \leq F \leq u_2$ so that $\hat{u}=F$ and $\mu(u_1)=0$

The demographic function satisfies:

$$\begin{aligned}\Delta(u_1,u_2,\xi) &= \Delta(u_1,F,\xi) + e^{-\xi(F-u_1)}\Delta(F,u_2,\xi) \\ &\leq \frac{1-e^{-\xi(F-u_1)}}{\xi} + \frac{e^{-\xi(F-u_1)}}{\xi+\mu(F)} = \frac{1}{\xi},\end{aligned}$$

which follows from the results above and the fact that $\mu(F)\geq 0$.

□

Proposition 2.2. *The demographic function has the following properties:*

(i) Positive, $\Delta(u_1,u_2,\xi)>0$

(ii) Decreasing in u_1, $\partial\Delta(u_1,u_2,\xi)/\partial u_1<0$

(iii) Increasing in u_2, $\partial\Delta(u_1,u_2,\xi)/\partial u_2>0$

(iv) Decreasing in ξ, $\partial\Delta(u_1,u_2,\xi)/\partial\xi<0$

Proof. Part (i) is obvious. The first derivatives of the demographic function are:

$$\begin{aligned}\frac{\partial\Delta(u_1,u_2,\xi)}{\partial u_1} &= [\xi+\mu(u_1)]\Delta(u_1,u_2,\xi)-1, \\ \frac{\partial\Delta(u_1,u_2,\xi)}{\partial u_2} &= e^{-\xi[u_2-u_1]}S(u_1,u_2), \\ \frac{\partial\Delta(u_1,u_2,\xi)}{\partial\xi} &= -\int_{u_1}^{u_2}[u-u_1]S(u_1,u)\,du.\end{aligned}$$

Part (iii) and (iv) are straightforward. Part (ii) follows from Lemma 2.1. □

2.C Computational details

2.C.1 Individual choices

In the steady state the optimal choices only depend on an individual's age $u \equiv t - v$, provided that we scale consumption and financial assets by the level of productivity at age M. We define:

$$\hat{c}^j(u) \equiv \frac{c^j(v, v+u)}{Z(v+M)}, \qquad \hat{l}^j(u) \equiv l^j(v, v+u),$$
$$\hat{a}^j(u) \equiv \frac{a^j(v, v+u)}{Z(v+M)}, \qquad \hat{h}^j(u) \equiv h^j(v, v+u).$$

We take as given the (constant) level of accidental bequests $\tilde{q} \equiv \bar{q}(t)/Z(t)$, tax rate on wage income τ, pension benefits $\tilde{p} \equiv \bar{p}(t)/Z(t)$, interest rate r and rental rates of effective labour $\tilde{w}^j \equiv w^j(t)/Z(t)$ and assume that the borrowing constraint only binds in the final years of life (we can check this ex-post).

(1) For any combination of the retirement age R^j and the age at which the borrowing constraint starts to bind $R^j \leq B^j \leq \bar{D}$ we can calculate the life-cycle profiles.

- Labour supply:

$$\hat{l}^j(u) = \begin{cases} 0 & \text{for } M \leq u < E^j \\ \bar{l} & \text{for } E^j \leq u < R^j \\ 0 & \text{for } R^j \leq u \leq \bar{D} \end{cases}$$

- Human capital:

$$\hat{h}^j(u) = \left[1 + \zeta d_s^j\right] e^{\int_{E^j}^{u} \left[\gamma_0^j \bar{l} - \delta_h^j(s)\right] ds}.$$

- Consumption:

$$\hat{c}^j(u) = \begin{cases} \dfrac{e^{(r-\rho)[u-M]} S(M,u)}{\Delta(M, B^j, \rho)} W^j(B^j) & \text{for } M \leq u < B^j \\ \left[\tilde{q} + \tilde{p}\right] e^{n_Z[u-M]} & \text{for } B^j \leq u \leq \bar{D} \end{cases}$$

where $W^j(B^j)$ is the discounted value of wage income earned and transfers received between ages M and B^j:

$$W^j(B^j) = \int_M^{B^j} e^{n_Z[u-M]} \Big[(1-\tau)\tilde{w}^j \hat{h}^j(u)\hat{l}^j(u) + \tilde{q} + \tilde{p}\,\mathbb{1}_{u \geq \bar{R}}\Big] e^{-r[u-M]}\, du,$$

with $\mathbb{1}_{u\geq \bar{R}}$ the indicator function that equals 1 if $u \geq \bar{R}$ and zero otherwise.

– Financial assets:

$$\hat{a}^j(u) = \int_M^{u} e^{n_Z[s-M]} \Big[(1-\tau)\tilde{w}^j \hat{h}^j(s)\hat{l}^j(s) + \tilde{q} + \tilde{p}\,\mathbb{1}_{s \geq \bar{R}} - \hat{c}^j(s)\Big] e^{-r[s-u]}\, ds.$$

(2) For any retirement age R^j we can find the optimal B^j by ensuring that at this age there is no jump in consumption:

$$\frac{e^{(r-\rho)[B^j-M]} S(M, B^j)}{\Delta(M, B^j, \rho)} W^j(B^j) = \big[\tilde{q} + \tilde{p}\big] e^{n_Z[B^j-M]}.$$

(3) The optimal retirement age R^j is the one that maximizes expected lifetime utility and is calculated using a minimization routine.

(4) We can find the threshold value for education as the difference between the expected lifetime utility of a skilled individual (ignoring the utility cost of education) and an unskilled individual.

2.C.2 Macroeconomic equilibrium

To calculate the macroeconomic equilibrium we start with a guess for the scaled capital stock $\tilde{K} \equiv K(t)/[Z(t)P(t)]$ and the two types of effective labour $\tilde{N}^j \equiv N^j(t)/P(t)$ for $j \in \{u, s\}$. Jointly they determine the factor prices $\tilde{w}^j$ and r. We find the optimal life-cycle profiles of skilled and unskilled individuals and the corresponding education threshold. Aggregating across individuals gives total consumption $\tilde{C} \equiv C(t)/[Z(t)P(t)]$, financial assets $\tilde{A} \equiv A(t)/[Z(t)P(t)]$ and labour supply $\tilde{L}^j \equiv L^j(t)/P(t)$.

We check whether the goods market is in equilibrium so that $\tilde{Y} = \tilde{C} + \tilde{I}$ where $\tilde{Y} \equiv Y(t)/[Z(t)P(t)] = \Phi \tilde{K}^{\phi} \tilde{N}^{1-\phi}$ and $\tilde{I} \equiv I(t)/[Z(t)P(t)] = (\delta_K + n_P + n_Z)\tilde{K}$. If so, then we have found the steady state. If not, then we change the level of accidental bequests and one of the parameters of the pension system using the respective balanced budget conditions. In addition we partially update the guess for the factor supplies in the direction of satisfying the capital market equilibrium condition $\tilde{K} = \tilde{A}$ and the labour market equilibrium conditions $\tilde{N}^j = \tilde{L}^j$.

CHAPTER 3

Life in shackles? The quantitative implications of reforming the educational loan system*

3.1 Introduction

> "...student loan systems [...] are often badly designed for an extended period of high unemployment. In contrast to the housing crash the risk from student debt is not of a sudden explosion in losses but of a gradual financial suffocation. The pressure needs to be eased."
>
> *The Economist* (October 29th, 2011)

Obtaining a college degree typically requires a large investment of time and money. In order to facilitate access to higher education most governments have instituted some kind of educational loan system. For example, in the United States there are four major federal sources of mortgage loans (subsidized and unsubsidized Stafford loans, the PLUS program and the Perkins loans) as well as private sector loans (Avery and Turner (2012)). In Australia higher education is financed with income-contingent loans (Chapman (1997)) that require an individual to start repaying study debt only after a certain income threshold is reached. Finally, whereas in the Netherlands basic grants to students are currently paid out of general tax revenue, there are plans to move to a so-called Social Borrowing System. This is essentially a system of mortgage loans similar to the United States.

The existence of these educational loan programs ensures that access to tertiary education in most developed countries is relatively good, but depending on the system in place college graduates may enter the labour force with a substantial amount of

*This chapter is based on Heijdra, Kindermann and Reijnders (2014).

study debt. The typical horror story is that of the National Consumer Law Centre's client who has a $300,000 debt resulting from a failed attempt to become an airline pilot (*The Economist*, October 29th, 2011). Although this is an extreme case, some commentators suggest that most educational loan systems produce generations of the educated 'indebted ones'. In their view the government is hanging a mill stone around the necks of those individuals who have to borrow funds in order to finance their tertiary education. The theoretical literature on this topic has suggested that the burden could be alleviated by moving away from a pure loan system to one involving graduate taxes. Under such a system individuals do not accumulate an explicit debt but instead have an implicit obligation to contribute to educational financing in the form of an additional tax on their labour income.

In a persuasive chapter in *Capitalism and Freedom*, Milton Friedman strongly favours a system of graduate taxes (1962, p. 105). The arguments with which he supports his position are worth repeating here. First, he argues that tertiary education is unlikely to feature significant external effects as it is "...a form of investment in human capital precisely analogous to investment in machinery, buildings, or other forms of non-human capital" (p. 100). With a perfect capital market there would be no role for government interference. Second, he notes that the rate of return on human capital investment is much higher than that on investment in physical capital and concludes that in the laissez faire economy there is underinvestment in human capital resulting from market imperfections. The main reason is that "in a non-slave state, the individual embodying the investment cannot be bought and sold" (p. 102). Third, he argues that private mortgage loans would be unattractive to borrowers because of the large risk-of-default premium that the lenders would require. What is needed is some kind of limited-liability equity financing scheme. For education it would be advantageous if it were possible "...to 'buy' a share in an individual's earning prospects; to advance him the funds needed to finance his training on condition that he agree to pay the lender a specified fraction of his future earnings. In this way, a lender would get back more than his initial investment from relatively successful individuals, which would compensate for the failure to recoup his original investment from the unsuccessful" (p. 103). Finally, he closes the case in favour of graduate taxes by noting that the government is able to institute such a system of equity investment in human beings at a much lower cost than the private sector could because it already possesses the power to tax individuals.

Despite Friedman's arguments in support of graduate taxes, many other systems of educational financing have been adopted around the world as pointed out above. Is this because Friedman's message was not well understood by policy makers, or are these systems, though theoretically distinct from graduate taxes, in practice more or less equivalent? To answer this inherently quantitative question we conduct a formal computational analysis of a number of educational loan systems. Taking the Subsidized Mortgage Loan (SML) system as our point of departure we investigate the micro- and macroeconomic effects of two reforms. The first is the introduction of a Graduate Labour Tax (GLT) levied on educated individuals. The second possible reform is a Comprehensive Labour Tax (CLT) system under which the educational tax has to be paid by all workers. In each case we compute the transitional and long-run effects of the policy change on the economic allocation and the consequences for the level of welfare of existing and future cohorts. In addition we decompose the total effect into several components in order to highlight the key mechanisms in the model.

The innovative features of our approach are in the modelling of individuals. Following the pioneering work by Bewley (1977), Aiyagari (1994) and Huggett (1993, 1997) we assume that individuals experience uninsurable idiosyncratic labour market risk during part of their life cycle. In the spirit of Krebs (2003), Abbott et al. (2013), Huggett et al. (2011), Kindermann (2012) and Krueger and Ludwig (2013) we enrich this workhorse model of modern quantitative macroeconomics by including features of the human capital accumulation process. We assume that at the start of adult life each individual chooses the optimal years of schooling given his or her talent for education. During the education phase the student receives funds from the government in order to pay tuition fees and to consume goods. Once education is completed, the graduate joins the labour force and finds out his or her ability to learn on the job. Despite the fact that the rental rate of effective labour is deterministic in the absence of aggregate risk, the wage rate received by an individual is stochastic due to productivity shocks. The different educational loan systems that we discuss in this chapter will affect the amount of financial distress associated with a bad run in the labour market.

Regarding the policy reform from SML and GLT we reach the following conclusions. First, in the long run the proportion of uneducated individuals stays roughly constant but the average educational attainment of students increases. Second, there are sizeable effects on the macroeconomy. In the long run, the capital stock and effective employment increase by, respectively, 2.72% and 0.23%. Since capital becomes relatively abundant in production its return drops by 0.14 percentage points while the wage rate increases by 0.56%. Steady-state consumption and output increase by, respectively, 0.30% and 0.79%. Third, there exists a considerable amount of transitional dynamics in the model and it takes about half a century before the economy is close to its new steady state. The realistic transition speed results from the fact that there are two slow-moving stocks in the model, namely physical and human capital (see Mankiw et al. (1992)). Fourth, because of the gradual transition the welfare consequences of the policy change differ by cohort. For adults economically active at the time of the reform ex-ante welfare invariably falls. For educated working-age individuals this result follows readily from the fact that they are, in a sense, paying the same bill twice. They must continue to redeem any existing study debt but are also faced with a higher tax on labour earnings. In contrast, all future cohorts gain from the policy change. Furthermore, their gains are large enough to, at least in principle, compensate those that lose out. From an ex-ante welfare perspective, therefore, we reach the conclusion that Friedman was right and that graduate taxes are better than mortgage loans.

Our quantitative results show that the consequences of a policy change from SML to CLT differ from the first scenario along a number of dimensions. First, there is a drop of 11.12 percentage points in the proportion of individuals without a college degree. Since they cannot avoid paying the educational tax anyway, more people will decide to get an education in order to reap at least some of the benefits of the system in the form of 'free' study grants. Second, whereas all new cohorts benefit from a move to the GLT system, under the CLT reform the cohorts that become economically active soon after the time of the policy change are worse off and the aggregate ex-ante welfare effect is negative.

This chapter relates to a growing literature. There are many theoretical contributions dealing with the financing of higher education, prominent examples of which include García-Peñaloza and Wälde (2000), Jacobs and van Wijnbergen (2007), Cigno and Luporini (2009) and Del Rey and Racionero (2010). These papers are invariably highly stylized in their description of economic decision making and are thus unsuitable for a quantitative analysis of educational loan systems. In recent years, however, a literature had emerged which uses the techniques of modern stochastic macroeconomics, in particular the incomplete markets model, to study education subsidies. Examples include Akyol and Athreya (2005), Ionescu (2009), Krueger and Ludwig (2013) and Abbott et al. (2013). Of these, the paper by Abbott et al. (2013) is most closely related to ours. Although the quantitative methodology used is similar, their focus is quite distinct. For example, Abbott et al. (2013) provide a more detailed description of how individuals decide about education and what exactly their resources are during the schooling period. They include in vivo transfers from parents to offspring and assume that there exists an intergenerational transmission of ability. In their computational implementation they restrict attention to steady-state comparisons. In contrast, we focus mainly on the design of repayment schemes for government loans, keeping resources of individuals at the beginning of life (and during the time of study) constant. By adopting a less detailed description of the education phase we are able to compute the transitional effects of policy reforms. In doing so we can demonstrate the rather uneven distribution of costs and benefits over the different cohorts. We thereby show that the actual implementation of policy reforms that improve long-run welfare may meet with a lot of political opposition in the short run.

The remainder of this chapter is structured as follows. In Section 3.2 we formulate the model. Section 3.3 discusses the calibration and visualizes some of its key features. Sections 3.4 and 3.5 present the quantitative results from our two reform scenarios. Section 3.6 summarizes and concludes. Technical issues are discussed in a number of appendices at the end.

3.2 Model

In this section we develop a stochastic general equilibrium model of a closed economy. We describe the behaviour of firms, individuals and the government and the market clearing conditions that should be satisfied in the macroeconomic equilibrium.

3.2.1 Firms

Perfectly competitive firms combine physical capital and efficiency units of labour in order to produce a homogeneous good, the price of which serves as the numeraire in the economy. In the absence of aggregate uncertainty and capital adjustment costs the representative firm essentially makes a sequence of static decisions regarding output supply and factor demands. The production function is of the Cobb-Douglas type:

$$Y_t = \Phi K_t^{\phi}[Z_t N_t]^{1-\phi}, \quad \Phi > 0, \quad 0 < \phi < 1, \tag{3.1}$$

where Y_t is output, K_t is the stock of physical capital at the start of year t and N_t is the amount of effective labour employed in production. The index of labour-augmenting technological change Z_t grows at a constant and exogenous rate $n_Z > 0$. The firm's stock of physical capital evolves according to $K_{t+1} = (1 - \delta_K)K_t + I_t$, where I_t is gross investment and δ_K is the rate of depreciation. The real profit flow in period t is given by $Y_t - w_t N_t - (r_t + \delta_K)K_t$, where r_t is the interest rate and w_t is the rental rate of effective labour. The profit-maximizing mix of inputs has to satisfy the following marginal productivity conditions:

$$r_t + \delta_K = \phi\Phi\left[\frac{K_t}{Z_t N_t}\right]^{\phi-1}, \quad \frac{w_t}{Z_t} = (1-\phi)\Phi\left[\frac{K_t}{Z_t N_t}\right]^{\phi}. \tag{3.2}$$

The representative firm makes zero profit because of the linear homogeneity of the production technology.

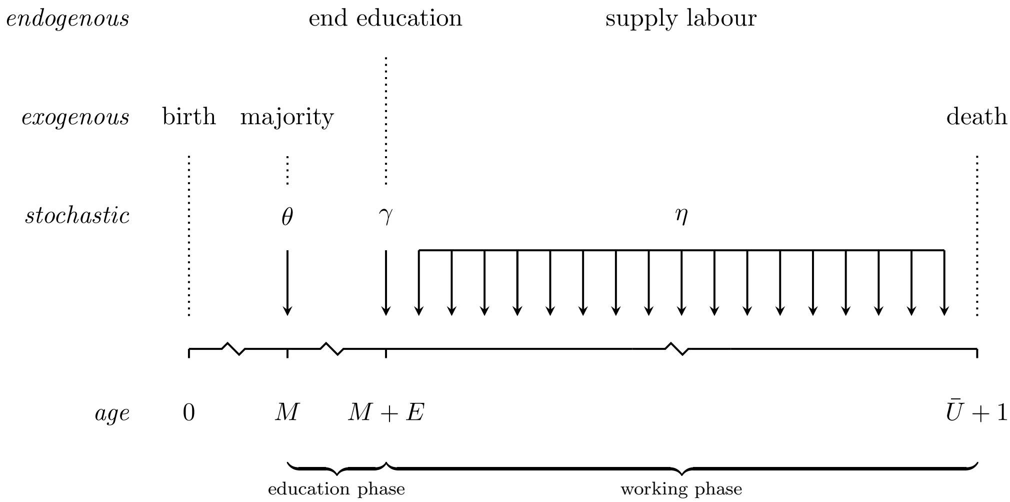
3.2.2 Individuals

Every individual lives for $\bar{U}+1$ years with certainty, such that age $u \in \{0, 1, \ldots, \bar{U}\}$. At the start of each period t a cohort of size $P_{0,t}$ is born. Under the assumption that there is no mortality risk the size of the cohort of age u in year t is given by $P_{u,t} = P_{0,t-u}$. The total population in a given year can be found by summing over all cohorts that are alive and is assumed to grow at a constant rate n_P.

Overview of the life cycle

The different phases in the life cycle of an individual and the stochastic shocks that occur at specific ages are illustrated in Figure 3.1.

Figure 3.1: The individual's life cycle



Individuals start to make economic decisions after attaining the age of majority M. They learn their innate talent for education, θ, which is drawn from a distribution with support $[0, 1]$. This affects the returns to education for the individual. In particular, the stock of human capital at labour market entry given talent for education θ and years of education E is given by $\Gamma(\theta, E)$ with $\partial\Gamma(\theta, E)/\partial E \geq 0$. By assuming that the amount of start-up human capital is deterministic for a given level of education we abstract from so-called 'input risk'. Someone who chooses no education at all enters the labour force with one unit of human capital so that $\Gamma(\theta, 0) = 1$. Upon completion of the chosen education phase at age $M + E$, nature reveals the individual's ability to learn on the job. We assume that the learning-by-doing parameter γ is (positively) correlated with θ. A person who is very talented at school is also likely to learn quickly on the job.

During the working phase the individual receives a draw of his or her labour productivity η in every year. For computational reasons we assume that the process for η takes the form of a four-state Markov chain with the following features. First, we capture the notion of (temporary) unemployment by setting the lowest realization equal to zero. Second, conditional on being productive ($\eta > 0$), the average productivity level is $\eta = 1$. The remaining possible values are η_l and η_h with $0 < \eta_l < 1 < \eta_h$. Third, we assume that the transition probabilities depend on the individual's education level. Fourth, we impose a lot of additional structure on the Markov process in that (i) any productive worker can become unemployed, (ii) barring moves to unemployment a productive worker can only move up or down a single state and (iii) an unemployed individual either remains unemployed or returns to average productivity. This is illustrated by the configuration of arrows in Figure 3.2. There is only a directed arrow from one state to another if there is a positive probability of transition between these states.

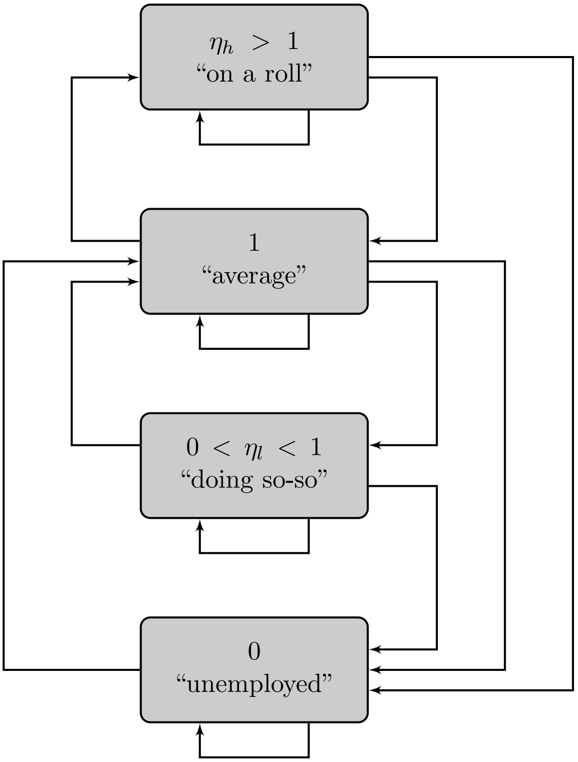
Individuals are assumed to be fully aware of the stochastic environment they live in and to formulate life-cycle plans which maximize their utility subject to the constraints they face. It is most convenient to describe an individual's optimization problem backwards, starting with the working phase and ending with the education phase.

Optimal decisions of a worker

Consider an individual who is of age u at the start of period t, has completed E years of education and is able to learn on the job at rate γ. He or she owns stocks of financial assets a and human capital h and has a labour productivity level η. The individual chooses this year's consumption c and labour supply l and next year's financial assets a^+ and human capital h^+ in order to maximize remaining lifetime utility. The optimization problem is characterized by the following Bellman equation:

$$V_{u,t}(E, \gamma, a, h, \eta) = \max_{c,l,a^+,h^+} \left\{ \left[c^\varepsilon (1-l)^{1-\varepsilon}\right]^{1-1/\sigma} + \beta \left[\mathbb{E}_{\eta^+|\eta,E} \left[V_{u+1,t+1}(E, \gamma, a^+, h^+, \eta^+)^{1-\zeta} \right] \right]^{\frac{1-1/\sigma}{1-\zeta}} \right\}^{\frac{1}{1-1/\sigma}}, \quad (3.3)$$

Figure 3.2: Markov process for labour productivity η



in combination with the laws of motion of the state variables and the constraints on the control variables:

$$a^{+} = [1 + (1 - \tau_t^r)r_t]a + (1 - \tau_t^w)w_t\,\eta\,h\,l + \nu_{u,t}\mathbb{1}_{\{\eta=0\}} - (1 + \tau_t^c)c$$
$$- \Upsilon_{u,t}(E, w_t\,\eta\,h\,l) \quad (3.4)$$
$$h^{+} = (1 - \delta_u^h)[1 + \gamma l^{\alpha}]h, \quad (3.5)$$
$$0 \leq l \leq 1, \quad c \geq 0, \quad a^{+} \geq 0. \quad (3.6)$$

where τ_t^r, τ_t^w, and τ_t^c are tax rates on, respectively, interest income, wage earnings and consumption, $\nu_{u,t}$ is the unemployment benefit, $\mathbb{1}_{\{\eta=0\}}$ is an indicator function which equals unity if $\eta = 0$ and zero otherwise and $\Upsilon_{u,t}(E, W)$ is the contribution to the educational loan system for someone with education E and gross wage income W.

Several things are worth noting. First, the preference structure above satisfies the King-Plosser-Rebelo conditions (see King et al. (2002)). This implies that, in the presence of ongoing labour-augmenting technological progress, we can obtain a stationary decision problem by scaling the individual's consumption and financial assets as well as wages, unemployment benefits and study loan payments by an index of productivity. For details see Appendix 3.B. Second, preferences are of the recursive form suggested by Epstein and Zin (1991) which allows us to disentangle the individual's attitude towards risk and intertemporal consumption smoothing. In the formulation adopted here, $\sigma > 0$ is the *inter*temporal substitution elasticity while $\zeta \geq 1$ captures the degree of relative risk aversion. Third, the trade-off between current consumption and leisure is governed by a Cobb-Douglas function with $0 < \varepsilon < 1$ the relative weight of consumption. This implies a unitary *intra*temporal substitution elasticity. Fourth, the individual faces uncertainty about the level of labour productivity in the next year η^{+} and uses information on its current level η and chosen years of education E to form an expectation about the future value function (discounted by $\beta \leq 1$).[1] Finally, since γ is revealed when individuals enter the labour force and E is predetermined both are constants throughout the working phase

Equation (3.4) states that the change in financial assets is equal to after-tax income net of spending on consumption and payments to the educational loan system. Expression (3.5) shows that the accumulation of human capital during the working phase is governed by two distinct mechanisms. The term γl^{α} captures *learning-by-doing.* Given the learning ability γ, an individual gains more experience the more hours he or she works but at a diminishing rate (with $0 < \alpha < 1$). The effect of *economic ageing* is

[1] We use the notation of Cai and Judd (2010) by writing the productivity levels in the current and the immediately following year as, respectively, η and η^{+}.

reflected in $1-\delta_u^h$ and results from the fact that the depreciation rate on human capital is assumed to be increasing in age (as in Chapter 2). The constraints in (3.6) show that labour supply, consumption and financial assets must be non-negative. We thus impose the restriction, conventional in the macroeconomic literature on uninsurable idiosyncratic risk, that individuals are not able to borrow for other purposes than financing their education. An often stated rationale for this borrowing constraint is the fact that the stock of human capital cannot be separated from the person who owns it and can therefore not serve as collateral for a loan (Friedman (1962), p. 102).

The solution to the worker's decision problem gives the optimal choices of the control variables conditional on the state variables. We write these policy functions for the working phase as:

$$\mathbf{c}_{u,t}(E,\gamma,a,h,\eta), \quad \mathbf{l}_{u,t}(E,\gamma,a,h,\eta), \quad \mathbf{a}^{+}_{u,t}(E,\gamma,a,h,\eta), \quad \mathbf{h}^{+}_{u,t}(E,\gamma,a,h,\eta). \tag{3.7}$$

Optimal decisions of a student

The working phase is preceded by the education phase. It is possible to enter into formal education upon attaining the age of majority M. This requires a fixed time input of $0<\bar{e}<1$ each year. Since the time endowment equals unity and working and studying are assumed to be mutually exclusive activities it follows that leisure during the education phase is given by $1-\bar{e}$. In the absence of labour income students finance their consumption expenses with government-provided educational loans. The annual loan amount q_t and the tuition fee f_t are exogenously given and increase over time at the rate of economic growth. In addition we assume that the borrowing limit is tight so that students do not want to save during the education phase but instead consume the remainder of the study loan after paying the tuition fee:

$$c_t = \frac{q_t - f_t}{1+\tau_t^c}. \tag{3.8}$$

There are four possible levels of education to choose from. In particular, the years of education $E \in \{0,2,4,6\}$ where $E=0$ means no (tertiary) education, $E=2$ is an associate degree, $E=4$ is a bachelor's degree and $E=6$ corresponds to a master's degree. We impose that individuals cannot return to school once they have started working in the labour market. Therefore the choice of education level can only be adjusted at the start of adulthood or while still in school. Consider a student with educational ability θ who is u years old at the start of year t. We can write this individual's expected remaining lifetime utility as follows:

$$S_{u,t}(\theta) = \max_{E\in\{0,2,4,6\}} \left[\sum_{s=t}^{t-u+M+E-1} \beta^{s-t} \left[(c_s)^{\varepsilon}(1-\bar{e})^{1-\varepsilon}\right]^{1-1/\sigma} \right.$$
$$\left. + \beta^{M+E-u} \left[\mathbb{E}_{\gamma|\theta} \left[V_{M+E,t-u+M+E}\big(E,\gamma,0,\Gamma(\theta,E),1\big)^{1-\zeta} \right] \right]^{\frac{1-1/\sigma}{1-\zeta}} \right]^{\frac{1}{1-1/\sigma}}, \quad (3.9)$$

provided that $E \geq u - M$. During the remainder of the education phase the student consumes fixed amounts of goods and leisure which gives rise to the first term on the right-hand side of (3.9). At labour market entry γ is revealed, savings are zero, the stock of human capital is equal to $\Gamma(\theta, E)$ and labour productivity equals $\eta = 1$. This explains the arguments entering the value function at the age of school leaving. Solving the student's problem gives a policy function for the education phase:

$$\mathbf{E}_{u,t}(\theta). \quad (3.10)$$

3.2.3 Educational loan systems

Thus far the details of the system of educational loans have been subsumed in the term $\Upsilon_{u,t}(E, W)$ that features in the worker's budget constraint. We consider three prototypical systems which differ in the way that (explicit or implicit) study debt is redeemed. The base case is that of Subsidized Mortgage Loans (SML) in which each individual pays off his or her own loan during the subsequent working career. In the second case we consider, the educational loans of students are financed by the revenue from a Graduate Labour Tax (GLT) that has to be paid by all *educated* individuals. The third system is called the Comprehensive Labour Tax (CLT) and requires all workers to pay an additional tax on labour earnings, including those who have chosen to remain uneducated.

Subsidized Mortgage Loans (SML)

In this system an individual has an explicit level of study debt which in a given year t depends on age u and years of education E and is denoted by $\Omega_{u,t}(E)$. Every individual starts adulthood without any debt so that $\Omega_{M,t}(E) = 0$. During the education phase study debt increases as a result of accrued interest on existing debt and exogenous loan inflows:

$$\Omega_{u+1,t+1}(E) = \left[1 + (1-\tau_t^r)r_t\right]\Omega_{u,t}(E) + q_t \quad \text{for } M < u+1 \leq M+E. \quad (3.11)$$

Note that the interest payments are tax deductible from asset income. However, since students do not earn any income by assumption they effectively borrow at a subsidized rate of interest. During the working phase debt decreases because loan repayments exceed interest obligations from then on:

$$\Omega_{u+1,t+1}(E) = \left[1+(1-\tau_t^r)r_t\right]\Omega_{u,t}(E) - \Upsilon_{u,t}(E,W) \quad \text{for } M+E < u+1 \leq \bar{U}. \quad (3.12)$$

There are only payments from ages $\underline{u}(E)$ up to and including $\overline{u}(E)$, the redemption period. If $\underline{u}(E) > E$ then there is a grace period. The size of the redemption payment $\Upsilon_{u,t}(E,W)$ is determined in such a way that, in the absence of unanticipated changes to the interest rate, the loan will be paid off at age $\overline{u}(E)+1$ if the payment stays constant during the remainder of the redemption period.

Under the SML system every individual settles his or her own account. Default does not happen because (i) there is a (small) social security system in place which covers periods with zero labour income ($\nu_{u,t}$ in a worker's budget constraint) and (ii) rational and forward-looking individuals accumulate precautionary savings in order to avoid getting confronted with very low consumption levels in the future.

Graduate Labour Tax (GLT)

Under the GLT system there is no explicit study debt so that $\Omega_{u,t}(E) = 0$. However, there exists an implicit obligation in the sense that the government imposes a tax on all educated workers. The redemption period is the entire working phase and the contribution in year t is:

$$\Upsilon_{u,t}(E,W) = \tau_t^e \mathbb{1}_{\{E>0\}} W, \quad (3.13)$$

where τ_t^e is the educational labour tax rate, $\mathbb{1}_{\{E>0\}}$ is an indicator function which equals unity provided E is positive and is zero otherwise and W is gross wage income. Note that in contrast to the SML system, under the GLT system an individual can avoid making any payments by not working ($l = 0$). Furthermore, a lucky worker (with a high realization of η) contributes more per effective work hour than an unlucky worker does.

Comprehensive Labour Tax (CLT)

The CLT system is very similar to the graduate tax, except that all workers contribute and not only those who went to school themselves:

$$\Upsilon_{u,t}(E,W) = \tau^e_t W. \tag{3.14}$$

However, since educated individuals tend to have more human capital and are less likely to be unemployed they will have higher gross wages and therefore greater expected contributions.

3.2.4 Aggregation

Consider a cohort that attains the age of majority M at the start of year t_0. Every individual in this cohort has an index $i \in \{1, 2, \ldots, P_{M,t_0}\}$ and the initial endowments are given by:

$$a^i_{M,t_0} = 0, \quad h^i_{M,t_0} = 1, \quad d^i_{M,t_0} = 0.$$

For each individual we draw a talent for education θ^i. Then we can follow him or her over time. First, the optimal years of education are given by policy function (3.10) evaluated at age M: $E^i = \mathbf{E}_{M,t_0}(\theta^i)$. Of course, if there are unexpected shocks (for example a change in the system of educational loans) then individuals who are still in school can re-optimize and choose a different education level than the one they planned before the shock. During the education phase consumption is fixed at $c^i_{u,t} = c_t$, labour supply $l^i_{u,t}$ is zero, there are no savings such that $a^i_{u,t} = 0$ and the level of study debt $d^i_{u,t}$ evolves exogenously according to the specific system in place.

At labour market entry individual i has a startup human capital stock equal to $\Gamma(\theta^i, E^i)$. We can then draw a learning ability parameter γ^i (correlated with θ^i) and a sequence of idiosyncratic productivity shocks $\{\eta^i_{u,t}\}$ (dependent on E^i). By applying the policy functions (3.7) iteratively we can obtain the profile of consumption $c^i_{u,t}$, labour supply $l^i_{u,t}$, financial assets $a^i_{u,t}$ and human capital $h^i_{u,t}$ during the working phase:

$$\begin{aligned} c^i_{u,t} &= \mathbf{c}_{u,t}(E^i, \gamma^i, a^i_{u,t}, h^i_{u,t}, \eta^i_{u,t}), & l^i_{u,t} &= \mathbf{l}_{u,t}(E^i, \gamma^i, a^i_{u,t}, h^i_{u,t}, \eta^i_{u,t}), \\ a^i_{u+1,t+1} &= \mathbf{a}^+_{u,t}(E^i, \gamma^i, a^i_{u,t}, h^i_{u,t}, \eta^i_{u,t}), & h^i_{u+1,t+1} &= \mathbf{h}^+_{u,t}(E^i, \gamma^i, a^i_{u,t}, h^i_{u,t}, \eta^i_{u,t}), \end{aligned}$$

while study debt still depends on the type of educational loan system.

Once individual choices have been determined cohort averages can be calculated as follows:

$$\bar{c}_{u,t} \equiv \frac{1}{P_{u,t}} \sum_{i=1}^{P_{u,t}} c^i_{u,t}, \quad \bar{l}_{u,t} \equiv \frac{1}{P_{u,t}} \sum_{i=1}^{P_{u,t}} \eta^i_{u,t} h^i_{u,t} l^i_{u,t},$$
$$\bar{a}_{u,t} \equiv \frac{1}{P_{u,t}} \sum_{i=1}^{P_{u,t}} a^i_{u,t}, \quad \bar{d}_{u,t} \equiv \frac{1}{P_{u,t}} \sum_{i=1}^{P_{u,t}} d^i_{u,t},$$

where $\bar{c}_{u,t}$ is average consumption, $\bar{l}_{u,t}$ is average effective labour, $\bar{a}_{u,t}$ is average financial assets and $\bar{d}_{u,t}$ is average study debt. Population totals are calculated by aggregating over cohorts, for example for total consumption C_t we have:

$$C_t \equiv \sum_{u=M}^{\bar{U}} P_{u,t} \bar{c}_{u,t}.$$

The definitions for total effective labour supply L_t, total financial asset holdings A_t and total study debt D_t are similar. Since there is no aggregate uncertainty, cohort averages and population totals (and thereby also factor prices) are deterministic quantities, provided that the population is sufficiently large. See Appendix 3.A for a discussion of how to obtain aggregate statistics from policy functions without the need to trace individual life-cycle choices.

3.2.5 Government

Apart from administering the educational loan system on a balanced budget basis, the government also collects taxes on consumption, wage income and interest income which it uses to finance (intrinsically useless) public spending and to fund the system of unemployment benefits. In the interest of clarity we split the governmental accounts into a regular budget and an educational loan budget.

Regular budget

There is an exogenous level of government spending G_t which increases in line with economic progress and population growth. Total tax revenue T_t is equal to:

$$T_t \equiv \tau^c_t C_t + \tau^w_t w_t L_t + \tau^r_t r_t \big[A_t - D_t\big]. \tag{3.15}$$

In case of mortgage loans the tax deductibility of interest payments on study debt shows up here so that the educational loan system is not completely separated from the regular budget. The total expenditure on unemployment benefits B_t is given by:

$$B_t \equiv \sum_{u=M}^{\bar{U}} \nu_{u,t} \sum_{i=1}^{P_{u,t}} \mathbb{1}_{\{\eta^i_{u,t}=0\}}. \tag{3.16}$$

The balanced budget requirement is then given by:

$$T_t = G_t + B_t. \tag{3.17}$$

Budget of the educational loan system

Under the SML system, study loans are redeemed by the students themselves and by its very design the repayment scheme ensures that all debt is paid back. In contrast, if educational loans are financed by taxes (the GLT or CLT system) then in every year t the contributions from workers should cover the total transfers to current students:

$$\sum_{u=M}^{\bar{U}} q_t \sum_{i=1}^{P_{u,t}} \mathbb{1}_{\{E^i>u-M\}} = \sum_{u=M}^{\bar{U}} \sum_{i=1}^{P_{u,t}} \Upsilon_{u,t}\big(E^i, w_t \eta^i_{u,t} h^i_{u,t} l^i_{u,t}\big). \tag{3.18}$$

3.2.6 Macroeconomic equilibrium

In the macroeconomic equilibrium all markets need to clear. The goods market equilibrium condition is given by:

$$Y_t = C_t + I_t + G_t + F_t, \tag{3.19}$$

where F_t is the total amount of tuition fees paid in year t:

$$F_t = \sum_{u=M}^{\bar{U}} f_t \sum_{i=1}^{P_{u,t}} \mathbb{1}_{\{E^i>u-M\}}. \tag{3.20}$$

The capital market equilibrium condition states that the productive capital stock is equal to the net stock of financial assets owned by individuals:

$$K_t = A_t - D_t. \tag{3.21}$$

Finally, the labour market equilibrium condition requires equality between the demand for and the supply of effective labour:

$$N_t = L_t. \tag{3.22}$$

In the steady state of the model A_t, B_t, C_t, D_t, F_t, G_t, I_t, K_t, T_t and Y_t grow at rate $(1+n_Z)(1+n_P)-1$, L_t and N_t grow at rate n_P and the wage rate w_t grows at rate n_Z. By scaling these variables appropriately they will be constant along the balanced growth path, see Appendix 3.B for more details.

3.3 Calibration

In this section we present and motivate the calibration of our model. In addition we visualize its main steady-state properties.

3.3.1 Distributions

We need to specify the distribution of the various stochastic model elements that have been discussed in Section 3.2.2. First, we assume that the talent for education θ follows a truncated normal distribution on $[0,1]$ with parameters μ_θ and σ_θ. This combines the convenience of a closed and bounded support with the flexibility of a bell-shaped curve. The second stochastic element is the learning-by-doing parameter γ which takes either the value γ_l or γ_h with $\gamma_l < \gamma_h$. We specify the probability of each outcome conditional on θ as:

$$\mathbb{P}\big(\gamma = \gamma_h|\theta\big) = 0.5 + \rho_{\gamma\theta}\big[F_\theta(\theta) - 0.5\big], \quad \mathbb{P}\big(\gamma = \gamma_l|\theta\big) = 1 - \mathbb{P}\big(\gamma = \gamma_h|\theta\big), \tag{3.23}$$

where F_θ is the cumulative distribution function of θ. If $\rho_{\gamma\theta} > 0$ then there is a positive correlation between the talent for education and the ability to learn on the job. By setting $\gamma_l = \mu_\gamma - \sigma_\gamma$ and $\gamma_h = \mu_\gamma + \sigma_\gamma$ we ensure that the unconditional mean and variance are given by $\mathbb{E}[\gamma] = \mu_\gamma$ and $\text{Var}(\gamma) = \sigma_\gamma^2$. Finally we have to determine the transition matrix for the Markov process that governs idiosyncratic labour productivity η. We assume that there is an education-specific probability to enter into 'unemployment' (that is, $\eta = 0$) denoted by $\pi(E)$. There is a probability κ of returning to $\eta = 1$ in the next year and a probability $1-\kappa$ of remaining unemployed for an additional year. Conditional on being productive (that is, $\eta > 0$) labour productivity should mimic a log-AR(1) process with autocorrelation ρ_η and a stochastic innovation term with variance σ_ϵ^2. We impose some additional restrictions on transitions between states, see Section 3.2.2. The resulting transition matrix is given by:

$$\Pi(E) = \begin{pmatrix} 1-\kappa & 0 & \kappa & 0 \\ \pi(E) & [1-\pi(E)]\rho_\eta & [1-\pi(E)](1-\rho_\eta) & 0 \\ \pi(E) & [1-\pi(E)]\frac{1-\rho_\eta}{4} & [1-\pi(E)]\frac{1+\rho_\eta}{2} & [1-\pi(E)]\frac{1-\rho_\eta}{4} \\ \pi(E) & 0 & [1-\pi(E)](1-\rho_\eta) & [1-\pi(E)]\rho_\eta \end{pmatrix}. \tag{3.24}$$

The corresponding states are $\{0, \eta_l, 1, \eta_h\}$ with:

$$\eta_l = e^{-\sqrt{3\sigma_\epsilon^2/(1-\rho_\eta)^2}}, \quad \eta_h = e^{\sqrt{3\sigma_\epsilon^2/(1-\rho_\eta)^2}}. \tag{3.25}$$

A given entry of $\Pi(E)$ represents the probability of moving from the state corresponding to the row to the one associated with the column. For example, the entry in row 2 and column 3 is $\mathbb{P}(\eta^+ = 1|\eta = \eta_l, E)$. The probability of the first state in the limiting distribution is $\pi(E)/[\kappa + \pi(E)]$ which captures the unemployment rate for a given education level.

3.3.2 Functional forms

We assume that the stock of human capital at labour market entry for a person with talent for education θ and years of education E is given by:

$$\Gamma(\theta, E) = 1 + \xi_1\theta E - \xi_2[1-\theta]E^2, \quad \xi_1 > 0, \quad \xi_2 > 0. \tag{3.26}$$

Note that individuals with a higher ability level experience weaker diminishing returns to education:

$$\frac{\partial\Gamma(\theta, E)}{\partial E} = \xi_1\theta - 2\xi_2[1-\theta]E, \quad \frac{\partial^2\Gamma(\theta, E)}{\partial E^2} = -2\xi_2[1-\theta] \leq 0. \tag{3.27}$$

Under this assumption the optimal number of college years is increasing in talent. In the absence of the diminishing-returns effect (with $\xi_2 = 0$) this need not be true because high-ability individuals also have a higher opportunity cost of time.

We postulate that the rate of human capital depreciation increases with age according to the following schedule:

$$\delta_u^h = 1 - \delta_0\left(\frac{\bar{U} - u}{\bar{U} - M}\right)^{\delta_1} \quad \text{for } M \leq u \leq \bar{U}, \quad 0 \leq \delta_0 \leq 1, \quad 0 < \delta_1 \leq 1. \tag{3.28}$$

3.3.3 Parameter values

We calibrate the model so that the benchmark steady state with subsidized mortgage loans matches some key features of the United States' economy. For this we use a two-step procedure. First we assign to a subset of the parameters values that are taken directly from the data or the literature, see Table 3.1. İmrohoroğlu and Kitao (2009) provide an overview of estimates for the intertemporal substitution elasticity σ and we choose a value within the range they report. The coefficient of relative risk aversion is set in accordance with Cecchetti et al. (2000), who find that it is reasonable to have a value between 1 and 5.

Data from the World Bank for 2012 gives a population growth rate of 0.74% for the United States. The maximum age is set equal to life expectancy at birth for the same year, rounded to the nearest integer. To obtain an estimate of the long run economic growth rate we collect data on GDP per capita from the Federal Reserve Economic Data of the St. Louis Federal Reserve Bank (measured in 2011 US dollars) and regress its log on a time variable. The resulting coefficient is 0.02 which corresponds to 2% growth per year.

The growth rate of wages in the model depends on an individual's ability to learn on the job and is therefore not the same for every person. We take an estimate for the autocorrelation of the labour productivity process ρ_η from Guvenen (2009), who allows for heterogeneity in income growth rates. In order to capture the fact that long-term unemployment (more than one year) is very uncommon in times of normal economic activity we choose a value close to 1 for the recovery rate κ. We set the probability of entering unemployment $\pi(E)$ such that the unemployment rate by education group approximately matches the average over the years 2000 up to and including 2006 as calculated from the March Current Population Survey (CPS, see King et al. (2010)). It follows that education offers some insurance against being out of work as more educated individuals are less likely to become unemployed.

We include a simple system of unemployment protection. In the absence of such a social security scheme individuals would work 'too hard' and save 'too much' in the years immediately following graduation compared to the data. As they enter the labour market without any savings but do face the risk of unemployment they have an incentive to accumulate precautionary savings at a quick rate. In addition, if there is no redistribution towards the unemployed in the benchmark case then we are likely to overstate the welfare gains from reforming the educational loan system in such a way that it offers more insurance against low income periods. We assume that all individuals between ages 18 and 60 whose labour productivity in a given year equals

Table 3.1: Parameters taken from data or literature

Parameter		Value	Source
Preferences			
Intertemporal substitution elasticity	σ	0.500	İmrohoroğlu and Kitao (2009)
Coefficient of relative risk aversion	ζ	4.000	Cecchetti et al. (2000)
Demography			
Age of majority	M	18.000	
Population growth rate	n_P	0.007	WB for 2012
Maximum age	$\overline{U}$	79.000	WB for 2012
Technology			
Economic growth rate	n_Z	0.020	FRED for 1970-2006
Wage uncertainty			
Autocorrelation of log productivity	ρ_η	0.821	Guvenen (2009)
Probability exiting unemployment	κ	0.990	
Probability entering unemployment	$\pi(0)$	0.048	March CPS for 2000-2006
Probability entering unemployment	$\pi(2)$	0.035	March CPS for 2000-2006
Probability entering unemployment	$\pi(4)$	0.027	March CPS for 2000-2006
Probability entering unemployment	$\pi(6)$	0.019	March CPS for 2000-2006
Educational loans			
Annual loan to average income		0.238	NCES for 2012
Tuition fee as fraction of loan		0.400	
Length of grace period		4.000	
Length of redemption period		15.000	
Unemployment protection			
Replacement rate unemployment		0.250	

Sources: CPS is the Current Population Survey. FRED is the Federal Reserve Economics Data of the St. Louis Federal Reserve Bank. NCES is the National Center for Education Statistics. WB is the World Bank.

zero receive a fixed benefit independent of their employment history. Data from the US Department of Labor indicate that the average replacement ratio (definition 1) in the United States is about 47%. However, since entitlements are typically capped at six months and unemployment lasts for one year in our model we have chosen to set the unemployment benefit equal to 25% of average income in the calibration.

Our modelling of the education phase is very stylized and therefore it is not straightforward to choose parameter values for the annual amount of study loan and the tuition fees. Our main goal is to have a realistic level of student debt. To that end we use the average annual loan take-up of undergraduate and graduate students in 2012 from the National Center for Education Statistics. This gives an amount of $\$11,887$ or approximately 24% of average income in the United States in the same year (which is about $\$50,000$). We set the tuition fee at 40% of this amount to capture the fact that part of the loan cannot be directly consumed. In the United States most types of study loans have no or a very brief grace period, but repayments can be deferred for up to 3 years during periods of unemployment or economic hardship. Although the standard repayment plan for federal loans is 10 years it is possible to arrange an extension up to 30 years. We simplify these provisions somewhat in the model by including a grace period of 4 years for everyone and by setting the redemption period equal to 15 years.

In the second step we calibrate the remaining parameters (Table 3.2) so as to match certain targets from the data (Table 3.3). Some of these targets are quite standard: a capital to output ratio of about 3, an average work week of 40 hours for those that work at least 5 hours and a net return to capital of 4% per year.[2] We impose that investment and government spending take up 19% and 17% of yearly output, respectively. In addition we normalize the (scaled) rental rate of effective labour to unity. The target for consumption tax revenue relative to output is taken from the OECD tax database.

The remaining targets require some more elaborate discussion. To calculate the education distribution we use information on educational attainment for individuals age 25 and above from the March Current Population Survey (CPS) of 2012. We exclude individuals without a high school diploma and group those with some college but no degree with the high school graduates ($E = 0$). An associate degree (whether occupational or academic) corresponds to $E = 2$ while a bachelor's degree is $E = 4$. For individuals with a master's degree or above we set $E = 6$. In the resulting distribution more than half of the population (52%) has no tertiary education at all, while most of

[2] We assume that the unit time endowment of individuals corresponds to about 14 hours a day (excluding sleep and personal care) or 100 hours a week. This means that a 40-hour work week equals 40% of the time endowment.

Table 3.2: Calibrated parameters

Parameter		Value
Preferences		
Time discount factor	β	0.983
Consumption share in felicity	ε	0.304
Technology		
Capital share in production	ϕ	0.227
Technology level	Φ	0.952
Capital depreciation rate	δ_K	0.036
Government		
Consumption tax rate	τ_t^c	0.070
Income tax rate	$\tau_t^w = \tau_t^r$	0.150
Education		
Location parameter talent for education	μ_θ	0.032
Scale parameter talent for education	σ_θ	0.402
Linear term in return to education	ξ_1	0.253
Quadratic term in return to education	ξ_2	0.001
Leisure cost of studying	$\bar{e}$	0.290
Learning ability		
Strength of experience effect	α	0.638
Location parameter learning ability	μ_γ	0.093
Scale parameter learning ability	σ_γ	0.019
Relation talent for education and learning ability	$\rho_{\gamma\theta}$	0.800
Human capital depreciation		
Level parameter human capital depreciation	δ_0	0.981
Curvature parameter human capital depreciation	δ_1	0.053
Wage uncertainty		
Standard deviation of innovation term	σ_ϵ	0.205

those that do attend college obtain a bachelor's degree.

From Krueger and Ludwig (2013) we take two age profiles for labour efficiency: one for individuals with no college education and one for individuals with some. These are normalized by the average productivity of a high school graduate at age 23. We include the efficiency level at ages 25, 35, 45 and 55 for each profile among our targets. This corresponds to the average of $h\,\eta$ in our model and will help us identify the parameters that govern the accumulation of labour market experience over the life cycle. We make sure that the implied college wage premium, the average hourly wage of individuals with at least 4 years of college education relative to that of individuals who are less educated, is comparable to the one calculated by Heathcote et al. (2010) for 2005. Finally, we include two measures of wage uncertainty. The first is the variance of the log of annual labour earnings at age 50 as reported by Storesletten et al. (2004). The second one comes from Guvenen (2009) and captures the variability among individuals in the extent to which wages increase with one more year of labour market experience. This corresponds to the variance of γl^{α} in our model.

The calibration then proceeds as follows. Parameters that are closely related to a specific target (those representing firm technology and government tax rates) or to which the model solutions are particularly sensitive (those affecting individual preferences) are updated in each iteration. These parameters and the corresponding moments are listed above the dashed line in Tables 3.2 and 3.3. The parameters below the dashed line are determined using the Method of Simulated Moments (MSM). Let x denote the vector of 15 targeted empirical moments and p the vector of 12 parameter values. For each choice of p we can solve the model and calculate the counterparts of the empirical moments from the simulated data. These are denoted by $X(p)$. Under the null hypothesis that the model has been correctly specified the following moment condition holds for the true parameter vector p^*:

$$\mathbb{E}\big[X(p^*) - x\big] = 0.$$

The MSM estimator $\hat{p}$ is then given by:

$$\hat{p} = \operatorname{argmin}\big[X(p) - x\big]'\bar{W}\big[X(p) - x\big],$$

where $\bar{W}$ is a weighting matrix. For $\bar{W}$ we use the matrix with on the diagonal the inverse of the square of the successive elements in x and zeros elsewhere. This means that effectively we minimize the sum of squared relative deviations of the simulated moments from their targets.

The resulting parameter values are reported in Table 3.2. We will briefly discuss some of them. As the ratio of government spending and investment to output are fixed and tuition fees are small, it follows that consumption will always constitute around 64% of income. As a consequence, setting the consumption tax at 7% will bring the resulting revenue close to the desired target of 4.35%. Note that we have imposed that the tax rate on wages and interest received and paid should be equal. The uniform income tax rate that is required to balance the government budget is around 15%. The parameter values for μ_γ, σ_γ and α imply that for young individuals the return to experience given a 40-hour work week ranges between 4% and 6% depending on the ability to learn on the job. For older individuals these figures decline because of the ageing effect in human capital depreciation. The return to one year of education for the marginal student (the one who is indifferent between no education at all and 2 years of college) is around 8.5% based on our estimates of ξ_1 and ξ_2.

The model does a good job in matching the targeted moments, as can be seen from Table 3.3 and Figure 3.3. First of all, the model is able to replicate the bimodal distribution of education levels. The corresponding density function for educational talent θ is depicted in panel (a) of Figure 3.3. It is single-peaked (by design) and features a lot of mass on the left-hand side and a thin tail at the right-hand side. The optimal education choice is increasing in ability as evidenced by panel (b). There are three cut-off values such that individuals for whom $0 \leq \theta < 0.292$ find it optimal not to enjoy any tertiary education, those with $0.292 \leq \theta < 0.384$ choose an associate degree, $0.384 \leq \theta < 0.607$ corresponds to a bachelor's degree and if $\theta \geq 0.607$ then the individual gets 6 years of schooling. Second, the cohort labour efficiency profiles generated by the model match their empirical counterparts quite well, especially up to age 55. Figure 3.3(c) shows that after age 55 the model underpredicts the relative productivity of college-educated individuals. Third, not only is the variance of log earnings at age 50 close to its target, the convexity of the age profile as shown in panel (d) is consistent with the empirical findings of Guvenen (2009).[3]

3.3.4 Visualization of the benchmark

In Figure 3.4 we visualize the most important life-cycle profiles for the calibrated benchmark economy. The first thing to note is that once all individuals have started to work (that is, from age 24 onwards) consumption, labour supply, financial assets and wage income are all monotonically increasing in education level for any given age.

[3] For higher ages the pattern becomes irregular as more and more individuals stop supplying hours to the labour market.

Table 3.3: Model fit on moments targeted by the calibration

	Model	Target	Source
Factor inputs and prices			
Capital to output ratio	2.983	3.000	
Average hours worked by employed	40.101	40.000	
Interest rate	0.040	0.040	
Rental rate of effective labour	1.000	1.000	
Investment to output	0.190	0.190	
Government			
Consumption tax revenue to output	4.454	4.350	OECD for 2012
Government spending to output	0.170	0.170	
Education			
Share with 0 years (in %)	52.020	53.200	March CPS for 2012
Share with 2 years (in %)	13.120	11.130	March CPS for 2012
Share with 4 years (in %)	21.810	22.890	March CPS for 2012
Share with 6 years (in %)	13.050	12.790	March CPS for 2012
Cohort labour efficiency profiles			
Efficiency no college age 25	1.059	1.060	Krueger and Ludwig (2013)
Efficiency no college age 35	1.311	1.287	Krueger and Ludwig (2013)
Efficiency no college age 45	1.457	1.398	Krueger and Ludwig (2013)
Efficiency no college age 55	1.427	1.407	Krueger and Ludwig (2013)
Efficiency college age 25	1.509	1.576	Krueger and Ludwig (2013)
Efficiency college age 35	2.119	2.243	Krueger and Ludwig (2013)
Efficiency college age 45	2.572	2.622	Krueger and Ludwig (2013)
Efficiency college age 55	2.672	2.700	Krueger and Ludwig (2013)
College wage premium (in %)	77.583	80.000	Heathcote et al. (2010)
Wage uncertainty			
Variance in income growth ($\times 10^3$)	0.357	0.380	Guvenen (2009)
Variance of log earnings at age 50	0.720	0.700	Storesletten et al. (2004)

Sources: BLS is the Bureau of Labor Statistics of the United States Department of Labor. CPS is the Current Population Survey. OECD is the Organisation for Economic Co-operation and Development.

Figure 3.3: Calibration outcomes

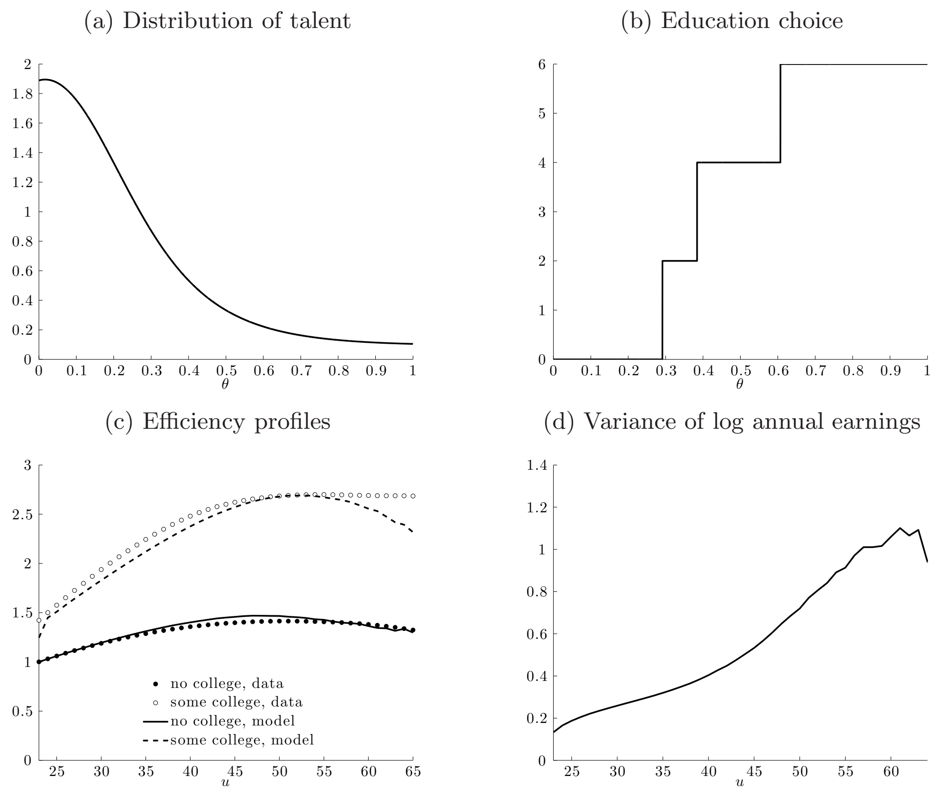
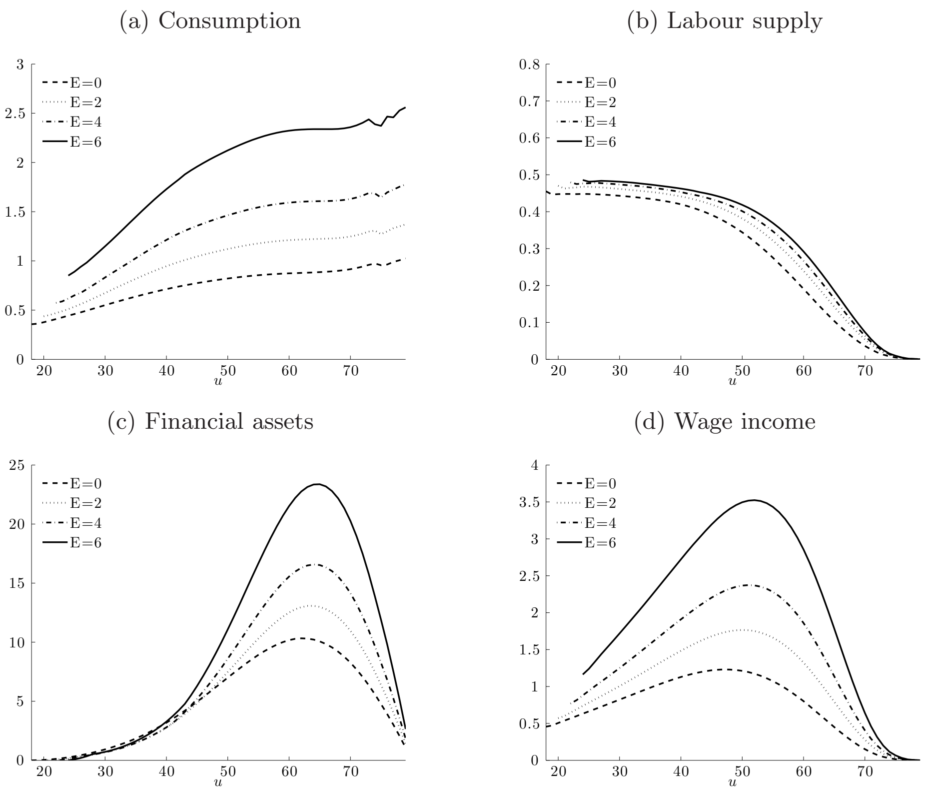
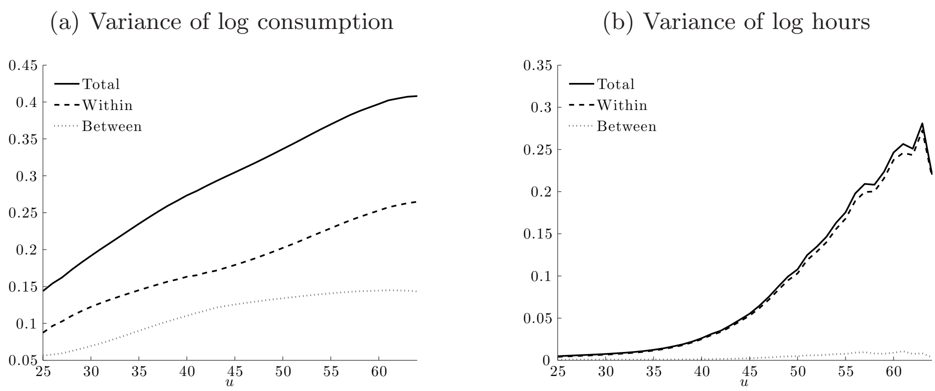


Figure 3.4: Age profiles of cohort averages



Naturally this only holds for the group averages, at the individual level it need not be true. Panel (a) shows that consumption is slightly hump-shaped during the working phase due to the non-separability between consumption and leisure in the felicity function. In the absence of uninsured mortality risk there is a counterfactual upward sloping profile during retirement. In panel (b) we observe that for each educational group average labour supply is roughly constant at first but decreases fast once middle age sets in. The hump shaped pattern of financial assets in panel (c) is a consequence of the fact that all individuals start life without financial assets and, in the absence of a bequest motive, plan to die with zero assets as well. During the working phase they have an incentive to accumulate precautionary savings in order to self-insure against productivity shocks and to smooth consumption over time. Finally, panel (d) shows that wage inequality is quite substantial during middle age but is reduced at the end of life as individuals cut back on labour hours.

Figure 3.5: Measures of economic inequality



In Figure 3.5 we show two commonly used measures of economic inequality by age, namely the variance of log consumption for the entire cohort (panel (a)) and the variance of log hours for those who work at least 5 hours per week (panel (b)). Using data for the United States, Heathcote et al. (2010) document that household consumption inequality rises until about age 50 and flattens out thereafter. The increasing part is clearly evident in our model too but the flattening out occurs later in life and is very mild, see the solid line in panel (a). In the data the age profile for the variance of log hours is U-shaped as the dispersion is especially high for young workers (due to high unemployment risk) and for older workers (due to early retirement). In contrast, in our model the profile is J-shaped because the unemployment rate is assumed to be age-independent, see the solid line in panel (b). In order to obtain more insight in the determinants of inequality we group individuals by their level of education and decompose the total dispersion in two components. The within-group variation is driven by the idiosyncratic labour productivity process η and the heterogeneity in the ability to learn on the job γ. The remainder is the between-group variation, which shows up in different levels of start-up human capital, the correlation between the talent for learning in school and on the job and the fact that more educated individuals face a smaller risk of becoming unemployed. Panel (a) shows that within-group inequality (dashed line) is about twice as high as between-group inequality (dotted line) for consumption. Interestingly, as is shown in panel (b), the dispersion in log hours consists almost entirely of within-group inequality. This result follows from the fact that the preference structure satisfies the growth-consistency conditions formulated by King et al. (2002), thereby ensuring that the substitution and income effect on labour supply of a proportional increase in wages at all ages exactly cancel out.

3.4 Policy reform 1: From SML to GLT

In this section and the next we study possible reforms of the educational loan system. Both of these reforms are initiated in the steady state of the benchmark economy with Subsidized Mortgage Loans (SML). Here we consider the introduction of a Graduate Labour Tax (GLT) while in Section 3.5 we discuss the move to a Comprehensive Labour Tax (CLT) system. In each case we compute the transitional and long-run effects of the policy change on the economic allocation and the consequences for the level of welfare of existing and future cohorts. In addition we decompose the total effect into several components in order to highlight the key mechanisms in the model.

3.4.1 Education choices

The policy reform is implemented at the start of period $t = 0$ and takes people unawares. Mortgage loans are no longer available as a means of education financing. Instead students receive a grant from the government during their education phase and face an additional tax on labour income while working. Any existing study debt accumulated before the policy reform will have to be redeemed according to the regulations of the original SML system.

In Table 3.4 we show how the distribution of education levels across the adult population changes between steady states. The column labeled SML corresponds to the calibrated benchmark. The next column gives the distribution under the GLT system. There is

Table 3.4: Steady-state education distribution

	SML	GLT	CLT
Share with 0 years	52.02%	52.55%	40.90%
Share with 2 years	13.12%	0.67%	12.84%
Share with 4 years	21.81%	23.10%	23.60%
Share with 6 years	13.05%	23.68%	22.66%

hardly any change in the proportion of uneducated individuals, the *extensive* margin of the education decision. In contrast, there are significant shifts in the shares of the remaining educational groups. Conditional on going to college, education is increased at the *intensive* margin as individuals stay in school longer.

3.4.2 Transitional dynamics

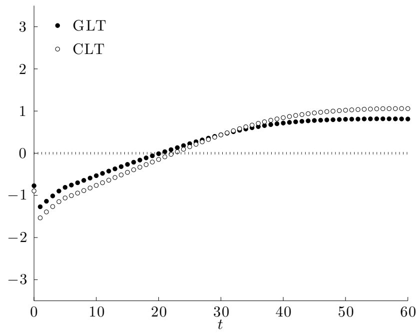
The relative changes in the macroeconomic variables during the transition period are illustrated by the black dots in Figure 3.6. At the time of the shock, output, consumption and effective employment drop by, respectively, 0.77%, 1.45%, and 1.00%. In the period immediately after these variables reach their maximum reductions at, respectively, 1.27%, 1.96%, and 1.65%. The capital stock initially stays fixed as it is predetermined by past investment decisions. In the long run it increases by 2.72% while effective labour rises by 0.23%. It follows that capital becomes relatively abundant in production so that its return drops by 0.14 percentage points and the rental rate of effective labour increases by 0.56%. Finally, consumption rises with 0.30% while output increases by 0.79% in the long run. These steady-state changes are also summarized in the column labeled GLT in Table 3.5. In addition we see that the graduate labour tax is equal to 2.37% of labour earnings and that the amount of tuition fees paid increases by 22.30% as the average educational attainment of the population goes up.

3.4.3 Welfare effects

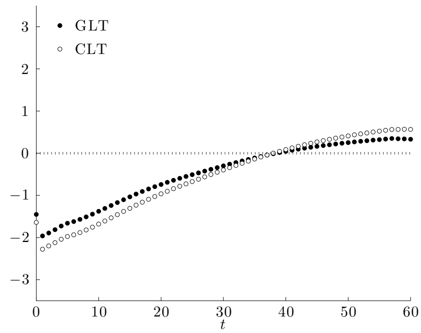
The fact that there are significant transitional dynamics in the model combined with the assumption that individuals are heterogeneous implies that the welfare consequences of the policy reform differ by cohort and talent for education. We study both dimensions by constructing two different measures of welfare changes.

Figure 3.6: Transitional changes

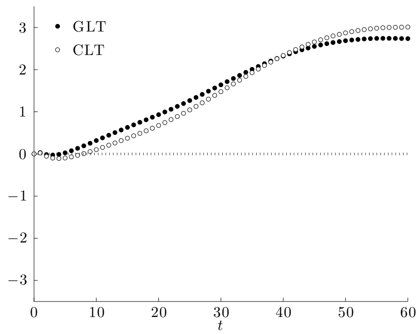
(a) Output (in %)



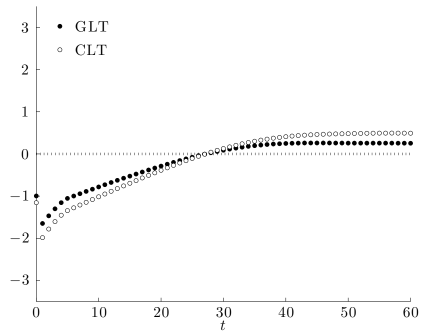
(b) Consumption (in %)



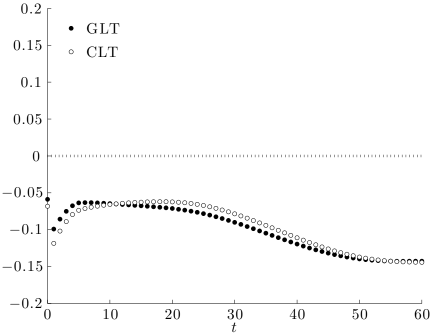
(c) Capital (in %)



(d) Effective labour (in %)



(e) Interest rate (in %p)



(f) Rental rate of effective labour (in %)

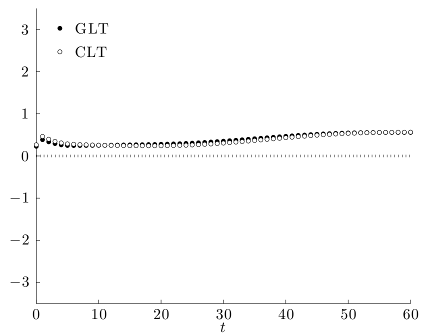


Table 3.5: Long-run macroeconomic changes

	GLT	CLT
Output	+ 0.79%	+ 1.03%
Consumption	+ 0.30%	+ 0.53%
Capital	+ 2.72%	+ 3.00%
Effective labour	+ 0.23%	+ 0.46%
Interest rate	− 0.14%p	− 0.15%p
Rental rate of effective labour	+ 0.56%	+ 0.57%
Income tax rate	− 0.14%p	− 0.21%p
Educational labour tax rate	+ 2.37%p	+ 1.56%p
Tuition fees	+22.30%	+33.39%

Ex-ante welfare changes along the transition path

In order to get a sense of the cohort-specific welfare changes along the transition path we adopt the approach suggested by Fehr and Kindermann (forthcoming). We take the factor prices and tax rates in each year as given. For every cohort we calculate the transfer they should receive in order to make them, from an ex-ante perspective, equally well off under the new policy regime as in the initial steady state. The level of ex-ante welfare is calculated one second before individuals attain the age of majority so that they still face uncertainty about their talent for education. Since everyone is identical 'behind the veil of ignorance' this implies that there is only one transfer needed for every cohort. Importantly, we do not want to provide the transfer at a moment in life when individuals are likely to be borrowing constrained (during the education phase or shortly thereafter). Therefore we impose that an individual cannot receive a transfer before age 27.

We first consider future cohorts, those reaching the age of majority at the time of the shock or in the years thereafter. We let $\Lambda_t^j(\theta, I)$ denote the lifetime utility of an individual of age M in year t under regime $j \in \{SML, GLT, CLT\}$. This person has educational talent θ and receives a transfer upon reaching age 27, the present value of which equals I at age M. Then we can calculate expected ex-ante welfare as:

$$\Psi_t^j(I) = \mathbb{E}_\theta \left[\Lambda_t^j(\theta, I)^{1-\zeta} \right]^{\frac{1}{1-\zeta}}, \tag{3.29}$$

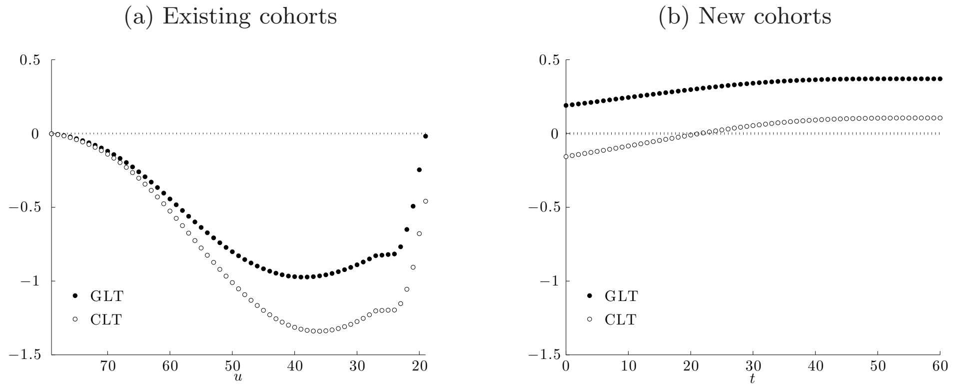
where we recall that ζ is the degree of relative risk aversion of individuals. The compensating transfer $I_{M,t}^j$ that this cohort should receive is the one that equalizes ex-ante welfare under subsidized mortgage loans to that under the new policy regime j:

$$\Psi_t^{SML}(0) = \Psi_t^j(I_{M,t}). \tag{3.30}$$

If $I^j_{M,t} < 0$ (a payment instead of a gift) then the cohort is better off after the policy change. In Figure 3.7(b) we plot the negative of the transfer under GLT (corrected for economic growth) as a percentage of the pre-reform level of aggregate consumption so that a positive number corresponds to a welfare gain. It follows that all future cohorts are better off as a result of the policy change.

For existing cohorts we make a similar calculation. Consider the cohort that is of age $u > M$ at the time of the policy reform $t = 0$. All decisions that have been made in the past are predetermined and cannot be changed. We calculate the transfer that, when paid out either immediately or at age 27 (whichever comes sooner) makes the individuals in this cohort indifferent between policy regimes in terms of ex-ante welfare (again calculated from the perspective of a second before the age of majority). The present value of this transfer at age u is denoted by $I^j_{u,0}$. In Figure 3.7(a) we show the negative of the transfer relative to pre-reform aggregate consumption for each existing cohort. We observe that all existing cohorts are worse off as a result of the policy change. For educated working-age individuals this result follows readily from the fact that they are, in a sense, paying the same bill twice. They must continue to redeem any existing study debt but are also faced with a higher tax on labour earnings. Students are negatively affected as well, but to a lesser extent the younger they are (and thus the lower is the incurred study debt). Middle-aged and old cohorts have paid off their study debts and are hurt mainly by the graduate tax.

Figure 3.7: Compensating transfers from SML to GLT



The above results indicate that some cohorts gain if the educational loan system is reformed while others are worse off (if uncompensated). To get an aggregate measure of the change in ex-ante welfare we calculate the present value of the negative of all the transfers using the (constant) interest rate in the initial steady state r for discounting. This will ensure that the weight given to each cohort is the same and does not depend on the factor price changes generated by the reform. The resulting expression is:

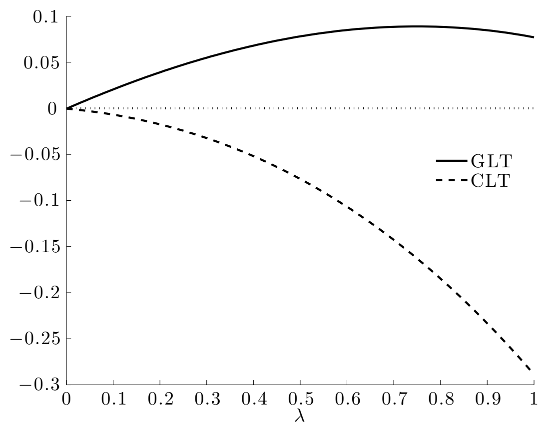
$$PV^j = -\left[\sum_{u=M+1}^{\bar{U}} P_{u,0} I_{u,0}^j + \sum_{t=0}^{\infty} \frac{P_{M,t} I_{M,t}^j}{(1+r)^t}\right]. \tag{3.31}$$

In order to facilitate interpretation we convert this present value into an annuity stream. That is, we determine a yearly annuity payment AP^j that is indexed by population growth and economic progress and has the same present value:

$$PV^j = AP^j \sum_{t=0}^{\infty} \left(\frac{(1+n_Z)(1+n_P)}{1+r}\right)^t. \tag{3.32}$$

We say that a policy reform leads to an aggregate welfare gain if the compensating annuity payment is positive. For the graduate labour tax system it is equal to 0.08% of aggregate consumption in the initial steady state. Although this number is quite small, it implies that everybody can be made better off (in an ex-ante sense) if the reform from SML to GLT takes place and cohorts are appropriately compensated.

Figure 3.8: Aggregate welfare change as a function of λ



Interestingly, as is shown in Figure 3.8 (solid line), the government could improve aggregate welfare even more under the graduate labour tax system by only partially taking over the student loans. Let $\lambda \in [0, 1]$ be the fraction of each student's allowance that is paid for out of government tax revenue so that the remainder $1-\lambda$ has to be financed by subsidized mortgage loans. Under GLT the optimal value of λ is 0.75. A hybrid reform thus outperforms the pure GLT system.

Ex-post welfare changes by educational talent

We are also interested in which individuals gain and lose by the policy reform within a cohort. To that end we want to compare ex-post steady-state welfare between policy regimes, conditional on the talent for education θ. Instead of calculating compensating transfers by type we use an alternative (in this case, simpler) welfare metric, similar to that discussed in Auerbach and Kotlikoff (1987). Suppose that we increase individual consumption in every year of life by a proportion ω, then due to the linear homogeneity of preferences we would find that the value function at the start of adulthood becomes $(1+\omega)^{\varepsilon}S_{M,t}(\theta)$. We want to obtain the value of ω that ensures that an individual of age M in period t is equally well off in the initial steady-state equilibrium with subsidized mortgage loans as in the steady state under the policy reform j:

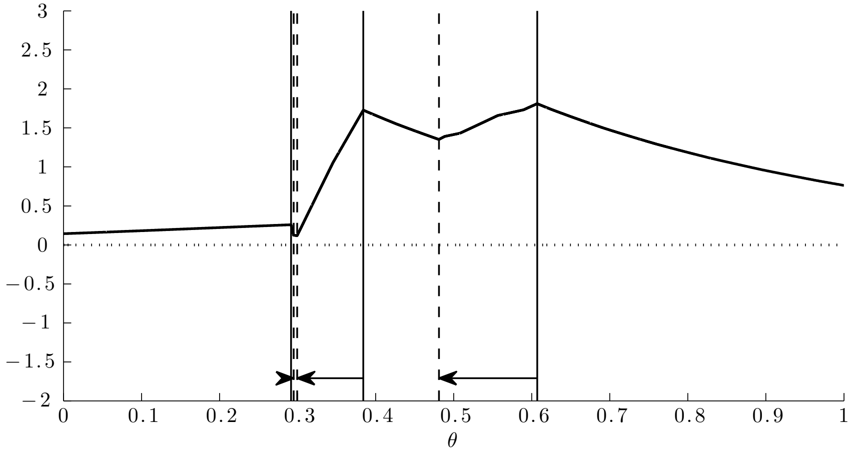
$$(1+\omega)^{\varepsilon}S_{M,t}^{SML}(\theta)=S_{M,t}^{j}(\theta). \tag{3.33}$$

It follows that:

$$\omega=\left(\frac{S_{M,t}^{j}(\theta)}{S_{M,t}^{SML}(\theta)}\right)^{\frac{1}{\varepsilon}}-1. \tag{3.34}$$

In Figure 3.9 we plot ω as a function of θ. The solid vertical lines indicate the cut-off values for education under the SML system while the dashed vertical lines represent the corresponding numbers for the GLT system. There are kinks at the old and new thresholds, reflecting the fact that the educational choice is a discrete one. Individuals located at either side of a kink differ by two years in educational attainment in the initial or the new steady state. For types who decide to extend the length of their education the welfare gain is increasing in θ while for those who do not the welfare gain is decreasing in θ (the uneducated excepted). Even though individuals without a college education do not directly contribute to the educational loan system either before or after the reform they are slightly better off under GLT.

Figure 3.9: Change in ex-post steady-state welfare from SML to GLT



3.4.4 Decomposition by key mechanisms

In order to highlight the key mechanisms that are operative in the model we decompose the long-run macroeconomic and aggregate welfare effects of the policy reform into several parts. To do so we initially shut down some adjustment channels and then open them one by one.

The starting point is the steady state equilibrium featuring subsidized mortgage loans. All long-run changes reported in Table 3.6 are with respect to this benchmark. We fix the interest rate and the rental rate of effective labour at their initial level in each year by assuming that we have a small open economy instead of a closed one. Any differences between output and domestic absorption are attributed to net exports and the discrepancy between domestic asset holdings and the capital stock determines net foreign asset holdings. In addition we keep the distribution of education levels constant over time so that each θ type makes the same schooling decision as in the benchmark. Finally we let individuals perceive the educational labour tax as being lump-sum, while in fact it is proportional to their gross labour income. Under these assumptions, the change in allocations and welfare as reported in column (a) of Table 3.6 can be attributed to a *redistribution effect*. There is an aggregate welfare gain equal to 0.14% of the initial level of aggregate consumption (the compensating annuity payment as explained above). By making contributions to the educational loan system proportional to labour income they automatically fall in periods of low productivity, in contrast to fixed mortgage loan payments. From an ex-ante perspective risk-averse individuals are better off with this kind of risk sharing. The required educational labour tax rate is 1.95% while the budget-balancing income tax rate remains virtually constant despite some shifts in the tax bases.

Table 3.6: Decomposition of long-run changes from SML to GLT

	(a)	(b)	(c)	(d)
Small open economy	yes	yes	yes	no
Fixed education	yes	yes	no	no
Individual lump-sum taxes	yes	no	no	no
Macroeconomic quantities				
Output	− 0.61%	− 1.51%	− 0.43%	+ 0.79%
Consumption	− 0.46%	− 1.67%	− 0.36%	+ 0.30%
Effective labour	− 0.61%	− 1.51%	− 0.43%	+ 0.23%
Capital	− 0.61%	− 1.51%	− 0.43%	+ 2.72%
Net financial assets	+ 5.03%	+ 2.98%	+ 4.95%	+ 2.72%
Factor prices				
Rental rate of effective labour	0.00%	0.00%	0.00%	+ 0.56%
Interest rate	0.00%p	0.00%p	0.00%p	− 0.14%p
Tax rates				
Income tax rate	0.00%p	+ 0.22%p	− 0.06%p	− 0.14%p
Educational labour tax rate	+ 1.95%p	+ 1.98%p	+ 2.41%p	+ 2.37%p
Education				
Share with 0 years	0.00%p	0.00%p	+ 0.62%p	+ 0.53%p
Share with 2 years	0.00%p	0.00%p	−12.74%p	−12.45%p
Share with 4 years	0.00%p	0.00%p	+ 1.11%p	+ 1.29%p
Share with 6 years	0.00%p	0.00%p	+11.01%p	+10.63%p
Aggregate welfare				
Compensating annuity payment	+ 0.14	− 0.04	+ 0.05	+ 0.08

In column (b) we keep the assumption of a small open economy and a fixed education distribution but assume that individuals are aware that the tax they pay to finance the educational loan system is not lump sum but a percentage of their labour income. As a consequence the tax not only has an income effect but also a substitution effect which distorts the labour supply decision. The budget-balancing educational tax rate now equals 1.98%. Even though the increase in the tax wedge on labour resulting from this source appears to be quite small, the disincentive effect is so strong that the regular income tax must also be increased by 0.22 percentage points. The compensating annuity payment is negative, indicating an aggregate welfare loss. The total welfare change that can be attributed to the *work incentive effect* is -0.18, the drop in the annuity payment when moving from column (a) to column (b).

In column (c) we allow individuals to optimally adjust their education decision. The share of uneducated individuals stays roughly constant while those of the two highest education categories go up. There clearly is a positive effect on average educational attainment. Compared to column (b) the long-run changes in macroeconomic quantities are strongly dampened. The aggregate welfare increase relative to the benchmark equals 0.05% of initial consumption so that the annuity payment associated with the *education incentive effect* is 0.09.

In the final step we reinstate the assumption of a closed economy, which means that factor prices adjust to changes in domestic demand and supply. The numbers reported in column (d) regarding the long-run consequences of the policy reform for macroeconomic quantities and the education distribution correspond to those in Table 3.4 and Table 3.5 above. The compensating annuity payment equals 0.08, of which 0.03 can be attributed to the *general equilibrium effect*.

We conclude that, in terms of aggregate welfare changes, the work incentive effect is the strongest and negative, followed by a positive redistribution effect and education incentive effect. The general equilibrium effect is quite small in the long run.

3.5 Policy reform 2: From SML to CLT

In this section we briefly discuss the most important consequences of a policy reform from SML to CLT and contrast them to the effects of the reform from SML to GLT studied above. Recall that the comprehensive labour tax system is very similar to the GLT system except for the fact that the educational labour tax is levied on all workers, even those who have chosen not to pursue any higher education themselves. Interestingly, however, the long-run educational composition of the adult population differs quite a lot between the two reform scenarios. Whereas the steady-state proportion of uneducated individuals hardly changes under GLT relative to the 52.02% in the benchmark (it even went up a bit), there is a clear drop of 11.12 percentage points under CLT. Since individuals cannot avoid paying the educational tax anyway, there is a strong response at the extensive margin. More people will decide to get an education in order to reap at least some of the benefits of the system in the form of ‘free’ study grants. In addition there is also an education-enhancing effect at the intensive margin:

comparing columns (a) and (c) in Table 3.4 shows that both the fraction of individuals with 4 years of college and the fraction with 6 years of college increase.

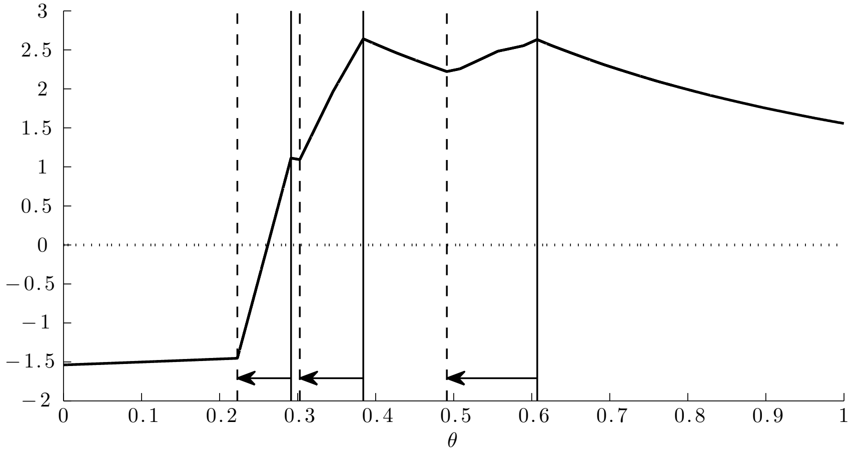
The long-run macroeconomic effects of the policy change are reported in the column labeled CLT of Table 3.5 while the transitional effects are illustrated by the white dots in Figure 3.6. At the time of the shock, output, consumption, and effective employment drop by, respectively, 0.90%, 1.64%, and 1.16%. One period later these variables reach their maximum reductions at, respectively, 1.53%, 2.28% and 1.99%. In the long run, the capital stock and effective employment both increase, the return to capital falls by 0.15 percentage points and the rental rate of effective labour increases by 0.57%. Finally, consumption rises with 0.53% while output increases by 1.03% in the long run. Comparing the two time paths in Figure 3.6 and the two rightmost columns of Table 3.5 we note that the transitional dynamics are very similar in the two reform scenarios, although the long-run effects are somewhat larger under CLT.

In Figure 3.7 we plot the ex-ante welfare effects along the transition path. As shown in panel (a), all existing cohorts are worse off. Whereas all new cohorts benefit from a move to the GLT system, under the CLT reform the cohorts attaining the age of majority soon after the time of the shock are worse off, see panel (b). Cohorts that arrive later do gain but their welfare increase is much smaller than under the GLT reform. Relative to the initial steady state with subsidized mortgage loans aggregate welfare falls by 0.29% of total consumption. Moreover, as is shown in Figure 3.8 (dashed line), even under a hybrid version of CLT and SML it is not possible to generate a welfare gain at the aggregate level.

The ex-post welfare changes by educational talent are plotted in Figure 3.10. The lowest ability types lose out as a result of the policy reform. They have to pay an additional tax on labour earnings but receive zero or only a small amount of educational transfers in return. For all other types the welfare effect is positive in the long run.

In Table 3.7 we present a decomposition of the macroeconomic and welfare effects into a redistribution effect (column (a)), a work incentive effect (column (b)), an education incentive effect (column (c)) and a general equilibrium effect (column (d)). For the sake of convenience, we report the aggregate welfare changes that can be attributed to each effect in Table 3.8 and contrast them to the GLT reform. Several things are worth noting. First, whereas redistribution yields a welfare gain under the GLT reform, the effect is negative (and relatively large) under the CLT system. There is not only redistribution from individuals with a high productivity draw to those who are less fortunate but also from uneducated individuals to educated ones. Second, the education incentive effect is positive under both scenarios but smaller for the CLT reform. This might suggest that

Figure 3.10: Change in ex-post steady-state welfare from SML to CLT



there are 'too many' individuals enrolled in higher education in the latter case. Finally, the size of the work incentive effect and the general equilibrium effect are nearly identical for the two cases.

3.6 Conclusion

In this chapter we have conducted a quantitative analysis of a number of educational loan systems. We have built a stochastic general equilibrium model of a closed economy with a competitive firm sector and a government that levies taxes and administers educational loans. Individuals are heterogeneous in their talent for education and ability to learn on the job and face uninsurable idiosyncratic labour productivity risk during their working career.

After calibrating the model to the mortgage loan system in the United States we have studied two possible reforms. The first is a Graduate Labour Tax (GLT) system whereby grants to students are financed by means of a tax on the labour income of educated individuals. In the long run the proportion of uneducated individuals stays roughly constant but the average educational attainment of students increases. As there exists a considerable amount of transitional dynamics in the model the welfare effects of the reform differ by cohort. Cohorts economically active at the time of the shock are worse off while ex-ante welfare of future cohorts increases. The gains to the latter are large enough to, at least in principle, compensate the losers from the policy reform and

Table 3.7: Decomposition of long-run changes from SML to CLT

	(a)	(b)	(c)	(d)
Small open economy	yes	yes	yes	no
Fixed education	yes	yes	no	no
Individual lump-sum taxes	yes	no	no	no
Macroeconomic quantities				
Output	− 0.62%	− 1.52%	− 0.21%	+ 1.03%
Consumption	− 0.47%	− 1.68%	− 0.14%	+ 0.53%
Effective labour	− 0.62%	− 1.52%	− 0.21%	+ 0.46%
Capital	− 0.62%	− 1.52%	− 0.21%	+ 3.00%
Net financial assets	+ 5.04%	+ 2.86%	+ 5.28%	+ 3.00%
Factor prices				
Rental rate of effective labour	0.00%	0.00%	0.00%	+ 0.57%
Interest rate	0.00%p	0.00%p	0.00%p	− 0.15%p
Tax rates				
Income tax rate	0.00%p	+ 0.22%p	− 0.06%p	− 0.21%p
Educational labour tax rate	+ 1.19%p	+ 1.20%p	+ 1.59%p	+ 1.56%p
Education				
Share with 0 years	0.00%p	0.00%p	−11.24%p	−11.12%p
Share with 2 years	0.00%p	0.00%p	− 0.35%p	− 0.28%p
Share with 4 years	0.00%p	0.00%p	+ 1.53%p	+ 1.79%p
Share with 6 years	0.00%p	0.00%p	+10.06%p	+ 9.61%p
Aggregate welfare				
Annuity payment	− 0.17	− 0.36	− 0.32	− 0.29

Table 3.8: Aggregate welfare changes in the two reform scenarios

	GLT	CLT
Redistribution effect	0.14	−0.17
Work incentive effect	−0.18	−0.19
Education incentive effect	0.09	0.04
General equilibrium effect	0.03	0.03
Total	0.08	−0.29

generate an overall welfare gain.

The second possible reform we have considered is a Comprehensive Labour Tax (CLT). It is very similar to the GLT except for the fact that the educational tax is levied on all workers, including those who are uneducated. In contrast to the GLT reform the proportion of uneducated individuals drops substantially. Generations that become economically active soon after the policy reform are worse off and the aggregate ex-ante welfare effect is negative.

Overall we conclude that Friedman was right and it might be advisable for policy makers in developed countries to consider introducing a graduate tax system to finance educational loans. However, as our analysis of the transitional dynamics shows, appropriate compensation of individuals who have already accumulated study debt is crucial in order to prevent them from paying the same bill twice. As always the devil is in the details.

Appendix

3.A Aggregation

We assume that there is no aggregate uncertainty so that for a sufficiently large population each cohort average takes on a deterministic value. Hence, if we are only interested in these kind of aggregate statistics then there is no need to trace the individual life-cycle choices as we do in Section 3.2.4. Instead of summing over individuals we can integrate over policy functions.

To calculate the cohort averages we have to determine the distribution of individuals over the state space of the model, which is the set of possible values for each state variable. The relevant state variables are the talent for education $\theta \in \mathcal{Z} = [0, 1]$, education $E \in \mathcal{E} = \{0, 2, 4, 6\}$, learning ability $\gamma \in \mathcal{G} = \{\gamma_l, \gamma_h\}$, financial assets $a \in \mathcal{A} = [0, \infty)$, human capital $h \in \mathcal{H} = [0, \infty)$ and labour productivity $\eta \in \mathcal{X} = \{0, \eta_l, 1, \eta_h\}$. The distribution of individuals will be mixed for two reasons. First, some state variables take on a finite number of values while for others there is an uncountable set of possible values. Second, even the state variables with an uncountable domain can have 'mass points' in their marginal distributions. For example, all individuals are born without financial assets so that even though a can take on any non-negative value all mass is concentrated at $a = 0$. Because the distribution is mixed, it is not possible to characterize it by the probability mass at each point of its domain only (as for a discrete distribution). Instead we specify the cumulative distribution function.

Let $\chi_{u,t}$ denote the proportion of individuals of age u that are in the working phase in year t. Given that we know the policy function for education and the distribution of the talent for education in a given cohort we can deduce:

$$\chi_{u,t} = \int_{\mathcal{Z}} \mathbb{1}_{\{\mathbf{E}_{u,t}(\theta) \leq u - M\}} \, \mathrm{d}F_\theta(\theta),$$

where F_θ is the cumulative distribution function of θ. Let $\Psi_{u,t}(E, \gamma, a, h, \eta)$ denote the cumulative distribution function of workers over the product space $\mathcal{E} \times \mathcal{G} \times \mathcal{A} \times \mathcal{H} \times \mathcal{X}$ for a given age u and period t. As for any probability distribution the total mass is equal to unity:

$$\int_{\mathcal{E}\times\mathcal{G}\times\mathcal{A}\times\mathcal{H}\times\mathcal{X}} \mathrm{d}\Psi_{u,t}(E,\gamma,a,h,\eta) = 1.$$

Every individual who starts working immediately upon entering adulthood ($E = 0$) has no financial assets ($a = 0$), one unit of human capital ($h = 1$) and an average level of productivity ($\eta = 1$). The initial distribution of workers is therefore characterized by:

$$\Psi_{M,t}(E,\gamma,a,h,\eta) = \frac{1}{\chi_{M,t}} \int_{\mathcal{Z}} \mathbb{1}_{\{E\geq 0\}} \mathbb{1}_{\{a\geq 0\}} \mathbb{1}_{\{h\geq 1\}} \mathbb{1}_{\{\eta\geq 1\}} \mathbb{1}_{\{\mathbf{E}_{M,t}(\theta)=0\}} F_{\gamma|\theta}(\gamma|\theta)\,\mathrm{d}F_{\theta}(\theta),$$

where $F_{\gamma|\theta}$ is the cumulative distribution function of γ conditional on θ. Note that $\chi_{M,t}$ is used as a normalizing constant to ensure that the total mass is indeed equal to unity. The evolution of the distribution over time is given by:

$$\Psi_{u+1,t+1}(E,\gamma,a^{+},h^{+},\eta^{+}) = \frac{1}{\chi_{u+1,t+1}} \Big\{ \chi_{u,t} \int_{\mathcal{A}\times\mathcal{H}\times\mathcal{X}} \mathbb{1}_{\{\mathbf{a}^{+}_{u,t}(E,\gamma,a,h,\eta)\leq a^{+}\}}$$
$$\times \mathbb{1}_{\{\mathbf{h}^{+}_{u,t}(E,\gamma,a,h,\eta)\leq h^{+}\}} F_{\eta^{+}|\eta,E}(\eta^{+}|\eta,E)\,\mathrm{d}\Psi_{u,t}(E,\gamma,a,h,\eta) + \int_{\mathcal{Z}} \mathbb{1}_{\{E\geq u+1-M\}}$$
$$\times \mathbb{1}_{\{a^{+}\geq 0\}} \mathbb{1}_{\{h^{+}\geq \Gamma(\theta,u+1-M)\}} \mathbb{1}_{\{\eta^{+}\geq 1\}} \mathbb{1}_{\{\mathbf{E}_{u+1,t+1}(\theta)=u+1-M\}} F_{\gamma|\theta}(\gamma|\theta)\,\mathrm{d}F_{\theta}(\theta) \Big\},$$

where $F_{\eta^{+}|\eta,E}$ is the cumulatieve distribution function of η^{+} conditional on η and E. The first part in curly brackets captures the mass of individuals that are working in year t. Their education level E and learning ability γ are given and constant over time. Conditional on a specific combination of state variables in the current year the optimal choice of next year's financial assets and human capital are described by the policy functions $\mathbf{a}^{+}_{u,t}$ and $\mathbf{h}^{+}_{u,t}$, respectively. The Markov process for labour productivity determines the probability of each possible draw of η^{+}. The second part reflects the entry of current students into the labour market in a similar way as for the initial distribution.

For large cohorts ($P_{u,t} \to \infty$) we find the cohort averages by integrating over the distribution of individuals just derived. For example:

$$\bar{c}_{u,t} = [1-\chi_{u,t}]c_t + \chi_{u,t} \int_{\mathcal{E}\times\mathcal{G}\times\mathcal{A}\times\mathcal{H}\times\mathcal{X}} \mathbf{c}_{u,t}(E,\gamma,a,h,\eta)\,\mathrm{d}\Psi_{u,t}(E,\gamma,a,h,\eta),$$
$$\bar{l}_{u,t} = \chi_{u,t} \int_{\mathcal{E}\times\mathcal{G}\times\mathcal{A}\times\mathcal{H}\times\mathcal{X}} \eta\, h\, \mathbf{l}_{u,t}(E,\gamma,a,h,\eta)\,\mathrm{d}\Psi_{u,t}(E,\gamma,a,h,\eta),$$

$$\bar{a}_{u,t} = \chi_{u,t} \int_{\mathcal{E}\times\mathcal{G}\times\mathcal{A}\times\mathcal{H}\times\mathcal{X}} a \, \mathrm{d}\Psi_{u,t}(E, \gamma, a, h, \eta).$$

In order to actually calculate these values on a computer it is necessary to 'discretize' the state space, see the discussion in Appendix 3.C.

3.B Scaling

The steady state or balanced growth path of the model has the property that all variables grow at a constant rate. This allows us to scale the variables in such a way that they will be time invariant in the long run equilibrium.

3.B.1 Macroeconomic level

At the macroeconomic level of the economy we have factor prices, policy variables and aggregate quantities. For each we state the growth rate in the steady state and, if different from zero, define the corresponding scaled variable which is distinguished by a tilde.

(1) The interest rate r_t and the tax rates τ_t^c, τ_t^r, τ_t^w and τ_t^e are constant.

(2) The level of consumption during the education phase c_t, the tuition fee f_t, the annual study loan q_t, the wage rate w_t and the unemployment benefit $\nu_{u,t}$ grow at rate n_Z.

$$\tilde{c}_t \equiv \frac{c_t}{Z_t}, \quad \tilde{f}_t \equiv \frac{f_t}{Z_t}, \quad \tilde{q}_t \equiv \frac{q_t}{Z_t}, \quad \tilde{w}_t \equiv \frac{w_t}{Z_t}, \quad \tilde{\nu}_{u,t} \equiv \frac{\nu_{u,t}}{Z_t}.$$

(3) Total effective labour supply L_t and effective labour demand N_t grow at rate n_P.

$$\tilde{L}_t \equiv \frac{L_t}{P_{M,t}}, \quad \tilde{N}_t \equiv \frac{N_t}{P_{M,t}}.$$

(4) Total asset holdings A_t, total unemployment benefits B_t, total consumption C_t, total study debt D_t, total tuition fees F_t, government spending G_t, gross investment I_t, the capital stock K_t, total tax receipts T_t and output Y_t grow at rate $(1+n_Z)(1+n_P)-1$.

$$\tilde{A}_t \equiv \frac{A_t}{Z_t P_{M,t}}, \quad \tilde{B}_t \equiv \frac{B_t}{Z_t P_{M,t}}, \quad \tilde{C}_t \equiv \frac{C_t}{Z_t P_{M,t}}, \quad \tilde{D}_t \equiv \frac{D_t}{Z_t P_{M,t}},$$
$$\tilde{F}_t \equiv \frac{F_t}{Z_t P_{M,t}}, \quad \tilde{G}_t \equiv \frac{G_t}{Z_t P_{M,t}}, \quad \tilde{I}_t \equiv \frac{I_t}{Z_t P_{M,t}}, \quad \tilde{K}_t \equiv \frac{K_t}{Z_t P_{M,t}},$$
$$\tilde{T}_t \equiv \frac{T_t}{Z_t P_{M,t}}, \quad \tilde{Y}_t \equiv \frac{Y_t}{Z_t P_{M,t}}.$$

3.B.2 Microeconomic level

At the microeconomic level we can also apply scaling in order to turn the decision problem of an individual into a stationary one. This means that in the steady state the solution to this problem only depends on an individual's age and not on the moment in time. It only works if the preference structure satisfies some conditions, see King et al. (2002). After scaling the problem of a worker can be written as:

$$\begin{aligned}\hat{V}_{u,t}(E,\gamma,a,h,\eta) = \max_{c,l,a^+,h^+} \Bigg\{ & \left[c^{\varepsilon}(1-l)^{1-\varepsilon}\right]^{1-1/\sigma} \\ & + \beta\left[\mathbb{E}_{\eta^+|\eta,E}\left[\hat{V}_{u+1,t+1}(E,\gamma,a^+,h^+,\eta^+)^{1-\zeta}\right]\right]^{\frac{1-1/\sigma}{1-\zeta}}\Bigg\}^{\frac{1}{1-1/\sigma}},\end{aligned}$$

subject to:

$$\begin{aligned} a^+ &= [1+(1-\tau_t^r)r_t]a + (1-\tau_t^w)\hat{w}_{u,t}\,\eta\, h\, l + \hat{\nu}_{u,t}\mathbb{1}_{\{\eta=0\}} - (1+\tau_t^c)c \\ &\quad - \hat{\Upsilon}_{u,t}(E,\hat{w}_{u,t}\,\eta\, h\, l), \\ h^+ &= (1-\delta_u^h)[1+\gamma l^{\alpha}]h, \\ & 0 \le l \le 1, \quad c \ge 0, \quad a^+ \ge 0. \end{aligned}$$

The growing factor prices and policy variables that appear in the constraints have been scaled by Z_{v+M}, which is the productivity level in the economy at the moment a person born at time v reaches the age of majority M:

$$\begin{aligned} \hat{w}_{u,t} &\equiv \frac{w_t}{Z_{t+M-u}} = \tilde{w}_t(1+n_Z)^{u-M}, \\ \hat{\nu}_{u,t} &\equiv \frac{\nu_{u,t}}{Z_{t+M-u}} = \tilde{\nu}_{u,t}(1+n_Z)^{u-M}, \\ \hat{\Upsilon}_{u,t}(E,W) &\equiv \frac{\Upsilon_{u,t}(E,WZ_{t+M-u})}{Z_{t+M-u}}. \end{aligned}$$

The solution to this problem gives a new set of policy functions indicated by a hat. The relationship with the unscaled policy functions as used in the main text is as follows:

$$\begin{aligned} \hat{\mathbf{c}}_{u,t}(E,\gamma,a,h,\eta) &\equiv \frac{\mathbf{c}_{u,t}(E,\gamma,aZ_{t+M-u},h,\eta)}{Z_{t+M-u}} \\ \hat{\mathbf{a}}^+_{u,t}(E,\gamma,a,h,\eta) &\equiv \frac{\mathbf{a}^+_{u,t}(E,\gamma,aZ_{t+M-u},h,\eta)}{Z_{t+M-u}}, \\ \hat{\mathbf{l}}_{u,t}(E,\gamma,a,h,\eta) &\equiv \mathbf{l}_{u,t}(E,\gamma,aZ_{t+M-u},h,\eta), \\ \hat{\mathbf{h}}^+_{u,t}(E,\gamma,a,h,\eta) &\equiv \mathbf{h}^+_{u,t}(E,\gamma,aZ_{t+M-u},h,\eta). \end{aligned}$$

Note that consumption and future financial assets are scaled because they grow over time, while labour supply and future human capital were already stationary in the original problem.

In order to determine how the new value function relates to the original one we start in the last year of life and write:

$$\begin{aligned}
\hat{V}_{\bar{U},t}(E,\gamma,a,h,\eta) &= \hat{\mathbf{c}}_{\bar{U},t}(E,\gamma,a,h,\eta)^{\varepsilon}\Big[1-\hat{\mathbf{l}}_{\bar{U},t}(E,\gamma,a,h,\eta)\Big]^{1-\varepsilon} \\
&= \left[\frac{\mathbf{c}_{\bar{U},t}(E,\gamma,aZ_{t+M-\bar{U}},h,\eta)}{Z_{t+M-\bar{U}}}\right]^{\varepsilon}\Big[1-\mathbf{l}_{\bar{U},t}(E,\gamma,aZ_{t+M-\bar{U}},h,\eta)\Big]^{1-\varepsilon} \\
&= \frac{V_{\bar{U},t}(E,\gamma,aZ_{t+M-\bar{U}},h,\eta)}{Z^{\varepsilon}_{t+M-\bar{U}}}.
\end{aligned}$$

Moving back in time using the recursive formulation of utility we find that this relationship holds in every year.

Similarly we can also scale the problem of a student:

$$\begin{aligned}
\hat{S}_{u,t}(\theta) = \max_{E\geq u-M}\Bigg[&\sum_{s=t}^{t-u+M+E-1}\beta^{s-t}\Big[\big(\hat{c}_{u+s-t,s}\big)^{\varepsilon}\big(1-\bar{e}\big)^{1-\varepsilon}\Big]^{1-1/\sigma} \\
&+\beta^{M+E-u}\Big[\mathbb{E}_{\gamma|\theta}\Big[\hat{V}_{M+E,t-u+M+E}\big(E,\gamma,0,\Gamma(\theta,E),1\big)^{1-\zeta}\Big]\Big]^{\frac{1-1/\sigma}{1-\zeta}}\Bigg]^{\frac{1}{1-1/\sigma}},
\end{aligned}$$

where:

$$\hat{c}_{u,t} \equiv \frac{c_t}{Z_{t+M-u}} = \tilde{c}_t(1+n_Z)^{u-M}.$$

This gives a policy function $\hat{\mathbf{E}}_{u,t}(\theta) \equiv \mathbf{E}_{u,t}(\theta)$ and value function $\hat{S}_{u,t}(\theta) \equiv S_{u,t}(\theta)/Z^{\varepsilon}_{t+M-u}$.

Stationary choices on the individual level automatically lead to stationary cohort averages:

$$\hat{\bar{c}}_{u,t} \equiv \frac{\bar{c}_{u,t}}{Z_{t+M-u}}, \quad \hat{\bar{l}}_{u,t} \equiv \bar{l}_{u,t}, \quad \hat{\bar{a}}_{u,t} \equiv \frac{\bar{a}_{u,t}}{Z_{t+M-u}}, \quad \hat{\bar{d}}_{u,t} \equiv \frac{\bar{d}_{u,t}}{Z_{t+M-u}}.$$

We can then directly compute the scaled aggregate quantities at the macroeconomic level. For example:

$$\tilde{C}_t = \sum_{u=M}^{\bar{U}} \frac{\hat{\bar{c}}_{u,t}}{(1+n_Z)^{u-M}(1+n_P)^{u-M}}.$$

3.C Computational details

3.C.1 Program structure

The structure of the computer program used to calculate the transition path from an initial steady state to a new one following a policy reform is visualized by means of a flow chart in Figure 3.C.1. We start at the top with a guess for the time path of the tax rates and the factor inputs. The marginal productivity conditions of the firms then imply what the interest rate and rental rate of effective labour should be. Given these prices we can solve for the policy functions and the value function of individuals of any given age in each year. We can then aggregate across individuals to compute the average consumption, labour supply, financial assets and study debt by cohort. More details about these computations are given in the next section. By summing over all cohorts alive at a given moment in time we obtain the macro aggregates.

The solution has been found if the goods market is in equilibrium in every year. If not, then we update the guesses. The educational tax is set in such a way that tax receipts exactly cover transfers to students while one of the other tax rates is used to balance the regular government budget. The factor supplies are partially updated using the Gauss-Seidel rule:

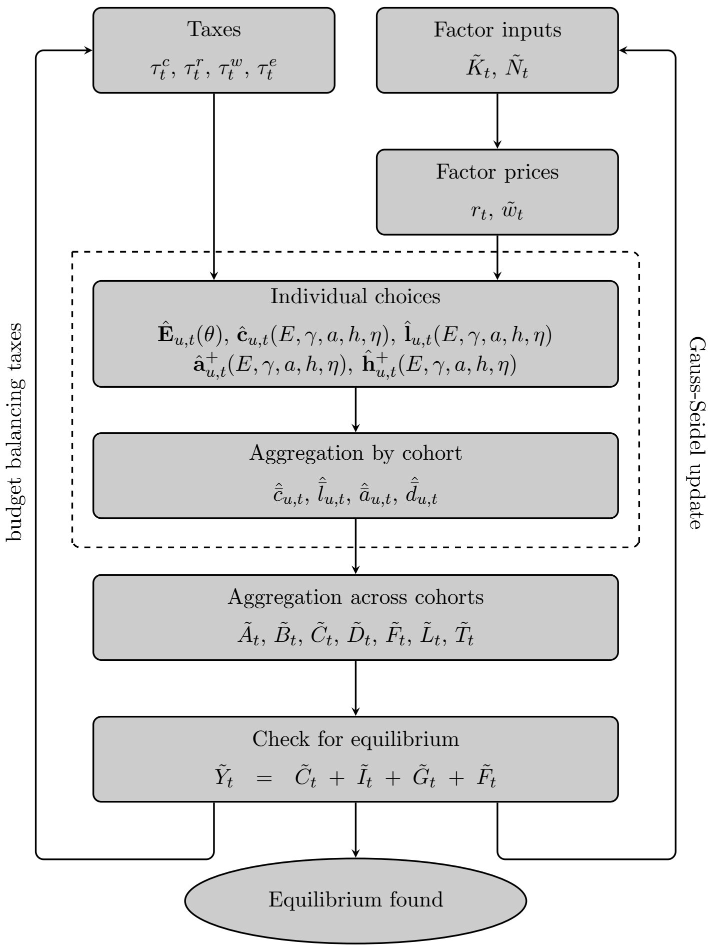
$$\tilde{K}_t^{\text{new}} = \varphi \tilde{K}_t^{\text{old}} + (1-\varphi)\big[\tilde{A}_t - \tilde{D}_t\big],$$
$$\tilde{N}_t^{\text{new}} = \varphi \tilde{N}_t^{\text{old}} + (1-\varphi)\tilde{L}_t,$$

where $0 < \varphi < 1$ is a dampening factor. Greater dampening makes the solution algorithm slower but also more stable. Note that if the program converges then the capital market and the labour market clear so that by Walras' Law the goods market should also be in equilibrium.

3.C.2 Individual choices and aggregation

In this section we provide more detail about the methods used to compute the optimal life-cycle choices of individuals and the cohort averages. This corresponds to the two steps framed by dashed lines in the flow chart of Figure 3.C.1.

Figure 3.C.1: Program structure



Set-up

We create a grid $\mathcal{U}$ for adult ages and a grid $\mathcal{T}$ for time periods:

$$u \in \mathcal{U} = \{M, M+1, \dots, \bar{U}\},$$
$$t \in \mathcal{T} = \{0, 1, \dots, \bar{T}\}.$$

In addition we set up a grid for the discrete variables in the model:

$$\gamma \in \mathcal{G} = \{\gamma_l, \gamma_h\},$$
$$E \in \mathcal{E} = \{0, 2, 4, 6\},$$
$$\eta \in \mathcal{X} = \{0, \eta_l, 1, \eta_h\}.$$

There are two state variables which can theoretically take on a continuum of values, namely financial assets a and human capital h. As the computer cannot handle this unboundedness we have to 'discretize' the set of possible values by setting up a grid with a finite number of elements:

$$a \in \mathcal{A} = \left\{\mathcal{A}^{(j_a)} : j_a = 1, 2, \dots, n_a\right\},$$
$$h \in \mathcal{H} = \left\{\mathcal{H}^{(j_h)} : j_h = 1, 2, \dots, n_h\right\}.$$

To improve the accuracy of the computations we let the grid for human capital depend on age. The points on the asset grid are not evenly spaced, instead they are more closely concentrated at low levels.

Finally we introduce a variable m which is uniformly distributed with support $[0, 1]$. There is a direct relation between this m and the talent for education θ in the model. Write $\theta = \Theta(m)$ with:

$$\Theta(m) = \mu_\theta + \sigma_\theta (\Phi^n)^{-1}\left(\Phi^n\left(-\frac{\mu_\theta}{\sigma_\theta}\right) + m\left[\Phi^n\left(\frac{1-\mu_\theta}{\sigma_\theta}\right) - \Phi^n\left(-\frac{\mu_\theta}{\sigma_\theta}\right)\right]\right),$$

where Φ^n is the cumulative distribution function of the standard normal distribution. This construction ensures that θ has a truncated normal distribution on $[0, 1]$. In order to discretize the variable m we create an equidistant grid on $[0, 1]$ with n_m elements:

$$m \in \mathcal{M} = \left\{\mathcal{M}^{(j_m)} : j_m = 1, 2, \dots, n_m\right\} = \left\{0, \frac{1}{n_m - 1}, \frac{2}{n_m - 1}, \dots, 1\right\},$$

and assign the same probability mass to each point on the grid:

$$\mathbb{P}(m = z) = \begin{cases} \dfrac{1}{n_m} & \text{if } z \in \mathcal{M} \\ 0 & \text{otherwise} \end{cases}$$

Value function and policy functions in the working phase

We use backward induction to compute the value function and the policy functions of an individual in the working phase. Consider a cohort that reaches the age of majority M at some time t_0. We start at the maximum age $u = \bar{U}$ with corresponding time period $t = t_0 + \bar{U} - M$.[4] At the end of this year the individual will die. Therefore we know that it is optimal to deplete the stock of financial assets ($a^+ = 0$) and that there is no human capital left at the end of the period irrespective of the labour supply decision ($h^+ = 0$, see the depreciation schedule in (3.28)). For a given state vector (E, γ, a, h, η) the individual's problem can be reduced to:

$$\hat{V}_{\bar{U},t}(E, \gamma, a, h, \eta) = \max_{l} \; c^{\varepsilon}(1 - l)^{1-\varepsilon},$$

where:

$$c = \frac{\big[1 + (1 - \tau_t^r)r_t\big]a + (1 - \tau_t^w)\hat{w}_{\bar{U},t}\eta\, h\, l + \hat{\nu}_{\bar{U},t}\mathbb{1}_{\{\eta=0\}} - \hat{\Upsilon}_{\bar{U},t}(E, \hat{w}_{\bar{U},t}\eta\, h\, l)}{1 + \tau_t^c}.$$

We already know that $\hat{\mathbf{a}}^+_{\bar{U},t}(E, \gamma, a, h, \eta) = 0$ and $\hat{\mathbf{h}}^+_{\bar{U},t}(E, \gamma, a, h, \eta) = 0$. We use Powell's algorithm to find the l that minimizes the negative of the objective function with an added penalty if one of the control variables is outside its domain. This gives us $\hat{\mathbf{l}}_{\bar{U},t}(E, \gamma, a, h, \eta)$, $\hat{\mathbf{c}}_{\bar{U},t}(E, \gamma, a, h, \eta)$ and the corresponding maximum level of utility $\hat{V}_{\bar{U},t}(E, \gamma, a, h, \eta)$. We repeat the procedure for every state vector in the product space $\mathcal{E} \times \mathcal{G} \times \mathcal{A} \times \mathcal{H} \times \mathcal{X}$.

The next step is to go one period back to age $u = \bar{U} - 1$ and year $t = t_0 + \bar{U} - M - 1$. For every possible state vector we have to solve the problem:

$$\hat{V}_{u,t}(E, \gamma, a, h, \eta) = \max_{l,a^+} \Bigg[\big[c^{\varepsilon}(1 - l)^{1-\varepsilon}\big]^{1-1/\sigma} + \beta\Big[\mathbb{E}_{\eta^+|\eta,E}\Big[\hat{V}_{u+1,t+1}(E, \gamma, a^+, h^+, \eta^+)^{1-\zeta}\Big]\Big]^{\frac{1-1/\sigma}{1-\zeta}} \Bigg]^{\frac{1}{1-1/\sigma}},$$

[4] If $t > \bar{T}$ (the maximum entry in the time grid) then we set $t = \bar{T}$ as we assume that the economy has converged to a (new) steady state in the final period.

where:

$$c = \frac{\big[1+(1-\tau_t^r)r_t\big]a + (1-\tau_t^w)\hat{w}_{u,t}\eta\, h\, l + \hat{\nu}_{u,t}\mathbb{1}_{\{\eta=0\}} - \hat{\Upsilon}_{u,t}(E, w_{u,t}\eta\, h\, l) - a^+}{1+\tau_t^c},$$
$$h^+ = (1-\delta_u^h)\big[1+\gamma l^\alpha\big]h.$$

Note that in the previous step we have derived next period's value function (as $u+1 = \bar{U}$). Given the realization of the productivity shock in the current period we can calculate for every combination $(E, \gamma, a^+, h^+) \in \mathcal{E} \times \mathcal{G} \times \mathcal{A} \times \mathcal{H}$ the expectation as follows:

$$\mathbb{E}_{\eta^+|\eta,E}\Big[\hat{V}_{u+1,t+1}(E,\gamma,a^+,h^+,\eta^+)^{1-\zeta}\Big] = \sum_{x\in\mathcal{X}} \mathbb{P}(\eta^+ = x|\eta, E)\hat{V}_{u+1,t+1}(E,\gamma,a^+,h^+,x)^{1-\zeta}.$$

However, we also want to allow for the possibility that the optimal choice of next period's financial assets or human capital is not on the grid. In case $a^+ \notin \mathcal{A}$ or $h^+ \notin \mathcal{H}$ we have to use a linear interpolation method. That is, we determine the index j_a such that $\mathcal{A}^{(j_a)} \le a^+ \le \mathcal{A}^{(j_a+1)}$ and the index j_h such that $\mathcal{H}^{(j_h)} \le h^+ \le \mathcal{H}^{(j_h+1)}$. Define:

$$\phi_a \equiv \frac{a^+ - \mathcal{A}^{(j_a)}}{\mathcal{A}^{(j_a+1)} - \mathcal{A}^{(j_a)}},$$
$$\phi_h \equiv \frac{h^+ - \mathcal{H}^{(j_h)}}{\mathcal{H}^{(j_h+1)} - \mathcal{H}^{(j_h)}}.$$

The expectation can then be approximated by:

$$\begin{aligned}
&\mathbb{E}_{\eta^+|\eta,E}\Big[\hat{V}_{u+1,t+1}\big(E,\gamma,a^+,h^+,\eta^+\big)^{1-\zeta}\Big]^{\frac{1}{1-\zeta}} \\
&\approx (1-\phi_a)(1-\phi_h)\mathbb{E}_{\eta^+|\eta,E}\Big[\hat{V}_{u+1,t+1}\big(E,\gamma,\mathcal{A}^{(j_a)},\mathcal{H}^{(j_h)},\eta^+\big)^{1-\zeta}\Big]^{\frac{1}{1-\zeta}} \\
&\quad + \phi_a(1-\phi_h)\mathbb{E}_{\eta^+|\eta,E}\Big[\hat{V}_{u+1,t+1}\big(E,\gamma,\mathcal{A}^{(j_a+1)},\mathcal{H}^{(j_h)},\eta^+\big)^{1-\zeta}\Big]^{\frac{1}{1-\zeta}} \\
&\quad + (1-\phi_a)\phi_h\mathbb{E}_{\eta^+|\eta,E}\Big[\hat{V}_{u+1,t+1}\big(E,\gamma,\mathcal{A}^{(j_a)},\mathcal{H}^{(j_h+1)},\eta^+\big)^{1-\zeta}\Big]^{\frac{1}{1-\zeta}} \\
&\quad + \phi_a\phi_h\mathbb{E}_{\eta^+|\eta,E}\Big[\hat{V}_{u+1,t+1}\big(E,\gamma,\mathcal{A}^{(j_a+1)},\mathcal{H}^{(j_h+1)},\eta^+\big)^{1-\zeta}\Big]^{\frac{1}{1-\zeta}}.
\end{aligned}$$

By solving the individual's problem for different state vectors we obtain the value function and the policy functions for this age and time period. We continue moving backwards in time until we reach age $u = M$ in period $t = t_0$.

Education choice

Given that we have determined the value functions for individuals in the working phase we can derive the optimal education decisions by means of a grid search. Suppose an individual considers entering the labour market at age $u \in \mathcal{U}$ in period $t \in \mathcal{T}$ so that $E = u - M$. At that moment he or she will get a draw for the ability to learn on the job γ. For every $m \in \mathcal{M}$ we have a corresponding $\theta = \Theta(m)$ and we can calculate:

$$\mathbb{E}_{\gamma|\theta}\Big[V_{u,t}(E,\gamma,0,\Gamma(\theta,E),1)^{1-\zeta}\Big] = \sum_{x\in\mathcal{G}} \mathbb{P}(\gamma = x|\theta)\hat{V}_{u,t}(E,x,0,\Gamma(\theta,E),1)^{1-\zeta}.$$

We find the optimal E for a given $m \in \mathcal{M}$ by searching over the grid of possible values $\mathcal{E}_u = \{E \in \mathcal{E} : E \geq u - M\}$:

$$\hat{S}_{u,t}(\theta) = \max_{E\in\mathcal{E}_u} \left[\sum_{s=t}^{t-u+M+E-1} \beta^{s-t} \Big[\big(\hat{c}_{u+s-t,s}\big)^{\varepsilon} \big(1-\bar{e}\big)^{1-\varepsilon}\Big]^{1-1/\sigma} \right. \\ \left. + \beta^{M+E-u} \Big[\mathbb{E}_{\gamma|\theta}\Big[\hat{V}_{M+E,t-u+M+E}\big(E,\gamma,0,\Gamma(\theta,E),1\big)^{1-\zeta}\Big]\Big]^{\frac{1-1/\sigma}{1-\zeta}} \right]^{\frac{1}{1-1/\sigma}}.$$

Then we create a dummy variable $s_{u,t}(m)$ that is equal to 1 if a person is in school at age u in period t and zero otherwise.

Distribution of individuals in the working phase

As a consequence of discretizing the domain of the continuous state variables the state space now has a finite number of grid points. This means that the distribution of workers over the state space is completely characterized by the 'mass' at every point $(E,\gamma,a,h,\eta) \in \mathcal{E} \times \mathcal{G} \times \mathcal{A} \times \mathcal{H} \times \mathcal{X}$ on the grid. Instead of specifying a cumulative distribution function (as in Appendix 3.A) it is sufficient to derive the corresponding probability density function which is denoted by $\psi_{u,t}$.

First we can determine for every age u and time period t the fraction of individuals in the cohort that are in the working phase:

$$\chi_{u,t} = \sum_{z\in\mathcal{M}} \mathbb{1}_{\{s_{u,t}(z)=0\}} \mathbb{P}(m = z).$$

This will be used as a normalizing constant to ensure that the total mass adds up to unity.

To find the probability density function we use forward iteration. Consider a cohort that reaches the age of majority M at some time $t = t_0$. Since everyone starts with $a = 0$, $h = 1$ and $\eta = 1$ the initial distribution of individuals in the working phase is given by:

$$\psi_{M,t}(0, x, 0, 1, 1) = \frac{1}{\chi_{u,t}} \sum_{z \in \mathcal{M}} \mathbb{1}_{\{s_{M,t}(z)=0\}} \mathbb{P}(\gamma = x | \theta = \Theta(z)) \mathbb{P}(m = z),$$

for $x \in \mathcal{G}$. The probability density function is zero everywhere else.

We move one period forward so that the cohort is of age $u = M+1$ in the year $t = t_0+1$. The aim is to determine $\psi_{u,t}$ for any $(E, \gamma, a, h, \eta) \in \mathcal{E} \times \mathcal{G} \times \mathcal{A} \times \mathcal{H} \times \mathcal{X}$. First of all, there will be a group of former students that enter the labour market. To the point $(E, x, 0, \Gamma(\Theta(z), 1), 1)$ we add:

$$\frac{1}{\chi_{u,t}} \mathbb{1}_{\{s_{u-1,t-1}(z)=1\}} \mathbb{1}_{\{s_{u,t}(z)=0\}} \mathbb{P}(\gamma = x | \theta = \Theta(z)) \mathbb{P}(m = z),$$

for $x \in \mathcal{G}$ and $z \in \mathcal{M}$. In addition there are those who were already in the working phase. Consider a point $(E, \gamma, a, h, \eta) \in \mathcal{E} \times \mathcal{G} \times \mathcal{A} \times \mathcal{H} \times \mathcal{X}$ on the grid with a certain mass $\psi_{u-1,t-1}(E, \gamma, a, h, \eta)$ in the previous period. We want to determine where on the grid this mass ends up in the current period. To that end we find the optimal choices of a^+ and h^+ using the policy functions:

$$a^+ = \hat{\mathbf{a}}^+_{u-1,t-1}(E, \gamma, a, h, \eta), \quad h^+ = \hat{\mathbf{h}}^+_{u-1,t-1}(E, \gamma, a, h, \eta).$$

If $a^+ \in \mathcal{A}$ and $h^+ \in \mathcal{H}$ then we can immediately allocate the mass onto the grid. If not, then we use a linear interpolation method interpolation to distribute it over neighbouring points, see Figure 3.C.2. The weights ϕ_a and ϕ_h are determined as described above such that the average amount of financial assets and stock of human capital are still correct:

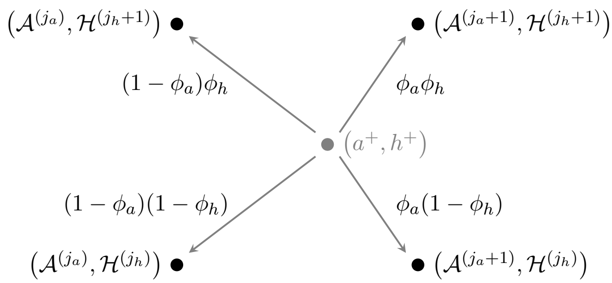
$$a^+ = (1 - \phi_a)\mathcal{A}^{(j_a)} + \phi_a \mathcal{A}^{(j_a+1)},$$
$$h^+ = (1 - \phi_h)\mathcal{H}^{(j_h)} + \phi_h \mathcal{H}^{(j_h+1)}.$$

For example, to the point $(E, \gamma, \mathcal{A}^{(j_a)}, \mathcal{H}^{(j_h)}, x)$ we add:

$$\frac{\chi_{u-1,t-1}}{\chi_{u,t}} (1 - \phi_a)(1 - \phi_h) \mathbb{P}(\eta^+ = x | \eta, E) \psi_{u-1,t-1}(E, \gamma, a, h, \eta),$$

for $x \in \mathcal{X}$.

Figure 3.C.2: Distributing mass over grid points



Cohort averages

Knowing the policy functions and the distribution of workers over the state space is sufficient to calculate the cohort averages. For example:

$$\hat{\hat{c}}_{u,t} = [1-\chi_{u,t}]\hat{c}_{u,t} + \chi_{u,t} \sum_{E\in\mathcal{E}} \sum_{\gamma\in\mathcal{G}} \sum_{a\in\mathcal{A}} \sum_{h\in\mathcal{H}} \sum_{\eta\in\mathcal{X}} \hat{\mathbf{c}}_{u,t}(E,\gamma,a,h,\eta)\, \psi_{u,t}(E,\gamma,a,h,\eta),$$

$$\hat{\bar{l}}_{u,t} = \chi_{u,t} \sum_{E\in\mathcal{E}} \sum_{\gamma\in\mathcal{G}} \sum_{a\in\mathcal{A}} \sum_{h\in\mathcal{H}} \sum_{\eta\in\mathcal{X}} \eta\, h\, \hat{\mathbf{l}}_{u,t}(E,\gamma,a,h,\eta)\, \psi_{u,t}(E,\gamma,a,h,\eta),$$

$$\hat{\hat{a}}_{u,t} = \chi_{u,t} \sum_{E\in\mathcal{E}} \sum_{\gamma\in\mathcal{G}} \sum_{a\in\mathcal{A}} \sum_{h\in\mathcal{H}} \sum_{\eta\in\mathcal{X}} a\, \psi_{u,t}(E,\gamma,a,h,\eta).$$

References Part I

Abbott, B., G. Gallipoli, C. Meghir and G. L. Violante (2013). *Education policy and intergenerational transfers in equilibrium.* Working Paper 18782. NBER.

Aiyagari, S. R. (1994). "Uninsured idiosyncratic risk and aggregate saving". *Journal of Political Economy* 109, pp. 659–684.

Akyol, A. and K. Athreya (2005). "Risky higher education and subsidies". *Journal of Economic Dynamics & Control* 29, pp. 979–1023.

d'Albis, H., S. Paul Lau and M. Sánchez-Romero (2012). "Mortality transition and differential incentives for early retirement". *Journal of Economic Theory* 147, pp. 261–283.

Auerbach, A. J. and L. J. Kotlikoff (1987). *Dynamic Fiscal Policy.* Camebridge University Press.

Avery, C. and S. Turner (2012). "Students loans: Do college students borrow too much – Or not enough?" *Journal of Economic Perspectives* 26, pp. 165–192.

Bell, F. C. and M. L. Miller (2005). *Life tables for the United States Social Security Area 1900-2100.* SSA Pub. 11-11536. Social Security Administration, Office of the Chief Actuary.

Bewley, T. (1977). "The permanent income hypothesis: A theoretical formulation". *Journal of Economic Theory* 16, pp. 252–292.

Boucekkine, R., D. de la Croix and O. Licandro (2002). "Vintage human capital, demographic trends, and endogenous growth". *Journal of Economic Theory* 104, pp. 340–375.

Cai, Y. and K. L. Judd (2010). “Stable and efficient computational methods for dynamic programming”. *Journal of the European Economic Association* 8, pp. 626–634.

Cannon, E. and I. Tonks (2008). *Annuity markets.* Oxford University Press.

Cecchetti, S. G., P.-S. Lam and N. C. Mark (2000). “Asset pricing with distorted beliefs: Are equity returns too good to be true?” *American Economic Review* 90, pp. 787–805.

Cervellati, M. and U. Sunde (2013). “Life expectancy, schooling, and lifetime labor supply: Theory and evidence revisited”. *Econometrica* 81 (5), pp. 2055–2086.

Chapman, B. (1997). “Conceptual issues and the Australian experience with income contingent charges for higher education”. *Economic Journal* 107, pp. 738–751.

Cigno, A. and A. Luporini (2009). “Scholarships or student loans? Subsidizing higher education in the presence of moral hazard”. *Journal of Public Economic Theory* 11, pp. 55–87.

Del Rey, E. and M. Racionero (2010). “Financing schemes for higher education”. *European Journal of Political Economy* 26, pp. 104–113.

Epstein, L. G. and S. E. Zin (1991). “Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An emprical analysis”. *Journal of Political Economy* 99, pp. 263–286.

Fehr, H. and F. Kindermann (forthcoming). “Taxing capital along the transition - Not a bad idea after all?” *Journal of Economic Dynamics & Control.*

Friedman, M. (1962). *Capitalism and Freedom.* Chicago, IL: University of Chicago Press.

García-Peñalosa, C. and K. Wälde (2000). “Efficiency and equity effects of subsidies to higher education”. *Oxford Economic Papers* 52, pp. 702–722.

Guvenen, F. (2009). “An empirical investigation of labor income processes”. *Review of Economic Dynamics* 12, pp. 58–79.

Heathcote, J., F. Perri and G. L. Violante (2010). “Unequal we stand: An empirical analysis of economic inequality in the United States, 1967–2006”. *Review of Economic Dynamics* 13, pp. 15–51.

Heckman, J. J., L. Lochner and C. Taber (1998). “Explaining rising wage inequality: Explorations with a dynamic general equilibrium model of labor earnings with heterogeneous agents”. *Review of Economic Dynamics* 1, pp. 1–58.

Heijdra, B. J., F. Kindermann and L. S. M. Reijnders (2014). *Life in shackles? The quantitative implications of reforming the educational loan system*. CESifo Working Paper 5013.

Heijdra, B. J., J. O. Mierau and L. S. M. Reijnders (2014). “A tragedy of annuitization? Longevity insurance in general equilibrium”. *Macroeconomic Dynamics* 18, pp. 1607–1634.

Heijdra, B. J. and L. S. M. Reijnders (2012). *Human capital accumulation and the macro-economy in an ageing society*. CESifo Working Paper 4046. University of Groningen.

Heijdra, B. J. and W. E. Romp (2009). “Retirement, pensions, and ageing”. *Journal of Public Economics* 93, pp. 586–604.

Huggett, M. (1993). “The risk-free rate in heterogeneous-agent incomplete-insurance economies”. *Journal of Economic Dynamics & Control* 17, pp. 953–969.

– (1997). “The one-sector growth model with idiosyncratic shocks: Steady states and dynamics”. *Journal of Monetary Economics* 39, pp. 385–403.

Huggett, M., G. Ventura and A. Yaron (2011). “Sources of lifetime inequality”. *American Economic Review* 101, pp. 2923–2954.

İmrohoroğlu, A. and S. Kitao (2009). “Labor supply elasticity and social security reform”. *Journal of Public Economics* 93, pp. 867–878.

Ionescu, F. (2009). “The Federal Student Loan Program: Quantitative implications for college enrollment and default rates”. *Review of Economic Dynamics* 12 (1), pp. 205–231.

Jacobs, B. and S. van Wijnbergen (2007). “Capital-market failure, adverse selection, and equity financing of higher education”. *FinanzArchiv* 63, pp. 1–32.

Jeong, H., Y. Kim and I. Manovskii (2014). *The price of experience*. Working Paper 20457. NBER.

Kalemli-Ozcan, S., H. E. Ryder and D. N. Weil (2000). "Mortality decline, human capital investment, and economic growth". *Journal of Development Economics* 62, pp. 1–23.

Kalemli-Ozcan, S. and D. N. Weil (2010). "Mortality change, the uncertainty effect, and retirement". *Journal of Economic Growth* 15, pp. 65–91.

Katz, L. F. and K. M. Murphy (1992). "Changes in relative wages, 1963-1987: Supply and demand factors". *Quarterly Journal of Economics* 107 (1), pp. 35–78.

Keane, M. P. (2011). "Labor supply and taxes: A survey". *Journal of Economic Literature* 49 (4), pp. 961–1075.

Kindermann, F. (2012). "Welfare effects of privatizing public education when human capital investments are risky". *Journal of Human Capital* 6, pp. 87–123.

King, M., S. Ruggles, J. T. Alexander, S. Flood, K. Genadek, M. B. Schroeder, B. Trampe and R. Vick (2010). *Integrated Public Use Microdata Series, Current Population Survey: Version 3.0*. machine-readable database. Minneapolis: University of Minnesota.

King, R. G., C. I. Plosser and S. T. Rebelo (2002). "Production, growth and business cycles: Technical Appendix". *Computational Economics* 20, pp. 87–116.

Krebs, T. (2003). "Human capital risk and economic growth". *Quarterly Journal of Economics* 118, pp. 709–744.

Krueger, D. and A. Ludwig (2013). "Optimal progressive labor income taxation and education subsidies when education decisions and intergenerational transfers are endogenous". *American Economic Review: Papers & Proceedings* 103, pp. 496–501.

Leung, S. F. (1994). "Uncertain lifetime, the theory of the consumer, and the life cycle hypothesis". *Econometrica* 62 (5), pp. 1233–1239.

Ludwig, A., T. Schelkle and E. Vogel (2012). "Demographic change, human capital and welfare". *Review of Economic Dynamics* 15, pp. 94–107.

Mankiw, N. G., D. Romer and D. N. Weil (1992). "A contribution to the empirics of economic growth". *Quarterly Journal of Economics* 107, pp. 407–437.

Prettner, K. and D. Canning (2014). "Increasing life expectancy and optimal retirement in general equilibrium". *Economic Theory* 56 (1), pp. 191–217.

Storesletten, K., C. I. Telmer and A. Yaron (2004). "Consumption and risk sharing over the life cycle". *Journal of Monetary Economics* 51, pp. 609–633.

Wallenius, J. (2011). "Human capital accumulation and the intertemporal elasticity of substitution of labor: How large is the bias?" *Review of Economic Dynamics* 14, pp. 577–591.

Part II
The role of gender and family formation

CHAPTER 4

The college gender gap reversal: Insights from a life-cycle perspective*

4.1 Introduction

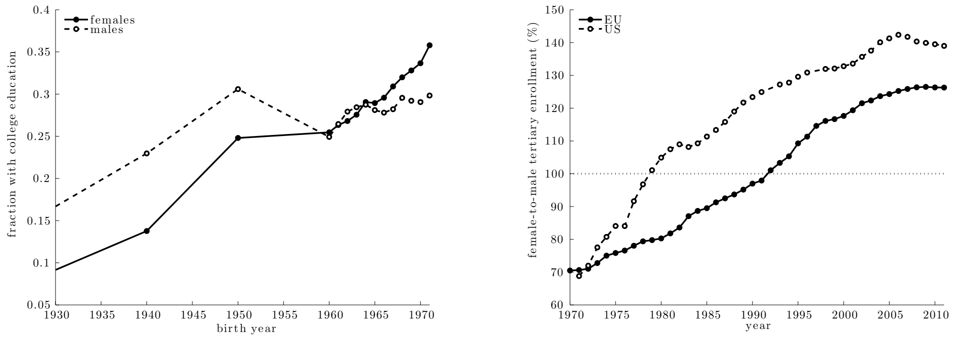
Over the last decades women have caught up with men in many domains, but nowhere has the change been so striking as for college education. Not only did they manage to close the gap, nowadays women even graduate in larger numbers in most developed countries. This is known as the 'college gender gap reversal', see for example Goldin et al. (2006). To illustrate this phenomenon for the United States, Figure 4.1(a) plots the fraction of females and males who have completed 4 years of college or more at the age of 40 by birth year. Whereas in the 1950 cohort about 30% of the men obtained a college education versus 25% of the women, by 1970 the fraction of educated women had surpassed that of men. The same pattern shows up in the enrolment rates for tertiary education, see Figure 4.1(b). Both in the United States and for the European Union countries the ratio of female to male enrolment has increased over time and nowadays exceeds 100%.

There are two main strands of literature that attempt to explain gender differences in educational attainment. The first assumes that parents make education decisions for their children. This may be particularly relevant in early stages of economic development or when the existence of borrowing constraints makes family income an important source of college funding. For example, Echevarria and Merlo (1999) develop a model in which men and women bargain over a binding prenuptial agreement which specifies the investment in education of children conditional on gender. As long as the time cost of child bearing is positive, girls receive less education than their brothers. In the

*This chapter is based on Reijnders (2014a).

Figure 4.1: The college gender gap reversal

(a) College graduates in US (b) Tertiary enrolment in US and EU



Source: Panel (a): Integrated Public Use Microdata Series (IPUMS) for 1970-2010. Panel (b): United Nations Educational, Scientific and Cultural Organization (UNESCO) Institute for Statistics.

benchmark model of Ríos-Rull and Sánchez-Marcos (2002), on the other hand, risk averse parents would invest more in the schooling of their daughters if women earn lower wages than men. In order to explain a higher college graduation rate for men they need to introduce an additional assumption which raises the returns to college for men relative to women. The ones that yield predictions closest to the data are a higher cost of education for women in terms of foregone home production or an altruistic parent that cares about the number of descendants. Sánchez-Marcos (2007) uses the same model in an attempt to explain the closing of the college gender gap based on observed changes in relative earnings, marital sorting and fertility.

The second strand of literature postulates that individuals make their own choice about whether or not to obtain a college degree. When education is viewed as a pure investment decision, the current gender imbalance is all the more puzzling since women appear to have fewer incentives to invest. On average they earn less in the labour market and spend more time on household work and child care than men, which lowers the return on their human capital. Several solutions to this conundrum have been proposed. First, some authors claim that the relative wage of educated versus uneducated workers (the college wage premium) is higher for women, see for example Dougherty (2005) or Goldin et al. (2006). A second set of theories points to the returns to education that extend beyond the labour market such as a higher probability of marriage and a greater marital surplus. For example, Iyigun and Walsh (2007a) show that if men are in short supply in the marriage market then women might invest more in education in order to compete for desirable husbands. A third explanation relies on uncertainty in income and marital status. DiPrete and Buchmann (2006) observe that education provides women with valuable insurance against poverty through three channels: higher wages, lower risks of divorce and less out-of-marriage childbearing.

However, a theory of why women invest more in education than men leaves open the question of why they did not do so in the past. Chiappori et al. (2009) propose that there has been a change in social norms about household roles which ensures that women are no longer tied up at home but can take advantage of a higher college wage premium in the labour market. The explanation put forward by Becker et al. (2010) relies on gender differences in the distribution of non-cognitive skills (such as self-motivation and discipline) which are inversely related to the cost of education. If women have a higher level of these skills on average and the variability among them is lower, then the supply of female college graduates is more responsive to changes in the economic environment that increase the payoff of a college education. Guvenen and Rendall (2013) find that the rise in divorce risk following a legislative reform increases the insurance value of education more for women than for men as they tend to get custody of the children.

The aim of this chapter is to investigate under what conditions a basic life-cycle model in which rational and forward-looking individuals make decisions about education can generate a college gender gap reversal. In order to derive analytical results where possible we assume that the probability of getting married and the chance of finding a spouse with or without education are exogenously given. This is in contrast to Chiappori et al. (2009) and Guvenen and Rendall (2013) who focus on endogenous matching at the expense of simplifying life-cycle choices about savings and fertility. However, we do impose that the beliefs that individuals hold about marriage probabilities are consistent with actual distributions in equilibrium. To study gender differences in educational attainment we decompose the return to education into two main components: a labour market benefit and a distortion due to the possibility of marriage.

The analysis yields two main contributions. First, we prove analytically that the labour market benefit of education for women can be higher than for men if there is a realistic amount of curvature in the utility function or if there are fixed costs. Intuitively this is because women earn lower wages and with strongly diminishing marginal utility of wealth they have more to gain by increasing their lifetime earnings through obtaining a college degree. This result does *not* rely on a higher college wage premium for women, the evidence for which is mixed (see Hubbard et al. (2011)). The distortions introduced through the marriage market tend to depress the overall benefit of education for women relative to men as they expect to marry a more wealthy spouse and to work less when a child is born.

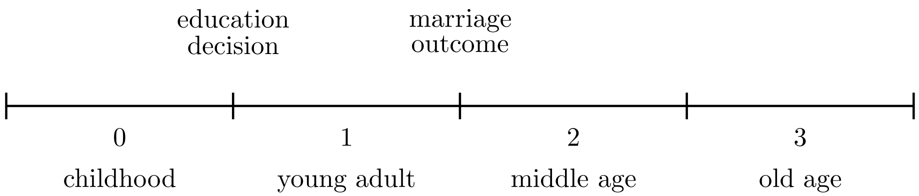
Second, parameterizing the model using US census data we show which changes in the economic and social environment can lead to a reversal in college graduation rates. It turns out that a drop in the probability of marriage of the magnitude observed in the data is sufficient. In the new equilibrium risk-averse women invest more in education than men because being single is more costly for them. Other factors that lead to a rise in the number of educated women relative to educated men, such as an increase in the common college wage premium, are quantitatively not strong enough to reverse the gap.

The remainder of this chapter is organized as follows. Section 4.2 discusses the benefits and costs of a college education and the general set-up of the model. Section 4.3 describes a fully specified example, which will be used to study gender differences in education choices in Section 4.4. In Section 4.5 we parameterize the model using US census data and illustrate how it can account for the college gender gap reversal. The last section concludes.

4.2 Trade-offs in the choice of education

Education is an investment in human capital, the costs of which are incurred early in life while the benefits materialize later. To study the intertemporal trade-offs associated with the choice of (tertiary) education we divide the life-cycle of an individual into four periods, see Figure 4.2. The first of these (period 0) is spent passively in the household of the parents and is ignored here.

Figure 4.2: Life cycle



At the start of period 1 an individual decides whether to obtain a college degree ($E = 1$, ‘educated’) or not ($E = 0$, ‘uneducated’). In this decision the pecuniary costs and benefits of a college education will play a role. There are costs in terms of tuition fees that have to be paid and wages that are foregone by delaying entry into the labour market. On the benefit side, a college graduate can earn a higher wage. At first glance, the benefit of education is that it increases the present value of wages earned over the life-cycle (known as human wealth). That is, $H_1^j(1) > H_1^j(0)$ where:

$$H_1^j(E) = w_1^j(E)[1 - \bar{e}E] + \frac{w_2^j(E)}{1+r} + \frac{w_3^j(E)[1-\bar{R}]}{(1+r)^2}, \tag{4.1}$$

with $w_t^j(E)$ the wage rate of a person of gender $j \in \{f, m\}$ with education level $E \in \{0, 1\}$ in period $t \in \{1, 2, 3\}$ and r the interest rate. We assume that obtaining a college degree requires an (exogenous) fraction $\bar{e}$ of the unit time endowment in period 1 and that a fraction $\bar{R}$ of the last period is spent in retirement. An individual might choose to obtain an education if the increase in wages more than compensates for the tuition fee $\bar{f}$, that is if $\bar{f} < H_1^j(1) - H_1^j(0)$.

However, this definition of the benefit of education does not take into account that educated and uneducated individuals make different life-cycle choices, for example regarding how much to consume in every period. What matters is therefore not the present value of earnings but the level of welfare that can be attained with them. A more comprehensive definition of the benefit of education would be that it increases the discounted utility of consumption. That is, $\mathcal{L}_1^j(1) > \mathcal{L}_1^j(0)$ where:

$$\begin{aligned}
\mathcal{L}_1^j(E) = \max_{c_1,c_2,c_3} \sum_{t=1}^{3} \left(\frac{1}{1+\rho}\right)^{t-1} u(c_t, 0) \\
\text{s.t.} \quad 0 = H_1^j(E) - \left[1 + \frac{1}{1+r} + \frac{1}{(1+r)^2}\right]\bar{c} - \bar{f}E - \sum_{t=1}^{3} \left(\frac{1}{1+r}\right)^{t-1} c_t,
\end{aligned} \tag{4.2}$$

where the flow of utility in each period t depends on consumption c_t and the number of children (assumed to be zero, see below) and ρ is the rate of time preference. We assume that there is a fixed cost $\bar{c}$ in every period. An individual might choose to obtain an education if the increase in utility from consumption more than compensates for the utility cost of studying for the degree θ, that is if $\theta < \mathcal{L}_1^j(1) - \mathcal{L}_1^j(0)$. This non-pecuniary or 'psychic' cost of education is inversely related to an individual's aptitude for learning. It reflects both cognitive skills (such as IQ) and non-cognitive skills (for example self-motivation and discipline).

Yet one important aspect of the trade-off is still missing. As individuals might get married in the future they could spent part of their life together with someone else. However, the education decision is generally made individually and non-cooperatively as the spouse-to-be has not yet been met. In order to understand how the prospect of marriage affects the benefit of a college degree we will first discuss the relevant differences between singles and married couples (Section 4.2.1) and the assumptions we make about the marriage market (Section 4.2.2). Subsequently we will define the notion of an equilibrium in this model (Section 4.2.3) and decompose the benefit of education into its constituent parts (Section 4.2.4).

4.2.1 Singles versus married couples

For now we only impose a few mild restrictions on preferences and the nature of marriage.[1] First, we assume that utility is time separable. In each period the individual derives felicity $u(c,b)$ from private consumption goods c and the number of children b (which is a public good within the household). For simplicity we ignore utility from leisure, but it would be straightforward to extend the framework with a labour-leisure choice. A second restriction is that singles cannot have children. As we will be comparing the welfare of singles and married individuals it follows that children should not be a necessary 'good', meaning that $u(c,0)$ is well-defined for any $c > 0$. Finally, we assume that marriage can only take place at the start of period 2 and that all relevant information about an individual at that moment in time can be summarized by his or her education level and accumulated savings alone.

Let $\mathcal{M}_2^j\big(E^j, E^{-j}, a_2^j, a_2^{-j}\big)$ denote remaining lifetime utility of a married individual of gender j at the start of period 2, as a function of the own level of education and financial assets $\big\{E^j, a_2^j\big\}$ and those of the spouse $\big\{E^{-j}, a_2^{-j}\big\}$. At this point we are agnostic about how this utility from marriage is defined or what kind of household decision-making process has given rise to it. Similarly, let $\mathcal{S}_2^j(E, a_2)$ be the value function of a single individual in period 2. The assumptions made so far imply that:

$$\begin{aligned} \mathcal{S}_2^j(E, a_2) &= \max_{c_2, c_3} \left\{ u(c_2, 0) + \frac{u(c_3, 0)}{1+\rho} \right\} \\ \text{s.t.} \quad 0 &= (1+r)a_2 + H_2^j(E) - c_2 - \frac{c_3}{1+r} - \left[1 + \frac{1}{1+r}\right]\bar{c}, \end{aligned} \tag{4.3}$$

where $H_2^j(E)$ is the net present value of wage income from the perspective of period 2:

$$H_2^j(E) = w_2^j(E) + \frac{w_3^j(E)[1-\bar{R}]}{1+r}. \tag{4.4}$$

[1]For the purpose of this chapter, marriage and cohabitation are equivalent. The term marriage is used by itself throughout for convenience.

4.2.2 The marriage market

As stated above, we assume that it is only possible for individuals to get married at the start of period 2. This leaves open the question of who is matched with whom. We will ensure that the model remains tractable by making the simplifying assumptions that (i) all matching probabilities are exogenously given (which is another way of saying that marriage decisions are driven by factors outside the model), (ii) the probability of getting married is independent of education level and (iii) there is no divorce.

Table 4.1: Matching probabilities

		E^f		
		0	1	
E^m	0	$\pi(0,0)$	$\pi(1,0)$	$\pi^m(0)$
	1	$\pi(0,1)$	$\pi(1,1)$	$\pi^m(1)$
		$\pi^f(0)$	$\pi^f(1)$	1

Let $\pi^j(E)$ denote the fraction of individuals of gender $j \in \{f, m\}$ with education level $E \in \{0, 1\}$. Naturally $\pi^j(0) + \pi^j(1) = 1$. Write $\pi(E^f, E^m)$ for the probability that a female with education E^f is matched with a male with education E^m. Despite the popular saying that 'opposites attract', most people tend to get married to someone with a similar level of education.[2] This type of marital sorting is known as positive assortative matching or homogamy and it might arise because of complementarities between spouses or simply because individuals who are alike are more prone to meet and fall in love. To allow for this kind of behaviour we define:

$$\pi(1,1) = (1-\lambda)\pi^f(1)\pi^m(1) + \lambda \min\left\{\pi^f(1), \pi^m(1)\right\}, \tag{4.5}$$

where the parameter λ is an index of the degree of marital sorting. If $\lambda = 0$ then matching is random. If $\lambda = 1$ then matching is perfectly positively assortative so that the probability of a match between two educated individuals is determined by whether educated men or educated women are in short supply. In the special case that $\pi^f(1) = \pi^m(1)$, λ equals the correlation coefficient between female and male education (as in Fernández and Rogerson (2001)). The expression for $\pi(0,0)$ is similar and the cross probabilities follow (see Table 4.1).

[2]See for example the matching patterns from the data in Section 4.5.3.

Finally, we write $\pi^{m|f}(E^m|E^f)$ for the probability that a woman is matched to a man with education E^m conditional on her own educational attainment E^f (and vice versa for $\pi^{f|m}(E^f|E^m)$). By Bayes' Rule:

$$\pi^{m|f}(E^m|E^f) = \frac{\pi(E^f, E^m)}{\pi^f(E^f)}. \tag{4.6}$$

If $0 < \lambda \leq 1$, so that there is positive assortative matching, then the probability of being matched to an educated husband is greater for an educated woman than for an uneducated one, that is $\pi^{m|f}(1|1) > \pi^{m|f}(1|0)$.

Figure 4.3: Matching process

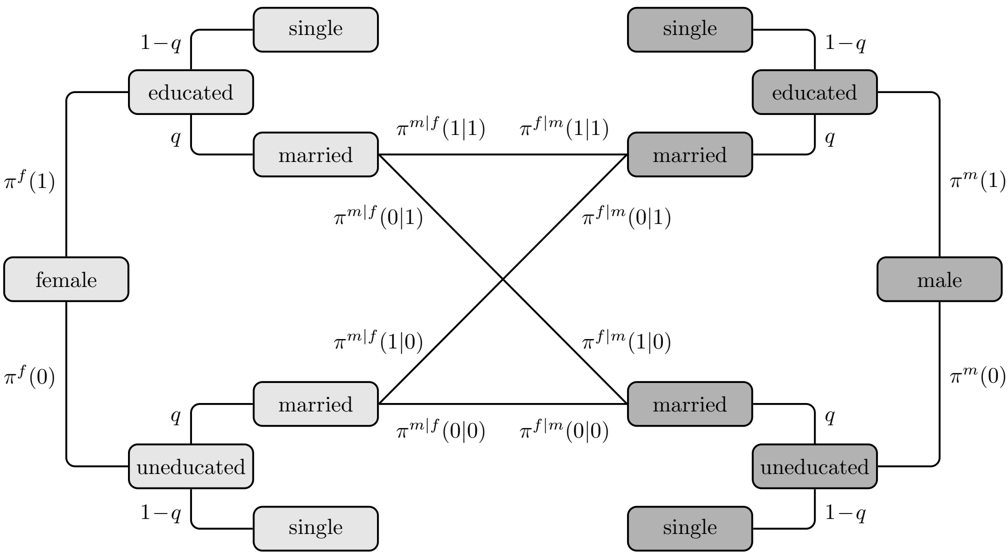


Figure 4.3 provides a schematic overview of the matching process. Starting from sides, women (left) and men (right) choose to become educated or not. A fraction q gets married while the remainder stays single. Given marriage, the conditional probabilities determine which type of woman ends up with which type of man.

4.2.3 Marriage market equilibrium

Upon entering adulthood at the start of period 1 each individual learns his or her utility cost of education θ which is drawn from a distribution F^j_θ. This heterogeneity in learning ability ensures that not everyone makes the same education decision. At the end of period 1 there will be four different types of individuals: educated females, uneducated females, educated males and uneducated males. Each person decides whether to go to school or not and how much to consume and save taking the choices made by all other individuals as given. In particular, the education frequencies $\pi^f(1)$ and $\pi^m(1)$ determine the conditional matching probabilities and the level of savings of individuals of the opposite sex $\mathbf{a}_2^{-j}(0)$ and $\mathbf{a}_2^{-j}(1)$ affect the utility of being married. The expected utility from consumption and fertility of an individual of gender j with education E is then:

$$\mathcal{S}_1^j(E) = \max_{c_1,a_2} \Bigg\{ u\big(c_1, 0\big) + \frac{1}{1+\rho} \Big[(1-q)\mathcal{S}_2^j(E, a_2) + q\Big[\pi^{-j|j}(0|E)\mathcal{M}_2^j\big(E, 0, a_2, \mathbf{a}_2^{-j}(0)\big) + \pi^{-j|j}(1|E)\mathcal{M}_2^j\big(E, 1, a_2, \mathbf{a}_2^{-j}(1)\big)\Big]\Big]\Bigg\}$$

$$\text{s.t.} \quad a_2 = w_1^j(E)[1-\bar{e}E] - c_1 - \bar{c} - \bar{f}E > -\frac{1}{1+r}\left\{ H_2^j(E) - \left[1 + \frac{1}{1+r}\right]\bar{c} \right\}, \tag{4.7}$$

The first term in the objective function is the immediate felicity from consumption in period 1. The remaining terms capture the expected discounted utility from period 2 onward. With probability $1-q$ the individual remains single and has value function $\mathcal{S}_2^j(E, a_2)$. If this person marries then there is a probability $\pi^{-j|j}(0|E)$ of being matched to an uneducated spouse and a probability $\pi^{-j|j}(1|E)$ of finding an educated partner. The constraint on financial assets shows that it is only possible to borrow against own human wealth net of fixed costs and not the income of a future spouse. The optimal choice of savings is denoted by $\mathbf{a}_2^j(E)$.

An individual will choose to obtain a college degree if the utility cost of education θ is below a gender-specific threshold $\bar{\theta}^j = \mathcal{S}_1^j(1) - \mathcal{S}_1^j(0)$. It follows that the fraction of individuals of gender j with a college education is $\pi^j(1) = F_\theta^j(\bar{\theta}^j)$, which corresponds to the mass in the left tail of the utility cost distribution. The marriage market equilibrium is then such that the beliefs about matching probabilities and savings are consistent with the actual choices.

Definition 4.1. *A marriage market equilibrium is a set of education frequencies* $\{\pi^f(1), \pi^m(1)\}$ *and a set of financial asset choices* $\{\mathbf{a}_2^j(0), \mathbf{a}_2^f(1), \mathbf{a}_2^m(0), \mathbf{a}_2^m(1)\}$ *such that for each gender* $j \in \{f, m\}$ *and each realization of the utility cost* θ *from the distribution* F_θ^j *the choice of education* E *and the corresponding level of assets* $\mathbf{a}_2^j(E)$ *jointly maximize expected lifetime utility.*

The maintained assumption in the remainder of this chapter is that the equilibrium is such that there is a (financial) benefit of being married to an educated spouse. That is, $(1+r)\mathbf{a}_2^j(1) + H_2^j(1) > (1+r)\mathbf{a}_2^j(0) + H_2^j(0)$.

4.2.4 Decomposition

In order to gain insight into the considerations that drive the individual's education choice, we decompose the threshold level $\bar{\theta}^j$ into two parts:

$$\bar{\theta}^j \equiv \mathcal{S}_1^j(1) - \mathcal{S}_1^j(0) = LMB^j + MMD^j. \tag{4.8}$$

The first part is the *labour market benefit*. In line with Chiappori et al. (2009) it is defined as the benefit of education for a person who knows for certain that he or she will never marry (referred to as a 'lifelong single' hereafter):

$$LMB^j \equiv \mathcal{L}_1^j(1) - \mathcal{L}_1^j(0), \tag{4.9}$$

where $\mathcal{L}_1^j(E)$ is the maximum level of lifetime utility for a person with education E in the absence of marriage prospects, see equation (4.2). Note that in this model such a lifelong single does not actually exist, as everyone faces uncertainty about whether they will marry or not (recall the specification of expected lifetime utility in (4.7)). The remaining part of the threshold consists of the *marriage market distortion*:

$$MMD^j \equiv \Delta SB^j + q\big[\Delta UG^j + \Delta MP^j\big]. \tag{4.10}$$

The possibility of marriage distorts the education decision in several ways. First, it affects the pre-marital savings decision of an individual. For example, the wealthier a future spouse is expected to be, the less incentive there is to save (provided the spouse is willing to share). For both educated and uneducated individuals the financial asset level in the marriage market equilibrium $\mathbf{a}_2^j(E)$ will differ from the optimal choice made by a lifelong single. If someone turns out to remain single after all, then actual lifetime utility will be less than $\mathcal{L}_1^j(E)$. The part of the threshold explained by differences in savings behaviour is:

$$\begin{aligned}\Delta SB^j \equiv & \left[u\big(w_1^j(1)[1-\bar{e}] - \bar{c} - \bar{f} - \mathbf{a}_2^j(1), 0\big) + \frac{1}{1+\rho}\mathcal{S}_2^j\big(1, \mathbf{a}_2^j(1)\big) - \mathcal{L}_1^j(1)\right] \\ & - \left[u\big(w_1^j(0) - \bar{c} - \mathbf{a}_2^j(0), 0\big) + \frac{1}{1+\rho}\mathcal{S}_2^j\big(0, \mathbf{a}_2^j(0)\big) - \mathcal{L}_1^j(0)\right].\end{aligned} \tag{4.11}$$

Secondly, the expected increase in welfare from being married relative to being single differs by education type. On the one hand an individual with a college degree might be able to capture a larger share of the gains from marriage, on the other hand he or she also has a higher material welfare as a single. The difference in the utility gain from being married is:

$$\begin{aligned}\Delta UG^j \equiv \frac{1}{1+\rho}\Big[&\pi^{-j|j}(1|0)\mathcal{M}_2^j\big(1,1,\mathbf{a}_2^j(1),\mathbf{a}_2^{-j}(1)\big) + \pi^{-j|j}(0|0)\mathcal{M}_2^j\big(1,0,\mathbf{a}_2^j(1),\mathbf{a}_2^{-j}(0)\big) \\ &- \mathcal{S}_2^j\big(1,\mathbf{a}_2^j(1)\big)\Big] - \frac{1}{1+\rho}\Big[\pi^{-j|j}(1|0)\mathcal{M}_2^j\big(0,1,\mathbf{a}_2^j(0),\mathbf{a}_2^{-j}(1)\big) \\ &+ \pi^{-j|j}(0|0)\mathcal{M}_2^j\big(0,0,\mathbf{a}_2^j(0),\mathbf{a}_2^{-j}(0)\big) - \mathcal{S}_2^j\big(0,\mathbf{a}_2^j(0)\big)\Big]. \end{aligned} \tag{4.12}$$

The first term bounded by square brackets is the expected gain from marriage when educated, keeping the matching probabilities fixed at those for an uneducated person. The second term is the corresponding expression for an uneducated individual.

The final part of the marriage market distortion can be ascribed to differences in matching probabilities:

$$\begin{aligned}\Delta MP^j \equiv \frac{1}{1+\rho}&\Big[\pi^{-j|j}(0|0) + \pi^{-j|j}(1|1) - 1\Big] \\ &\times \Big[\mathcal{M}_2^j\big(1,1,\mathbf{a}_2^j(1),\mathbf{a}_2^{-j}(1)\big) - \mathcal{M}_2^j\big(1,0,\mathbf{a}_2^j(1),\mathbf{a}_2^{-j}(0)\big)\Big]. \end{aligned} \tag{4.13}$$

If there is positive assortative matching then being educated has the advantage of increasing the probability of marrying an educated spouse, assuming that this is desirable. On the other hand, if the matching process is completely random then this term disappears because the conditional probabilities in the first set of brackets coincide with the unconditional ones and sum to unity. As in Guvenen and Rendall (2013), the prevalence of sorting in the marriage market gives rise to an externality in the choice of education. For example, suppose that there are fewer educated women than educated men such that $\pi^f(1) < \pi^m(1)$. This implies that, even with perfect assortative matching, some educated men will marry uneducated women. When more women decide to get a college education then an uneducated woman becomes less likely to find an educated spouse, which leads to an increase in ΔMP^f and thereby (ceteris paribus) a higher benefit of education for women.

The last two components of the distortion taken together (that is, $\Delta UG^j + \Delta MP^j$) comprise what is sometimes thought of as a 'marriage market benefit' of education (as in Chiappori et al. (2009), for example). It can be either positive or negative, depending on how much surplus a married couple generates relative to two single individuals and how this surplus is divided between the spouses.

4.3 A fully specified example

In this section we will develop a fully specified version of the general model outlined above. It will serve to illustrate gender differences in the benefit of education (Section 4.4) and thereby help to explain the college gender gap reversal (Section 4.5).

4.3.1 Assumptions

Assume that the individual's felicity function is of the isoelastic form:

$$u(c,b) = \begin{cases} \dfrac{\left[c^{\varepsilon}(1+b)^{1-\varepsilon}\right]^{1-1/\sigma} - 1}{1-1/\sigma} & \text{if } \sigma \neq 1 \\ \varepsilon \ln c + (1-\varepsilon)\ln(1+b) & \text{if } \sigma = 1 \end{cases} \tag{4.14}$$

The felicity function features a constant intertemporal substitution elasticity $\sigma > 0$ with respect to a Cobb-Douglas composite of consumption and an index of fertility, with $0 < \varepsilon \leq 1$ representing the weight of consumption. These preferences are quasi-homothetic because (i) there is a fixed cost, which drives a wedge between total expenditures $\bar{c} + c$ and utility-generating consumption c and (ii) children are not a necessary 'good', as $1 + b$ enters the felicity function and not b itself.

For future reference we define the intertemporal substitution elasticity of consumption:

$$\sigma^* \equiv -\frac{u_c(c,b)}{u_{cc}(c,b)c} = \frac{1}{1-\varepsilon(1-1/\sigma)}, \quad \sigma^* \gtreqless 1 \quad \Leftrightarrow \quad \sigma \gtreqless 1. \tag{4.15}$$

The value function of a single at the start of period 2, as introduced in (4.3), can then be written as:

$$\mathcal{S}_2^j(E,a_2) = \begin{cases} \dfrac{\varepsilon}{\Gamma_2(1)} \dfrac{\Gamma_2(1)\Gamma_2(\sigma)^{-1}\left[\Gamma_2(\sigma)W_2^j(E,a_2)\right]^{1-1/\sigma^*} - 1}{1-1/\sigma^*} & \text{if } \sigma \neq 1 \\ \dfrac{\varepsilon}{\Gamma_2(1)} \ln\left[\Gamma_2(1)W_2^j(E,a_2)\right] + \Psi_2 & \text{if } \sigma = 1 \end{cases} \tag{4.16}$$

where Ψ_2 is a constant[3] and $W_2^j(E,a_2)$ is the individual's wealth net of fixed costs from

[3] The parameter Ψ_2 is defined as:

$$\Psi_2 \equiv \frac{\varepsilon}{1+\rho} \ln\left(\frac{1+r}{1+\rho}\right).$$

the perspective of period 2:

$$W_2^j\big(E, a_2\big) = (1+r)a_2 + H_2^j(E) - \left[1 + \frac{1}{1+r}\right]\bar{c}. \tag{4.17}$$

The marginal propensity to consume out of wealth in period 2 is captured by $\Gamma_2(\sigma)$:

$$\Gamma_2(\sigma) = \left[1 + \frac{1}{1+r}\left(\frac{1+r}{1+\rho}\right)^{\sigma^*}\right]^{-1}. \tag{4.18}$$

To derive the value function of a married individual we need to specify the process of decision-making within a couple. In line with a large part of the literature we postulate that the resulting allocation choices are Pareto efficient.[4] This implies that the couple acts *as if* it maximizes a weighted average of the individual utility functions of the husband and the wife (see for example Chiappori (1992)). The couple's periodic welfare function can be written as:

$$U\big(c_t^f, c_t^m, b\big) = \alpha u\big(c_t^f, b\big) + (1-\alpha)u\big(c_t^m, b\big), \tag{4.19}$$

where $0 < \alpha < 1$ is the Pareto weight of the woman. The larger is α, the more the allocation on the Pareto frontier that is chosen by the couple tends to favour the wife. By taking this weight as exogenously given and constant we assume that the couple acts as a 'unitary' household and that there is full commitment to the marriage. In Appendix 4.B we relax this assumption and allow for bargaining within the family.

If a married couple decides to have b children then all of these are born at the start of period 2 and they remain in the household for exactly 1 period. The cost of having children is threefold. First, they increase the fixed consumption cost that the household has to incur. Second, parents are required to spend a certain amount of time with their children. Child care is created according to a linear homogeneous production function with a constant elasticity of substitution between father and mother time $\xi > 1$. The child care constraint is then:

$$\Omega\big(n^f, n^m\big) = \Big[\big(n^f\big)^{1-1/\xi} + \big(n^m\big)^{1-1/\xi}\Big]^{\frac{1}{1-1/\xi}} = N^b b, \tag{4.20}$$

where n^j is the time input of parent j and N^b is the time requirement of a single child. Finally, the mother has to incur an additional time cost related to child birth of T^b.

[4] Although reasonable for repeated day-to-day decisions such as the division of consumption expenditure, it is arguably less realistic for 'big' choices like how many children to raise.

Under the assumption that there are no bequests to children, the consolidated budget constraint for the household is given by:

$$\begin{aligned} 0 = (1+r)\big[a_2^f + a_2^m\big] + H_2^f\big(E^f\big) + H_2\big(E^m\big) - w_2^f\big(E^f\big)\big[n^f + T^b b\big] - w_2\big(E^m\big)n^m \\ - c_2^f - c_2^m - \frac{c_3^f + c_3^m}{1+r} - \left[Q^a + Q^b b + \frac{Q^a}{1+r}\right]\bar{c}, \end{aligned} \quad (4.21)$$

where $1 < Q^a \leq 2$ is the equivalence scale for two adults and $0 < Q^b \leq 1$ is the adult equivalent of a child. With $Q^a < 2$ and $Q^b < 1$ there are economies of scale for a multi-person household in providing the fixed consumption cost (think of sharing a house, kitchen equipment, etcetera). Labour supply in period 2 is $1 - n^m$ for the husband and $1 - n^f - T^b b$ for the wife and can either be interpreted as hours worked or the fraction of the period worked full-time. Note that although there is no leisure in the utility function, labour supply of married individuals is endogenous as it depends on the chosen number of children and the child care allocation.

The problem of the household is to maximize the sum of discounted felicity (4.19) for period 2 and 3, subject to the budget constraint (4.21) and the child care requirement (4.20). Assuming an interior solution, the most efficient allocation of child care between the parents for a given number of children is the one which minimizes the associated cost in terms of foregone wages. The unit cost function is defined as:

$$\omega\big(E^f, E^m\big) = \min_{n_2^f, n_2^m} \left[w_2^f\big(E^f\big)n^f + w_2^m\big(E^m\big)n^m\right] \quad \text{s.t.} \quad \Omega\big(n^f, n^m\big) = 1. \quad (4.22)$$

The total cost of a child is then given by:

$$\Upsilon^b\big(E^f, E^m\big) = Q^b\bar{c} + N^b\omega\big(E^f, E^m\big) + T^b w_2^f\big(E^f\big), \quad (4.23)$$

which depends positively on the education levels (or wages) of the parents. The optimal intra-family sharing rule is such that in every period a fraction $\beta(\alpha, \sigma) = [1 + ((1 - \alpha)/\alpha)^{\sigma^*}]^{-1}$ of total spending on private consumption goods goes to the wife while the remainder is dedicated to the husband. It follows that the share of a woman is increasing in her Pareto weight α.

The value function for a married woman can then be written as:

$$\mathcal{M}_2^f\big(E^f, E^m, a_2^f, a_2^m\big) =$$
$$\begin{cases} \dfrac{1}{\Gamma_2(1)} \dfrac{\Gamma_2(1)\Gamma_2(\sigma)^{-1}\left[\bar{\varepsilon}\dfrac{\beta(\alpha,\sigma)^{\varepsilon}\Gamma_2(\sigma)^{\varepsilon}}{\Upsilon^b(E^f,E^m)^{1-\varepsilon}}W_2(E^f,E^m,a_2)\right]^{1-1/\sigma} - 1}{1-1/\sigma} & \text{if } \sigma \neq 1 \\ \dfrac{1}{\Gamma_2(1)} \ln\left[\bar{\varepsilon}\dfrac{\beta(\alpha,\sigma)^{\varepsilon}\Gamma_2(1)^{\varepsilon}}{\Upsilon^b(E^f,E^m)^{1-\varepsilon}}W_2(E^f,E^m,a_2)\right] + \Psi_2 & \text{if } \sigma = 1 \end{cases} \tag{4.24}$$

where $a_2 = a_2^f + a_2^m$ is joint savings, $\bar{\varepsilon} \equiv \varepsilon^{\varepsilon}(1-\varepsilon)^{1-\varepsilon}$ is a constant and $W_2\big(E^f, E^m, a_2\big)$ is household wealth net of subsistence costs:

$$\begin{aligned} W_2\big(E^f, E^m, a_2\big) = {} & (1+r)a_2 + H_2^f\big(E^f\big) + H_2^m\big(E^m\big) + \Upsilon^b\big(E^f, E^m\big) \\ & - \left[1 + \frac{1}{1+r}\right] Q^a \bar{c}. \end{aligned} \tag{4.25}$$

The household allocation only depends on total financial and human wealth and not its distribution over the spouses. This is known as income pooling and it is a consequence of the assumption of a unitary household with fixed Pareto weights. Joint wealth is higher than the sum of individual wealth as given in (4.17) due to (i) economies of scale, provided that $Q^a < 2$ and (ii) the possibility to produce children, in combination with a negative 'subsistence level'.[5] An individual's utility when married increases with the level of own and spousal wealth and decreases with the cost of children.

Below we will sometimes refer to the case without fertility, by which we mean that $\varepsilon = 1$ such that $b = 0$ (a corner solution) and $\Upsilon^b\big(E^f, E^m\big)$ drops out of (4.25). The 'value' of a spouse can then be summarized as the sum of his or her financial and human wealth. This is not possible when there are children involved, as then the education level by itself also matters in determining the opportunity cost of child care.

4.3.2 Marriage market equilibrium

The optimal choice of financial assets to take along to the second period, $\mathbf{a}_2^j(E)$, depends negatively on the savings of educated and uneducated individuals of the opposite sex. This means that financial assets are strategic substitutes in the marriage game. In general it is not possible to solve for the equilibrium amounts analytically. As a special case, consider a person of gender j who is single for certain such that $q = 0$. Then the

[5] That is, b enters the utility function through the term $1 + b$. This can be seen as a subsistence level of -1 for children.

optimal asset choice is independent of that of all other individuals and given by:

$$\mathbf{a}_2^j(E) = w_1^j(E)[1 - \bar{e}E] - \bar{c} - \bar{f}E - \Gamma_1(\sigma)W_1^j(E), \tag{4.26}$$

where $W_1^j(E)$ is net total wealth from the perspective of period 1:

$$W_1^j(E) = H_1^j(E) - \left[1 + \frac{1}{1+r} + \frac{1}{(1+r)^2}\right]\bar{c} - \bar{f}E, \tag{4.27}$$

and $\Gamma_1(\sigma)$ is the corresponding propensity to consume:

$$\Gamma_1(\sigma) \equiv \left[1 + \frac{1}{1+r}\left(\frac{1+r}{1+\rho}\right)^{\sigma^*} + \frac{1}{(1+r)^2}\left(\frac{1+r}{1+\rho}\right)^{2\sigma^*}\right]^{-1}. \tag{4.28}$$

Note that this is the solution to the problem expressed in (4.2) given the functional form of the felicity function. The value function of a lifelong single in period 1 can therefore be written as:

$$\mathcal{L}_1^j(E) = \begin{cases} \dfrac{\varepsilon}{\Gamma_1(1)} \dfrac{\Gamma_1(1)\Gamma_1(\sigma)^{-1}\left[\Gamma_1(\sigma)W_1^j(E)\right]^{1-1/\sigma^*} - 1}{1 - 1/\sigma^*} & \text{if } \sigma \neq 1 \\ \dfrac{\varepsilon}{\Gamma_1(1)} \ln\left[\Gamma_1(1)W_1^j(E)\right] + \Psi_1 & \text{if } \sigma = 1 \end{cases} \tag{4.29}$$

where Ψ_1 is a constant.[6]

4.4 Gender differences in the education choice

The next two sections describe potential gender differences in the labour market benefit of education (Section 4.4.1) and the marriage market distortion (Section 4.4.2). The implications for the marriage market equilibrium are derived numerically in Section 4.4.3.

4.4.1 The labour market benefit

Recall that the labour market benefit of education is the difference in lifetime utility with and without education for a person who remains single for certain. Using (4.29)

[6]The parameter Ψ_1 is defined as:

$$\Psi_1 \equiv \varepsilon\left[\frac{1}{1+\rho}\ln\left(\frac{1+r}{1+\rho}\right) + \frac{1}{(1+\rho)^2}\ln\left(\frac{1+r}{1+\rho}\right)^2\right]$$

we obtain:

$$LMB^j \equiv \mathcal{L}_1^j(1) - \mathcal{L}_1^j(0) = \begin{cases} \varepsilon\Gamma_1(\sigma)^{-1/\sigma^*} \dfrac{W_1^j(1)^{1-1/\sigma^*} - W_1^j(0)^{1-1/\sigma^*}}{1-1/\sigma^*} & \text{if } \sigma \neq 1 \\ \dfrac{\varepsilon}{\Gamma_1(1)} \ln\left(\dfrac{W_1^j(1)}{W_1^j(0)}\right) & \text{if } \sigma = 1 \end{cases} \tag{4.30}$$

where total wealth $W_1^j(E)$ consists of human wealth $H_1^j(E)$ net of tuition fees $\bar{f}E$ and the discounted value of fixed costs $\bar{c}$ as in (4.27). Without loss of generality, we set $w_1^j(E) = w^j(E)$ and define $\eta_t^j(E)$ to be the net growth rate of wages from period $t-1$ to period t. This growth rate could in general depend on both gender and education level and captures factors such as experience build-up or human capital depreciation. Human wealth can then be written as:

$$H_1^j(E) = w^j(E)\left[1 - \bar{e}E + \frac{1+\eta_2^j(E)}{1+r} + \frac{\left[1+\eta_3^j(E)\right]\left[1+\eta_2^j(E)\right][1-\bar{R}]}{(1+r)^2}\right]. \tag{4.31}$$

Gender differences in the labour market benefit of education (4.30) can stem from several sources. First of all, there is ample evidence for the existence of a 'gender wage gap': after accounting for measurable skills, women earn less than men (see Jarrell and Stanley (2004) for a meta-analysis for the United States). This implies that men and women might not receive the same college wage premium, which is a measure of how much more a college graduate earns compared to a person without such a degree. Usually it is defined as the log difference in wages $\ln(w^j(1)/w^j(0))$. It corresponds to the coefficient on a college dummy in a Mincerian-style semi-log wage regression. Several authors have argued that the college wage premium is higher for women as obtaining an education gives them a double dividend: not only does it increase their productivity, but it also reduces the gender gap in wages. Dougherty (2005) attributes this gap to 'discrimination, tastes and circumstances', all of which might be inversely related to a woman's educational attainment. Similarly, Chiappori et al. (2009) postulate that discrimination is weaker against educated women because they are expected to show more labour market commitment and to invest more on the job. Hubbard (2011) on the other hand, claims that the gender difference in the college wage premium is actually a statistical fluke which is the result of censoring of the highest wages in the data. As these top wages are disproportionally earned by men, ignoring the fact that they are only recorded up to a maximum tends to depress the male college wage premium. After correcting for this 'topcoding bias' he finds that there has not been a significant gender difference in the college wage premium for at least a decade.

In the context of the model, with $\sigma = 1$ (log felicity) and $\bar{c} = \bar{f} = 0$ (no fixed costs or tuition fees) the labour market benefit is linearly increasing in the college wage premium. In this case only *relative* wages of educated and uneducated workers matter for the education threshold $\bar{\theta}^j$ and a higher college wage premium for one of the sexes immediately translates into a higher labour market benefit (given equal wage growth). However, with $\sigma \neq 1$ also the *absolute* wage levels matter, as the following proposition illustrates.

Proposition 4.1. *Assume there are no fixed costs, no tuition fees and equal wage growth for both sexes over time and for each level of education. If the college wage premium is the same for both sexes but females earn less then equally qualified males, then:*

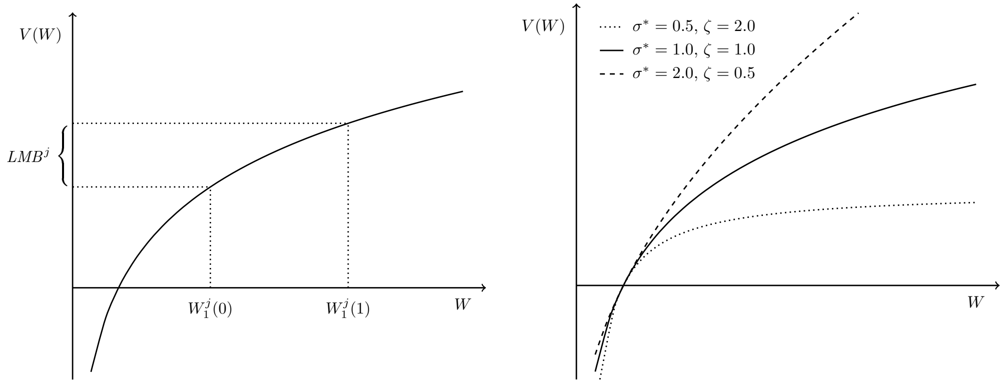
$$LMB^f \gtreqless LMB^m \quad \Leftrightarrow \quad \sigma \lesseqgtr 1.$$

Differences in the labour market benefit between the sexes depend positively on the common college wage premium.

Proof. See Appendix 4.A. □

Figure 4.4: Curvature of the utility function

(a) Labour market benefit (b) Different elasticities



Hence, even with a common college wage premium there might be a gender difference in the labour market benefit of education which is solely attributable to the curvature of the utility function. To understand this, note that under the assumption of perfect capital markets the utility level of a lifelong single is an increasing function of individual wealth, say $\mathcal{L}_1^j(E) = V\big(W_1^j(E)\big)$. The labour market benefit is defined as the difference in lifetime utility with and without education for a lifelong single, see Figure 4.4(a). By a first-order approximation it can be written as:

$$\begin{aligned} LMB^j &\approx V'\big(W_1^j(0)\big)\Big[W_1^j(1) - W_1^j(0)\Big] \\ &= V'\big(H_1^j(0) - \delta\bar{c}\big)H_1^j(0)\left[\frac{H_1^j(1)}{H_1^j(0)} - \frac{\bar{f}}{H_1^j(0)} - 1\right], \end{aligned} \tag{4.32}$$

where δ is the cumulative interest discount factor:

$$\delta \equiv 1 + \frac{1}{1+r} + \frac{1}{(1+r)^2}. \tag{4.33}$$

Assuming equal wage growth for both sexes, human wealth is proportional to the wage by a factor that is gender independent, see (4.31). Therefore, if the college wage premium is the same for men and women then so is the ratio of human wealth with and without education. By taking the derivative of (4.32) with respect to $H_1^j(0)$ while keeping $H_1^j(1)/H_1^j(0)$ fixed we obtain:

$$\begin{aligned} \frac{\partial LMB^j}{\partial H_1^j(0)} = V'\big(W_1^j(0)\big)\Bigg\{&\left[1 - \zeta\big(W_1^j(0)\big)\frac{H_1^j(0)}{H_1^j(0) - \delta\bar{c}}\right]\frac{H_1^j(1) - \bar{f} - H_1^j(0)}{H_1^j(0)} \\ &+ \frac{\bar{f}}{H_1^j(0)}\Bigg\}, \end{aligned} \tag{4.34}$$

where ζ is the elasticity of the marginal utility of wealth or the degree of relative risk aversion:

$$\zeta(W) = -\frac{V''(W)W}{V'(W)}. \tag{4.35}$$

Under the assumption that the utility function is of the isoelastic form it follows from (4.29) that $\zeta = 1/\sigma^*$ and therefore independent of wealth. Figure 4.4(b) depicts the utility function for several values of ζ in order to illustrate the difference in curvature. The higher is ζ, the faster marginal utility declines as wealth increases. If $\bar{c} = \bar{f} = 0$ as in Proposition 4.1 then the sign of the derivative in (4.34) depends solely on whether ζ is greater or smaller than unity. Estimates based on US consumption data indicate that $0 < \sigma^* < 1$ such that $\zeta > 1$, see Attanasio and Weber (1995). In the presence of a gender wage gap this implies that women, for whom $w^j(0)$ and thereby $H_1^j(0)$ is lower, have a higher labour market benefit of education than men. This difference is greater the larger is the common college wage premium. If $\bar{c} > 0$ then the result also holds with $\sigma = 1$ (such that $\sigma^* = \zeta = 1$), as summarized in Proposition 4.2.

Proposition 4.2. *Let $0 < \sigma \leq 1$. Assume there are positive fixed costs, no tuition fees and equal wage growth for both sexes over time and for each level of education. If the college wage premium is the same for both sexes but females earn less then equally qualified males, then $LMB^f > LMB^m$.*

Proof. See Appendix 4.A. □

This proposition underlines the important role of fixed costs in the model. As these costs are the same for every individual, irrespective of gender or education level, they weigh heaviest on those that receive the lowest wages. These are likely to be uneducated women. For them education offers a possibility to escape poverty, in line with the insights from the empirical work of DiPrete and Buchmann (2006).[7] With $\sigma > 1$ the above result might still be valid but not necessarily for all parameter values. The presence of a tuition fee mitigates the result, as it is also the same for both sexes but does not have to be paid by those who choose to remain uneducated.

Taken together Propositions 4.1 and 4.2 show that, in the context of this model, the labour market benefit of education can be greater for women than for men even when they do not have a higher college wage premium (the evidence for which is mixed).

A final potential source of gender difference in the labour market benefit is the relative wage growth of educated versus uneducated workers over the life cycle. However, to the best of our knowledge evidence for this is absent and it is also not obvious which gender would be favoured. For example, on the one hand it could be argued that educated women start at a lower level of wages than men but have a tendency to 'catch up' after they show labour market commitment. Alternatively it might be the case that they run into 'glass ceilings', preventing them from attaining the highest-paid jobs. It gets even more complicated if wage growth captures returns to experience in proportion to hours worked, because then the child care allocation at home matters for the relative experience build-up of husband and wife and wage growth becomes endogenous. In the remainder of this chapter we will simplify matters by assuming that wages are constant over the life cycle.

[7] If leisure would be included in the utility function then assuming $\bar{c} > 0$ gives two counterfactual results, namely (i) that single women work more than single men and (ii) that uneducated singles work more than educated singles. The first could be remedied by allowing single women to have children but not single men, the second by introducing a social security system that pays lump-sum transfers to uneducated individuals.

4.4.2 The marriage market distortion

If wages are taken as given then finding potential gender differences in the labour market benefit of education is relatively straightforward. The marriage market distortion is much more complex to analyze, as it depends on the matching probabilities which are themselves determined in the overall marriage market equilibrium. By keeping the probabilities constant across the sexes we derive some partial equilibrium insights.

Consider the simplest possible case with log felicity, no children, no fixed costs and no savings in period 1. Then the level of utility during marriage is increasing in the log of household wealth. The difference in utility gain can be written as:

$$\begin{aligned}\Delta UG^j = & \frac{1}{1+\rho}\frac{1}{\Gamma_1(1)}\Bigg\{\pi^{-j|j}(1|0)\ln\left(1+\frac{H_2^j(1)-H_2^j(0)}{H_2^j(0)+H_2^{-j}(1)}\right)\\ & +\pi^{-j|j}(0|0)\ln\left(1+\frac{H_2^j(1)-H_2^j(0)}{H_2^j(0)+H_2^{-j}(0)}\right)-\ln\left(1+\frac{H_2^j(1)-H_2^j(0)}{H_2^j(0)}\right)\Bigg\}. \quad (4.36)\end{aligned}$$

The first two terms capture the expected return to education when married, which is compared to the corresponding return in the single state. Note that the married individual's allocated share of total wealth drops out of the expression with log felicity as it is independent of the education level of the spouse. It is clear that under these assumptions $\Delta UG^j < 0$. The *absolute* increase in human wealth that comes from an education $H_2^j(1) - H_2^j(0)$ does not depend on whether a person ends up being married or single from period 2 onward, yet if there is already some spousal wealth $H_2^{-j}(E^{-j})$ then this yields *relatively* less additional utility. In other words, with diminishing marginal utility of wealth an educated person has less to gain by being married than an uneducated individual. If the college wage premium is the same for both sexes but women earn less than equally qualified men then $\Delta UG^f < \Delta UG^m$ because (i) a woman has more to gain from an education in relative terms than a man if she remains single and (ii) a woman can expect to marry a richer spouse, which provides her with fewer incentives to accumulate wealth.[8]

Concerning the part of the marriage market distortion attributable to matching

[8] Write:

$$H_2^j(1) - H_2^j(0) = w^j(0)\left[\frac{w^j(1)}{w^j(0)} - 1\right]\left[1 + \frac{1-\bar{R}}{1+r}\right].$$

Then if $w^f(1)/w^f(0) = w^m(1)/w^m(0)$ but $w^f(0) < w^m(0)$ it follows that:

$$H_2^f(1) - H_2^f(0) < H_2^m(1) - H_2^m(0) \quad \text{and} \quad H_2^f(1) + H_2^m(0) < H_2^f(0) + H_2^m(1).$$

probabilities it is exactly the other way around. Since wages for women are lower they have more to gain than men by marrying an educated spouse compared to an uneducated one:

$$\Delta MP^j = \frac{1}{1+\rho}\frac{1}{\Gamma_1(1)}\left[\pi^{-j|j}(0|0)+\pi^{-j|j}(1|1)-1\right]\ln\left(1+\frac{H_2^{-j}(1)-H_2^{-j}(0)}{H_2^j(1)+H_2^{-j}(0)}\right). \quad (4.37)$$

These insights are not straightforward to generalize to the case with $\sigma \neq 1$, as then the sharing rule for wealth also matters. With endogenous fertility matters get even more complicated, because a college education not only implies a greater amount of human wealth but also a higher opportunity cost of time spent on child care. Biological differences between men and women then play an important role. The change in the cost of a child if a woman decides to become educated can be written as:

$$\Upsilon^b\big(1,E^m\big)-\Upsilon^b\big(0,E^m\big)=N^b\Big[\omega\big(1,E^m\big)-\omega\big(0,E^m\big)\Big]+T^b\Big[w_3^f(1)-w_3^f(0)\Big]. \quad (4.38)$$

For men the expression is similar but the increase in the childbearing cost component is not present. This implies that in this model, all other gender differences aside, educated women are less desirable partners than educated men.

Overall, it is likely that $MMD^f < MMD^m$ so that marriage market distortion lowers the education threshold of women relative to men. There is one notable exception. Under the restrictive assumptions that felicity is linear in consumption, fertility is exogenous and there are no savings in period 1, the marriage market distortion reduces to $MMD^j = \Delta MP^j$ which might be higher for women.[9]

4.4.3 Numerical results

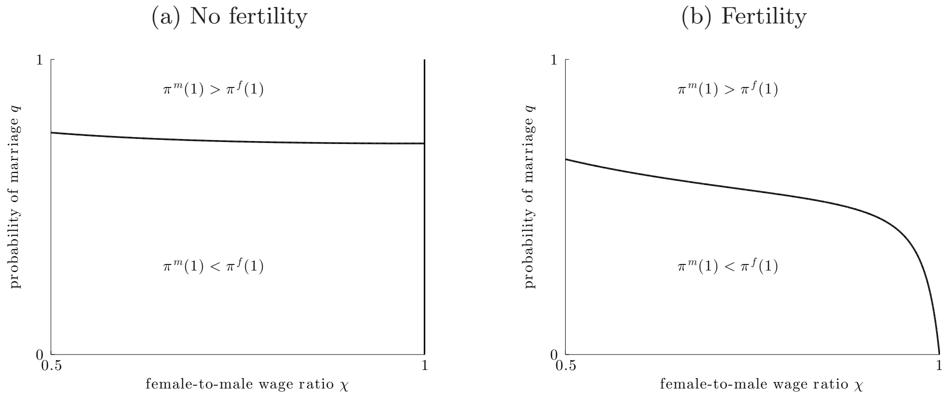
In order to see how gender differences in the benefit of a college education translate into differences in educational choices of men and women we solve for the marriage market equilibrium numerically (computational details are given in Appendix 4.E). The focus here is on the sign of $\pi^m(1)-\pi^f(1)$, the actual levels are not of much interest until the next section.[10]

We restrict attention to the case that both the utility cost distribution and the college wage premium are the same for men and women. Wage levels might differ and the female-to-male wage ratio is denoted by $\chi \leq 1$. Figure 4.5 shows the interaction between

[9]Incidentally, these are the type of assumptions often made in models with endogenous matching.
[10]The parameter values used for this illustrative exercise are similar to the ones described in Section 4.5.2 below.

this gender wage gap and the probability of marriage q in determining relative education frequencies. In both panels the solid line depicts combinations of q and χ that yield a symmetric equilibrium with $\pi^m(1) = \pi^f(1)$. Consider first panel (a), where it is assumed that there are no children and no fixed costs. In the absence of a gender wage gap men and women are then exactly the same. Regardless of the probability of marriage the education frequencies will be equal, which is why there is a vertical line at $\chi = 1$. Under the assumption that $\sigma < 1$ the labour market benefit of education for women is greater than that for men as long as $\chi < 1$ by Proposition 4.1. This explains why $\pi^f(1) > \pi^m(1)$ if the probability of marriage is low. On the other hand, if the probability of marriage is high (above the solid horizontal line) then educated men outnumber educated women. In this case a man knows that he is likely to end up marrying a wife who earns less than he does, which gives him an incentive to invest in his own education in order to generate a higher level of household income. For a woman it is the other way around, she is almost certain to marry a richer husband.

Figure 4.5: Relative education frequencies



Notes: The solid line shows the combinations of the probability of marriage q and the female-to-male wage ratio χ for which the equilibrium is symmetric ($\pi^m(1) = \pi^f(1)$). In panel (a) felicity depends only on consumption and there are no fixed costs. In panel (b) felicity depends on both consumption and the number of children, fixed costs are positive and there is a time cost of child birth for the mother. In both cases we assume that $\sigma < 1$.

The horizontal line of demarkation between the two types of equilibria is downward sloping because a higher value of χ means that the labour market benefit of education for women is closer to that of men. The relative education frequencies will then switch sign for a lower value of q. The line shifts if one of the model parameters other than q or χ changes. For example, if the intertemporal substitution elasticity σ decreases or the fixed cost $\bar{c}$ increases then the return to education for women tends to rise relative to that of men (in line with Proposition 4.1 and 4.2) and the line shifts up. An increase in the degree of marital sorting λ has a similar effect, as it implies that the part of the marriage market distortion due to matching probabilities goes up for women. If the tuition fee $\bar{f}$ is positive instead of zero then the line shifts down but its slope increases. Since $\sigma < 1$ an increase in the common college wage premium favours women more than men (Proposition 4.1) and the line goes up. An overall increase in the level of wages does not change the position of the line but decreases the education frequencies of both men and women.[11]

Consider now panel (b), which includes fertility. Even without a gender wage gap men and women are not the same as there is a positive time cost of child birth for mothers. With $\chi = 1$ there will only be a symmetric equilibrium if $q = 0$, as in the absence of marriage children do not play a role. If the preference for consumption relative to children ε goes down then the line rotates in a counter-clockwise direction around the point $(1, 0)$.

4.5 The college gender gap reversal

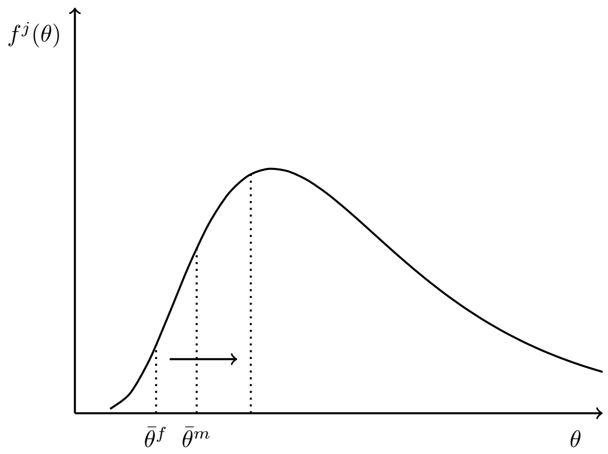
In this section we will use the model in order to understand why women have overtaken men in terms of educational attainment. The main argument is as follows. Suppose that the distribution of utility costs of education is the same among men as it is among women. For illustrative purposes we have drawn a unimodal probability density curve in Figure 4.6. Recall that the fraction of individuals of a given gender that obtain education is represented by the area to the left of the threshold level $\bar{\theta}^j$. Initially the benefits are lower for women, that is $\bar{\theta}^f$ is to the left of $\bar{\theta}^m$. Over time the threshold levels shift in such a way that $\bar{\theta}^f > \bar{\theta}^m$. For this explanation to be valid there must be (i) gender differences in the benefit of education and (ii) a change in the relative benefits for men and women over time. The aim here is to show that there exists a reasonable set of parameter values under which the model indeed generates a college gender gap reversal.

4.5.1 Data

We use census data from the Integrated Public Use Microdata Series (IPUMS) for the United States (Ruggles et al. (2010)), see Appendix 4.D for a description. We pick two

[11]Higher wages imply higher consumption and lower marginal utility. The education threshold, which is defined as the difference between utility levels of educated and uneducated individuals, will be smaller. With an unchanged distribution of utility costs this means that less individuals choose to get an education.

Figure 4.6: College gender gap reversal with equal cost distributions



cohorts, one born in 1950 and one born in 1970. When they are 40 years of age (in 1990 and 2010, respectively) we obtain some key statistics. The first two lines of Table 4.2 give the proportion of women and men who have obtained at least a Bachelor's degree for each cohort. While for the 1950 cohort the fraction of educated men exceeded that of educated women, by 1970 this inequality had been reversed. For men the graduation rate even decreased a little.

Table 4.2: Graduation rates and matching parameters

	Cohort 1950	Cohort 1970
$\pi^f(1)$	0.252	0.337
$\pi^m(1)$	0.304	0.291
q	0.905	0.830
λ	0.563	0.564

Source: Integrated Public Use Microdata Series (IPUMS) for 1990 and 2010.

Table 4.3 shows how men and women are matched in the data. Unfortunately it is not possible to retrieve information about the characteristics of the former spouse for individuals who are separated or divorced, so the matching pattern is solely based on those who are (still) married at age 40 or are cohabiting.[12] The second row and second column do not add up exactly to $\pi^m(1)$ and $\pi^f(1)$, respectively, because in reality

[12] We define a cohabiting couple as one in which a 'household head' lives together with an 'unmarried partner' with a maximum age difference of 10 years. See Appendix 4.D.

Table 4.3: Matching patterns

(a) Cohort 1950

		E^f 0	E^f 1	
E^m	0	0.596 [2.310]	0.075 [1.779]	0.671
	1	0.148 [2.082]	0.181 [1.888]	0.329
		0.744	0.256	1

(b) Cohort 1970

		E^f 0	E^f 1	
E^m	0	0.519	0.129	0.648
	1	0.094	0.258	0.352
		0.613	0.387	1

Source: Integrated Public Use Microdata Series (IPUMS) for 1990 and 2010.
Notes: Average number of children in square brackets for the cohort of 1950.

educated people are more likely to get and stay married and because spouses need not be of the exact same age. From these cross tables we compute the index of marital sorting λ, see Table 4.2. It is greater than zero, indicating that there is positive assortative matching in education. Over time λ has remained virtually constant. Other studies have claimed that marital sorting has become stronger (see for example Fernández et al. (2005)), yet this conclusion is often based on the correlation coefficient between male and female education which indeed increased from 0.472 for the 1950 cohort to 0.523 for individuals born in 1970.[13] The rise in correlation can be entirely explained by the increased supply of educated women $\pi^f(1)$ without any change in the underlying preferences for assortative matching λ (a similar conclusion is reached by Chiappori et al. (2011) in the context of a different model). We calculate the probability of marriage q as one minus the fraction of people who are classified as 'never married/single' and are not cohabiting. Table 4.2 shows that this probability has declined over time.

As the year 1990 is the last one for which there are records of completed fertility per woman in the census data, we can calculate the average number of children for each type of household only for the cohort born in 1950.[14] These are given in square brackets in

[13]The correlation coefficient is given by:

$$cor = \lambda \frac{\min\{\pi^f(1), \pi^m(1)\} - \pi^f(1)\pi^m(1)}{\sqrt{\pi^f(1)[1-\pi^f(1)]}\sqrt{\pi^m(1)[1-\pi^m(1)]}}.$$

[14]After 1990 there is a variable that gives the number of children currently living in the household, but this one is much less useful for our purposes.

Table 4.3. Completed fertility is highest for uneducated women married to uneducated men and lowest for educated women married to uneducated men.

4.5.2 Parameterization

We assume that the length of each period is 18 years. Obtaining a college degree requires a fraction $\bar{e} = 0.25$ of period 1, while the retirement phase is a share $\bar{R} = 0.3$ of the final period. The interest rate is set at 4% and the impatience discount factor at 2.5% per annum which translates into $r = (1.04)^{18} - 1$ and $\rho = (1.025)^{18} - 1$. Wages remain constant over the life-cycle so that $\eta_t^j(E) = 0$. The wage rate for an uneducated male is normalized to $w^m(0) = 1$. Spouses have equal Pareto weights $\alpha = 1 - \alpha = 0.5$ in the household welfare function which implies that the share of wealth allocated to the wife is $\beta(\alpha, \sigma) = 0.5$ irrespective of the value of σ. We assume that child care requires $N^b = 0.25$ per child. If performed by one parent this would correspond to the loss of a quarter of each working day on average during 18 years. With a substitution elasticity between parents of $\xi = 4$ the actual time burden will be less.[15] The cost of child birth for the mother is set at $T^b = 0.02$ or about 4 months with no wages. The equivalence scale of two adults is $Q^a = 1.7$ while each additional child requires $Q^b = 0.5$ (in line with the so-called 'Oxford scale'). We abstract from tuition fees ($\bar{f} = 0$) and assume that fixed costs are constant at $\bar{c} = 0.1$ (or 10% of $w^m(0)$).

The remaining parameters are set in such a way as to match some of the key statistics for the 1950 cohort as described in Section 4.5.1 above. The marital sorting index is $\lambda = 0.563$ and the probability of marriage $q = 0.905$ as in the data. Uneducated women earn $w^f(0) = 0.75$ so that the male-to-female wage ratio is $1/\chi \approx 1.33$, which is in line with the average findings for the gender wage gap in the United States as reported in the meta-analysis of Jarrell and Stanley (2004). The college wage premium is equal for both sexes and set to a value of $\ln(w^j(1)/w^j(0)) = 0.47$, within the range estimated by Hubbard (2011). With an intertemporal substitution elasticity of $\sigma = 0.5$ this configuration satisfies the premises of Proposition 4.2 and the labour market benefit of education is greater for women than for men. The value of the preference parameter $\varepsilon = 0.671$ ensures that the number of children born to parents who are both uneducated equals the average of 2.310 reported in Table 4.3. The intertemporal substitution elasticity for consumption then equals $\sigma^* = 0.598$ and the elasticity of the marginal utility of wealth is $\zeta = 1.672$. After calculating the threshold values for education (with

[15] As parents are imperfect substitutes the sum of father time and mother time will be less than $N^b b$ as long as the care burden is allocated between them.

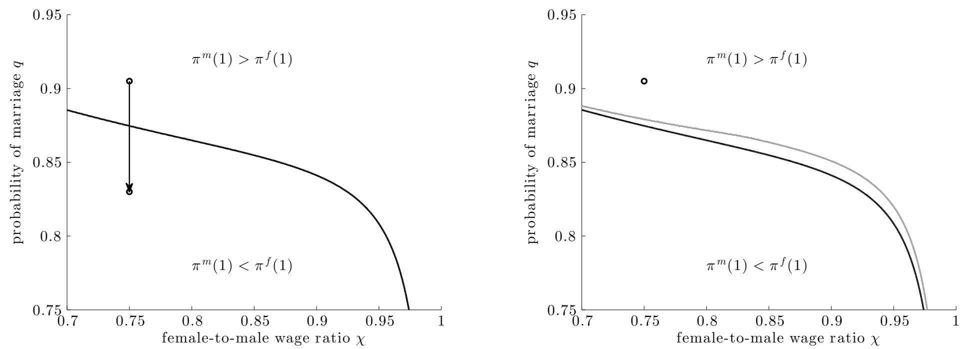
$\bar{\theta}^f < \bar{\theta}^m$) we choose the parameters of a lognormal utility cost distribution to ensure that the cumulative density equals $\pi^f(1) = 0.252$ at $\bar{\theta}^f$ and $\pi^m(1) = 0.304$ at $\bar{\theta}^m$.

4.5.3 Numerical results

For the chosen parameter values, the solid line in Figure 4.7 depicts combinations of the probability of marriage q and the female-to-male wage ratio χ that yield a symmetric equilibrium with $\pi^m(1) = \pi^f(1)$ (as in Section 4.4.3). The initial equilibrium that represents the 1950 cohort is indicated with a dot at the point where $\chi = 0.75$ and $q = 0.905$. The model will generate a college gender gap reversal if the equilibrium for the 1970 cohort is below the solid line in Figure 4.7 instead of above it. There are two conditions under which this may occur, which need not exclude each other. First, the dot may shift following a change in the probability of marriage q and/or the female-to-male wage ratio χ. From Table 4.2 it is clear that q has decreased from 0.905 for the 1950 cohort to 0.830 for the generation born in 1970. Given the current parameterization this would be sufficient to let women overtake men in educational attainment, see the arrow in Figure 4.7(a). The reversal would still occur if there is a moderate decrease in the gender wage gap (an increase in χ). The second possibility is that the line itself shifts up. This would happen, for example, if the common college wage premium increases over time or fixed cost $\bar{c}$ rises (see Section 4.4.3). However, even an increase in the college wage premium to about 0.6 does not result in a sufficient upward shift of the line to obtain a reversal, see Figure 4.7(b).

Figure 4.7: College gender gap reversal

(a) Decrease in probability of marriage (b) Increase in college wage premium



For the case that there is only a decrease in q the numerical decomposition of the benefit of education is given in Table 4.4. The bottom line gives the education frequencies $\pi^j(1)$ for each gender and cohort. The row above reports the corresponding threshold level $\bar{\theta}^j$. This number by itself is not very informative as it depends on the scaling of wages.[16] As only the relative magnitudes matter we have chosen to rescale the numbers in such a way that $\bar{\theta}^f = 100$ for the 1950 cohort. The threshold can be decomposed into the labour market benefit of education LMB^j and the marriage market distortion MMD^j (the two lines above). The latter is itself made up of three separate components, namely the parts due to differences in the utility gain ΔUG^j, matching probabilities ΔMP^j and savings behaviour ΔSB^j (recall Section 4.2.4).

Wages remain constant across cohorts, which means that the labour market benefit is also unchanged. It is significantly higher for women than for men. As women earn lower wages and expect to marry a more wealthy spouse they have less incentive to add to household income by obtaining an education (given diminishing marginal utility of wealth) as is evidenced by the large negative number reported for ΔUG^j. On the other hand the part of the marriage market distortion attributable to matching probabilities is greater for women, in line with the insights from Section 4.4.2. Initially the negative effect of marriage expectations on the education threshold for women outweighs the female advantage in the labour market benefit, but as q decreases over time the inequality is reversed. The percentage of women that obtain education increases from 25.2% to 31.3% while for men it decreases from 30.4% to 28.1%. As a consequence, there is a reversal of the college gender gap. If the drop in the marriage probability is accompanied by a small increase in the common college wage premium then both education frequencies go up, which brings them close to the values for the 1970 cohort reported in Table 4.2.

Robustness

There are four comments to be made with respect to the robustness of the numerical results obtained above.

First of all, the ability of the model to generate a college gender gap reversal depends critically on the choice of parameters. For example, with a higher intertemporal substitution elasticity σ or a lower fixed cost $\bar{c}$ the difference in labour market benefit between men and women is smaller and the drop in q alone might not be sufficient

[16] For example, if all wages are multiplied by 100 then consumption levels change accordingly. Marginal utility is then lower and the threshold level, which is defined as the difference between utility levels of educated and uneducated individuals, will be smaller.

Table 4.4: Decomposition of the benefit of education

		Cohort 1950		Cohort 1970	
		female	male	female	male
	$q\Delta UG^j$	−75.434	−8.474	−59.848	−10.894
+	$q\Delta MP^j$	16.043	3.574	13.246	3.899
+	ΔSB^j	19.972	0.563	12.181	0.764
=	MMD^j	−39.419	−4.337	−34.421	−6.231
+	LMB^j	139.419	108.625	139.419	108.625
=	$\bar{\theta}^j$	100.000	104.288	104.998	102.394
	$\pi^j(1)$	0.252	0.304	0.313	0.281

to induce women to catch up. In addition, the choice of parameters for the lognormal distribution affects the extent to which education choices respond to changes in benefits. If the threshold levels are close together for the 1950 cohort, then the standard deviation will have to be low in order to be consistent with a gap of 0.052 in college graduation rates. Consequently, the change in education frequencies as a result of threshold shifts will be greater.

Secondly, many factors are taken as given in the model that need not be independent. For example, an increase in female wages might make them more 'picky' in choosing a partner and therefore less prone to marry, see Caucutt et al. (2002). In addition they are more likely to 'choose love over money' which would decrease the degree of sorting, as in Fernández et al. (2005). If wages depend on work experience then the increase in female labour supply following a drop in marriage rates might lower the gender wage gap.

Third, we have abstracted from the possibility of divorce. Other studies, such as Guvenen and Rendall (2013) and Fernández and Wong (2011) point to the increase in divorce rates over the last decades as a potential explanation for the surge in female college enrolment. Divorce tends to be more costly for women than for men as custodial arrangements are usually such that the children reside with their mother. This may incentivize risk-averse women to invest in their financial independence. In this chapter we have chosen not to include divorce risk in order to able to get a closed-form solution for the value function of a married individual. However, a similar incentive mechanism is still at work. In the presence of a gender wage gap it is more costly for women to be single than for men because they have to cover the fixed cost with a lower level of wages. A drop in the probability of marriage then yields a similar response to an increase in the rate of marital dissolution: the ratio of female to male college graduates goes up.

Finally, the fully specified example used here makes a few strong assumptions about the behaviour of married couples that can be relaxed. In Appendix 4.B the 'unitary' household is replaced by one in which spouses bargain over allocations, while Appendix 4.C shows what happens if individuals care for the welfare of their partner. In both cases it is possible to find reasonable parameter values under which the model generates a college gender gap reversal if there is a decrease in the probability of marriage.

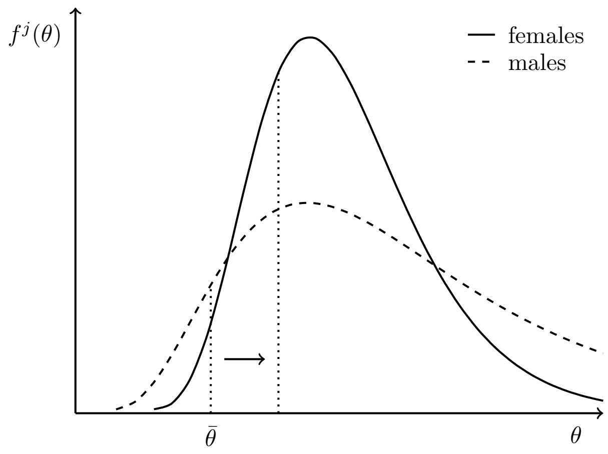
The role of costs

So far we have ignored possible gender differences in the utility cost of education. This cost is probably inversely related to a person's level of cognitive skills (such as IQ) and non-cognitive skills (for example self-motivation and discipline). Both Becker et al. (2010) and Jacob (2002) provide evidence to support the claim that whereas there are only minor gender differences in cognitive skills, women have on average better non-cognitive skills than men and among them there is less variability.[17] If so, then there is an alternative explanation for the college gender gap reversal which relies exclusively on cost differences, see Becker et al. (2010). Suppose that men and women have similar benefits from a college education but that the distribution of psychic costs is gender-specific as in Figure 4.8. In particular, men have both a higher mean and a greater standard deviation. Initially the threshold level is such that the mass of men to the left of it is greater than the share of women. Over time the benefits of a college education increase and the threshold shifts to the right. The fraction of college educated women grows faster than the fraction of college educated men and the college gender gap is reversed.

Becker et al. (2010) focus on gender differences in costs as they argue that nowadays the benefits of a college education are the same for men and women, if not still higher for men. Part of this argument is based on the observation that the college wage premium has been similar in recent years, as found by Hubbard (2011). However, we have shown that despite a common college wage premium it is still possible for women to have a

[17]This leaves open the question, however, of whether this is an innate biological difference between the sexes, the result of conscientious investment decisions, or the by-product of a culture that rewards and condones different types of behaviour in men and women.

Figure 4.8: College gender gap reversal with unequal cost distributions



higher (labour market) return to education. Hence there is a role for gender differences in the benefit of education in explaining the college gender gap reversal, which could be complemented by differences in costs.

4.6 Conclusion

In this chapter we have shown under which conditions a basic life-cycle model in which rational and forward-looking individuals make decisions about education can generate a college gender gap reversal. The analysis has yielded two main contributions. First, we have proved analytically that the labour market benefit of education for women can be higher than for men if there is a realistic amount of curvature in the utility function or there are fixed costs. Intuitively this is because women earn lower wages and with strongly diminishing marginal utility of wealth they have more to gain by increasing their lifetime earnings through obtaining a college degree. This result does *not* rely on a higher college wage premium for women, the evidence for which is mixed. The distortions introduced through the marriage market tend to depress the overall benefit of education for women relative to men as they expect to marry a more wealthy spouse and to work less when a child is born.

Second, after parameterizing the model using US census data we have showed which changes in the economic and social environment can lead to a reversal in college graduation rates. A drop in the probability of marriage of the magnitude observed in the data would be sufficient. In the new equilibrium risk-averse women invest more in education than men because being single is more costly for them. Other factors that lead to a rise in the number of educated women relative to educated men, such as an increase in the common college wage premium, are quantitatively not strong enough to reverse the gap.

Appendix

4.A Proofs

From (4.30) it follows that the labour market benefit of education can be written as:

$$LMB^j = \begin{cases} \varepsilon\Gamma_1(\sigma)^{-1/\sigma^*} \dfrac{W_1^j(0)^{1-1/\sigma^*}}{1-1/\sigma^*} \left[\left(\dfrac{W_1^j(1)}{W_1^j(0)} \right)^{1-1/\sigma^*} - 1 \right] & \text{if } \sigma \neq 1 \\ \dfrac{\varepsilon}{\Gamma_1(1)} \ln \left(\dfrac{W_1^j(1)}{W_1^j(0)} \right) & \text{if } \sigma = 1 \end{cases}$$

where $1 - 1/\sigma^* = \varepsilon(1 - 1/\sigma)$. Total wealth $W_1^j(E)$ consists of human wealth $H_1^j(E)$ net of tuition fees $\bar{f}E$ and the present value of fixed costs $\delta\bar{c}$, where:

$$H_1^j(E) = w^j(E) \left[1 - \bar{e}E^j + \frac{1 + \eta_2^j(E)}{1+r} + \frac{\left[1 + \eta_3^j(E)\right]\left[1 + \eta_2^j(E)\right]\left[1 - \bar{R}\right]}{(1+r)^2} \right].$$

Proposition 4.1. *Assume there are no fixed costs, no tuition fees and equal wage growth for both sexes over time and for each level of education. If the college wage premium is the same for both sexes but females earn less then equally qualified males, then:*

$$LMB^f \gtreqless LMB^m \quad if \quad \sigma \lesseqgtr 1.$$

Differences in the labour market benefit between the sexes depend positively on the common college wage premium.

Proof. According to the premises of the proposition:

$$\bar{c} = 0, \quad \bar{f} = 0, \quad \eta_t^f(1) = \eta_t^m(1), \quad \eta_t^f(0) = \eta_t^m(0), \quad \frac{w^f(1)}{w^f(0)} = \frac{w^m(1)}{w^m(0)}.$$

Under these assumptions $W_1^j(E) = H_1^j(E)$. If $w^j(1)/w^j(0)$ is the same for both sexes then so is $H_1^j(1)/H_1^j(0)$. Taking the derivative of LMB^j with respect to $H_1^j(0)$ while keeping $H_1^j(1)/H_1^j(0)$ constant gives:

$$\frac{\partial LMB^j}{\partial H_1^j(0)} = \varepsilon\Gamma_1(\sigma)^{-1/\sigma^*} H_1^j(0)^{-1/\sigma^*}\left[\left(\frac{H_1^j(1)}{H_1^j(0)}\right)^{1-1/\sigma^*} - 1\right] \lesseqqgtr 0 \quad \Leftrightarrow \quad \sigma \lesseqqgtr 1,$$

since $H_1^j(1)/H_1^j(0) > 1$. The result follows because $H_1^f(0) < H_1^m(0)$ if $w^f(0) < w^m(0)$. Note:

$$\frac{\partial^2 LMB^j}{\partial H_1^j(0)\partial\left(\frac{H_1^j(1)}{H_1^j(0)}\right)} = (1 - 1/\sigma^*)\varepsilon\Gamma_1(\sigma)^{-1/\sigma^*} H_1^j(1)^{-1/\sigma^*} \lesseqqgtr 0 \quad \Leftrightarrow \quad \sigma \lesseqqgtr 1.$$

□

Proposition 4.2. *Let $0 < \sigma \leq 1$. Assume there are positive fixed costs, no tuition fees and equal wage growth for both sexes over time and for each level of education. If the college wage premium is the same for both sexes but females earn less then equally qualified males, then $LMB^f > LMB^m$.*

Proof. According to the premises of the proposition:

$$\bar{c} > 0, \quad \bar{f} = 0, \quad \eta_t^f(1) = \eta_t^m(1), \quad \eta_t^f(0) = \eta_t^m(0), \quad \frac{w^f(1)}{w^f(0)} = \frac{w^m(1)}{w^m(0)}.$$

Under these assumptions $W_1^j(E) = H_1^j(E) - \delta\bar{c}$. If $w^j(1)/w^j(0)$ is the same for both sexes then so is $H_1^j(1)/H_1^j(0)$. Taking the derivative of LMB^j with respect to $H_1^j(0)$ while keeping $H_1^j(1)/H_1^j(0)$ constant gives:

$$\begin{aligned}\frac{\partial LMB^j}{\partial H_1^j(0)} &= \varepsilon\Gamma_1(\sigma)^{-1/\sigma^*}\left[H_1^j(0) - \delta\bar{c}\right]^{-1/\sigma^*}\left\{\left[\left(\frac{H_1^j(1) - \delta\bar{c}}{H_1^j(0) - \delta\bar{c}}\right)^{1-1/\sigma^*} - 1\right]\right.\\ &\quad \left. - \left(\frac{H_1^j(1) - \delta\bar{c}}{H_1^j(0) - \delta\bar{c}}\right)^{-1/\sigma^*}\frac{\delta\bar{c}}{H_1^j(0) - \delta\bar{c}}\left[\frac{H_1^j(1)}{H_1^j(0)} - 1\right]\right\} < 0,\end{aligned}$$

since $H_1^j(1)/H_1^j(0) > 1$ and $\sigma \leq 1$ such that $\sigma^* \leq 1$. The result follows because $H_1^f(0) < H_1^m(0)$ if $w^f(0) < w^m(0)$.

□

4.B Household bargaining

Suppose that a married couple does not act as a unitary whole. Instead future husbands and wives bargain cooperatively over household allocations just before they get married but fully commit to them afterwards. Provided that the bargaining outcome is Pareto efficient the household still acts as though it maximizes a weighted average of individual utility functions as in (4.19) but now the weight attached to the wife α is endogenous. Let $\hat{\mathcal{M}}_2^j\big(E^j, E^{-j}, a_2\big|\alpha\big)$ denote the value function of a married individual conditional on α, which is the same as (4.24) above.

Previously it was possible to ignore any utility gain from being married other than that derived from the sharing of resources and children because it would cancel out in the decomposition of the education threshold (assuming it is independent of the education level of the spouse). In the context of bargaining, however, it influences how sensitive allocations are to changes in education and assets. From now on ν denotes the discounted flow of 'conjugal bliss' at the start of period 2.

A disadvantage of the bargaining approach is that the solutions will in large part be driven by the specification of threat points, the choice of which may not be obvious. Here we take the utility when single $\mathcal{S}_2^j(E, a_2)$ as specified in (4.16) as the outside option of each individual.[18] We focus on generalized Nash bargaining, in which case α is given by:

$$\alpha\big(E^f, E^m, a_2^f, a_2^m\big) = \underset{\alpha}{\operatorname{argmax}} \left\{ \Big[\hat{\mathcal{M}}_2^f\big(E^f, E^m, a_2^f + a_2^m\big|\alpha\big) + \nu - \mathcal{S}_2^f\big(E^f, a_2^f\big)\Big]^{\psi} \right.$$
$$\left. \times \Big[\hat{\mathcal{M}}_2^m\big(E^m, E^f, a_2^m + a_2^f\big|\alpha\big) + \nu - \mathcal{S}_2^m\big(E^m, a_2^m\big)\Big]^{1-\psi} \right\},$$

where the parameter $\psi \in [0, 1]$ captures the relative bargaining strength of the woman. We will restrict attention to the classical case with $\psi = 0.5$ so that the overall bargaining position of each individual is determined by his or her outside option. Note that α is a function of individual education levels and savings.

The value of being married is now defined as:

$$\mathcal{M}_2^j\big(E^j, E^{-j}, a_2^j, a_2^{-j}\big) = \hat{\mathcal{M}}_2^j\big(E^j, E^{-j}, a_2^j + a_2^{-j}\big|\alpha\big(E^f, E^m, a_2^f, a_2^m\big)\big) + \nu.$$

In this context, individuals have a strategic reason for obtaining education and

[18] Other possibilities include the non-cooperative household allocation or the utility derived from remaining single with the possibility of marriage to a different person (but this would require a different model). For more examples see Browning et al. (2014).

accumulating financial assets. A college degree or a greater stock of savings enhances their position within the household and thereby increases the share of wealth that they will extract. This allows for a further decomposition of the benefit of education. In particular, the part of the marriage market distortion attributable to differences in utility gain can now be written as:

$$\Delta UG^j = \Delta MU^j + \Delta PW^j.$$

The first term ΔMU^j is related to the marginal utility of wealth and is obtained by keeping the Pareto weights fixed when comparing one marital state with another. The second one ΔPW^j captures the strategic effect of changing Pareto weights. The remainder of the decomposition remains unchanged. An important difference, however, is that the part due to matching probabilities ΔMP^j might be negative: Having an educated spouse is now less desirable as this will deteriorate the own Pareto weight.

As women earn lower wages and therefore have a less favourable outside option, they have a greater incentive for strategic investment in education than men (see also Iyigun and Walsh (2007b)). In order to get a consistent parameterization with $\bar{\theta}^f < \bar{\theta}^m$ we increase the intertemporal substitution elasticity to $\sigma = 0.7$ (thereby narrowing the gap between men and women in the labour market benefit of education) and set $\nu = 3$. The model then still generates a college gender gap reversal following a drop in the probability of marriage, see Table 4.B.1 for a decomposition of the benefit of education in the two equilibria.

Table 4.B.1: Decomposition with household bargaining

		Cohort 1950		Cohort 1970	
		female	male	female	male
	$q\Delta MU^j$	−47.271	−11.570	−40.726	−11.718
+	$q\Delta PW^j$	6.783	4.346	5.966	3.995
=	$q\Delta UG^j$	−40.488	−7.224	−34.760	−7.723
+	$q\Delta MP^j$	13.179	1.725	11.772	1.986
+	ΔSB^j	4.995	0.380	3.448	0.351
=	MMD^j	−22.314	−5.119	−19.540	−5.386
+	LMB^j	122.314	107.491	122.314	107.491
=	$\bar{\theta}^j$	100.000	102.372	102.775	102.105
	$\pi^j(1)$	0.252	0.304	0.313	0.298

4.C Caring preferences

As a second extension we revert to the unitary model but now endogenize the level of conjugal bliss ν introduced in Appendix 4.B. Up to this point we have assumed that each individual's preferences are completely egotistic: he or she only cares about his or her private consumption and the own utility derived from the presence of children. It is easy to generalize this to the case of 'caring preferences' by assuming that after marriage the felicity function of spouse j becomes:

$$\bar{u}^j\left(c_t^j, c_t^{-j}, b\right) = u\left(c_t^j, b\right) + \nu^j u\left(c_t^{-j}, b\right), \quad 0 \leq \nu^j < 1.$$

In order to be able to meaningfully compare the welfare of a married person with that of a single (for whom $\nu^j = 0$) we need to ensure that $u\left(c_t^{-j}, b\right) \geq 0$. This is not necessarily the case for the chosen felicity function if $\sigma \leq 1$ but can be achieved by a suitable scaling of endowments, for example by multiplying all wages and the fixed cost by a factor 100.

The periodic household welfare function (4.19) can now be written as:

$$\begin{aligned} U\left(c_t^f, c_t^m, b\right) &= \alpha \bar{u}^f\left(c_t^f, c_t^m, b\right) + (1-\alpha)\bar{u}^m\left(c_t^m, c_t^f, b\right) \\ &= \left[\alpha + (1-\alpha)\nu^m\right] u\left(c_t^f, b\right) + \left[(1-\alpha) + \alpha\nu^f\right] u\left(c_t^m, b\right). \end{aligned}$$

From the structure of the welfare function it follows that the optimal allocations are similar to the ones with egotistic preferences, only the wife's Pareto weight α is replaced by:

$$\bar{\alpha} = \frac{\alpha + (1-\alpha)\nu^m}{1 + \alpha\nu^f + (1-\alpha)\nu^m}.$$

The more a person is cared for, the higher is his or her 'adjusted' Pareto weight. For a woman the corresponding value function (with $\sigma \neq 1$) is given by:

$$\begin{aligned} \mathcal{M}_2^f\left(E^f, E^m, a_2^f, a_2^m\right) = \frac{1}{\Gamma_2(1)} \Bigg\{ & \Gamma_2(1)\Gamma_2(\sigma)^{-1} \left[\bar{\varepsilon} \frac{\Gamma_2(\sigma)^{\varepsilon}}{\Upsilon^b\left(E^f, E^m\right)^{1-\varepsilon}} W_2\left(E^f, E^m, a_2\right)\right]^{1-1/\sigma} \\ & \times \frac{\beta(\bar{\alpha}, \sigma)^{\varepsilon(1-1/\sigma)} + \nu^f\left[1 - \beta(\bar{\alpha}, \sigma)\right]^{\varepsilon(1-1/\sigma)}}{1-1/\sigma} - \frac{(1+\nu^f)}{1-1/\sigma} \Bigg\}. \end{aligned}$$

The wife does not only derive utility from her own share of wealth $\beta(\bar{\alpha}, \sigma)$ but to a lesser degree also from that of her husband. The anticipated positive effect on the welfare of the spouse provides additional incentives for investment in education and saving in the first period. Again this allows for a further decomposition of the benefit of education. We write:

$$\Delta UG^j = \Delta MU^j + \nu^j \Delta CP^j.$$

where ΔMU^j is the difference in the own utility from consumption and children, while ΔCP^j captures that of the future spouse. The overall marriage market distortion is now more likely to be positive (in contrast to the benchmark case and the extension with household bargaining).

We repeat the parameterization procedure with $\nu^f = \nu^m = 0.1$. The results are reported in Table 4.C.1. The marriage market distortion is positive for men in the 1950 cohort but negative for women. The model still generates a college gender gap reversal following a drop in the probability of marriage but the education frequencies are somewhat closer together for the 1970 cohort.

Table 4.C.1: Decomposition with caring preferences

		Cohort 1950		Cohort 1970	
		female	male	female	male
	$q\Delta MU^j$	−71.812	−7.972	−57.201	−9.930
+	$q\nu^j \Delta CP^j$	1.870	4.098	1.740	3.515
=	$q\Delta UG^j$	−69.942	−3.874	−55.461	−6.415
+	$q\Delta MP^j$	16.794	3.945	14.617	4.282
+	ΔSB^j	17.927	0.177	10.794	0.362
=	MMD^j	−35.221	0.248	−30.050	−1.771
+	LMB^j	135.221	105.091	135.221	105.091
=	$\bar{\theta}^j$	100.000	105.339	105.171	103.320
	$\pi^j(1)$	0.252	0.304	0.302	0.284

4.D Data

We use data from the Integrated Public Use Microdata Series (IPUMS) for the United States (Ruggles et al. (2010)).

To create Figure 4.1(a) we take the default sample for every available year from 1970 up to and including 2011. We only select individuals who are 40 years of age. We create a dummy for college education which takes the value of 1 if a person has 4 years of college or more and 0 otherwise. (From 1990 onwards a more detailed education variable is available which explicitly includes the highest degree earned.) Then we calculate the proportion of college-educated individuals of each sex using the person weights present in the data.

In order to obtain the matching probabilities and marriage patterns reported in Table 4.2 and 4.3 we take the 1% sample for 1990 and 2010. We start by selecting individuals from age 30 up to and including age 50. We create a cohabitation dummy that takes the value of 1 if a household head lives together with an unmarried partner and 0 otherwise. For all married and cohabiting couples we make college dummies for the male and the female in the household. We then restrict the sample to those individuals who are 40 years of age (but whose partner might have any age between 30 and 50). Individuals of 40 years old that are not included in the final sample are those that live with a partner of the same sex and those that have a partner with whom the difference in age is more than 10 years. We calculate the probability of marriage q as one minus the proportion of individuals who have never been married and are not currently cohabiting. The degree of sorting λ can be obtained from:

$$\lambda = \frac{\pi(1,1) - \pi^f(1)\pi^m(1)}{\min\left\{\pi^f(1), \pi^m(1)\right\} - \pi^f(1)\pi^m(1)}.$$

4.E Computational details

The equilibrium of the model can be found by solving the following subproblems.

(1) *The value function of a single at the start of stage* 2

We assume that the solution to the optimization problem of a single at the start of stage 2 is interior and verify this ex-post. This implies that we have an analytical expression for all allocation choices and the corresponding value function for both sexes $j \in \{f, m\}$ and any set of state variables $\{E, a_2\}$.

(2) *The value function of a married individual at the start of stage* 2

We assume that the solution to the optimization problem of a couple at the start of stage 2 is interior and verify this ex-post. Given the Pareto weight α there is an analytical expression for all allocation choices and the corresponding value function of each spouse for any set of state variables $\left\{E^f, E^m, a_2^f, a_2^m\right\}$. If the weight is endogenous then we have to numerically find the value of α that maximizes the Nash bargaining objective function.

(3) *The equilibrium in savings conditional on education*

For any given set of education frequencies $\left\{\pi^f(1), \pi^m(1)\right\}$ we calculate the equilibrium choices of savings $\left\{\mathbf{a}_2^f(0), \mathbf{a}_2^f(1), \mathbf{a}_2^m(0), \mathbf{a}_2^m(1)\right\}$ numerically. Under the assumption of fixed Pareto weights this amounts to solving 4 first-order conditions in 4 endogenous variables using rootfinding techniques.

In the case of Nash bargaining it is more complicated. We set up a grid of feasible values for a_2 for any combination of gender and education level. Then we find the best response for a person of gender $j \in \{f, m\}$ with education level $E \in \{0, 1\}$ for any set of choices $\left\{\mathbf{a}_2^{-j}(0), \mathbf{a}_2^{-j}(1)\right\}$ made by individuals of the opposite gender with numerical optimization. We use spline interpolation and rootfinding techniques to find a consistent equilibrium.

(4) *The marriage market equilibrium*

Obtaining the marriage market equilibrium amounts to finding a fixed point. For any guess regarding the education frequencies $\left\{\pi^f(1), \pi^m(1)\right\}$ we calculate the equilibrium in savings and the corresponding values functions. This yields an education threshold $\bar{\theta}^j$, which provides a new guess $\pi^j(1) = F_\theta^j(\bar{\theta}^j)$. We iterate over the education frequencies until the solution converges.

CHAPTER 5

Child care subsidies with endogenous education and fertility*

5.1 Introduction

What are the effects of child care subsidies on education decisions, fertility and the sectoral allocation of the labour force in an economy? Making professional care more affordable lowers the cost of bringing up a child and allows parents to work additional hours. Everything else equal this leads to an increase in the desired number of children and the return to education. However, in order to finance the subsidization program the government might have to levy distorting taxes. In addition, the demand for formal child care will draw uneducated workers away from production and into the service sector. This will affect the wage premium earned by a college educated worker and thereby fertility decisions and education choices.

A proper investigation of the economic consequences of a child care subsidy program needs to take all these feedback effects into account and that is exactly the aim of this chapter. The existence of such a program is taken as given. There can be several reasons why policy makers choose to offer child care subsidies, that need not be restricted to aims of increasing material welfare or economic efficiency. For example, they might wish to stimulate the labour force participation of women and ensure equality of opportunity for both sexes. Blau and Robins (1988) provide empirical evidence that the use of market care is responsive to its price and that subsidies therefore indeed have the intended effect of encouraging labour supply. Attanasio et al. (2008) show that declining costs of child care can explain a large part of the rise in participation rates of married women in the United States as observed in recent decades. A second reason could be that policy

*This chapter is based on Reijnders (2014b).

makers desire to provide incentives to increase fertility in order to avoid the adverse effects associated with population ageing, such as a low support ratio and a strained pension system. As a final example, policy makers might want to give children from disadvantaged backgrounds access to professional care in order to aid their development early in life (see Brilli et al. (2013)).

The contribution of this chapter is twofold. First, it adds to the theoretical literature on child care subsidies. Most papers in this field focus on the implications for (female) labour supply decisions while keeping the number of children and the productivity type of the parents constant. For example, Domeij and Klein (2013) derive that in an economy with pre-existing distortionary taxes on labour it can be efficiency improving to subsidize day care. In a numerical application with German data they show that under an optimal subsidy rate of 50% the labour supply of mothers with small children nearly doubles. A second example is the recent contribution by Guner et al. (2013). They develop a quantitative model with heterogeneous households and calibrate it to the United States economy in order to study the welfare effects of an expansion of current subsidy arrangements. Their conclusion is that child care subsidies lead to a substantial reallocation of hours worked from males to females and generate aggregate welfare losses.

In contrast to the above mentioned studies, in this chapter the choice of education and fertility are endogenous. We take into account that the first is usually an individual decision while the latter is made by a couple. In addition, we carefully model the time inputs required for the provision of child care services in a general equilibrium setting with an endogenous wage premium for educated labour. We find that if there is an ad valorem subsidy on child care financed by a proportional tax on wage income then fertility is higher for all households. As more uneducated workers are employed in the service sector the college wage premium goes down and college graduation rates drop. If the aim of the subsidy is to stimulate fertility, then this can be more effectively done by providing a specific subsidy per child. However, this reduces the supply of labour, especially by uneducated married women.

The second strand of research to which this chapter is related is the economics of fertility, see Hotz et al. (1997) for a survey. The starting point of the pioneering work by Becker (1960) is the observation that empirical studies tend to find a negative relationship between the number of children and a measure of income (usually a proxy for male wages), both in cross-section and over time. This might be a statistical fluke resulting from a missing variable that can explain both low income and high fertility, such as knowledge of contraceptive methods or a strong preference for children over consumption goods. Nevertheless, economic models of fertility have attempted to explain this ‘stylized fact’ (see Jones et al. (2010)). In a static setting these explanations rely on the existence of time costs of child care (which are higher for high-wage parents) or a trade-off between the quantity and ‘quality’ of the offspring (the latter of which is assumed to be relatively cheap for high-wage parents). We add to this discussion by looking at the role of intertemporal dynamics (such as marriage expectations and savings choices) and institutional features (such as taxes and subsidies) in explaining the cross-sectional fertility pattern.

In the context of our model we find that in the absence of taxes and subsidies the marriage market equilibrium is such that a couple with an uneducated wife and an educated husband has the most children, while parents who are both educated have the least. An ad valorem subsidy on child care tends to favour the birth rates of high-wage individuals because they find it easier to afford professional child care. If there is a fixed subsidy per child instead then the relative fertility of uneducated parents increases as for them the subsidy is largest in comparison to household wealth.

The remainder of this chapter is organized as follows. Section 5.2 describes in detail the model. In Section 5.3 we will discuss the model’s implications for the cross-sectional fertility pattern, followed by a numerical assessment of different child care subsidy policies in Section 5.4. The last section concludes.

5.2 Model

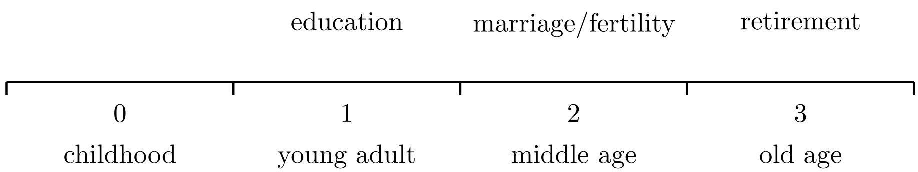
We construct a general equilibrium model of a closed economy with overlapping generations of households, two sectors of production and a government. In order to answer the central question of this chapter, three model elements are crucial. First, fertility is endogenous. Couples optimally decide about the number of children they want to have, taking into account that child care requires time from either parents or professional caregivers. Second, individuals make choices about education based on marriage expectations and the college wage premium. Third, the general equilibrium framework requires individual decisions to be consistent at the aggregate level. For example, any child care subsidies provided by the government have to be financed by (potentially distorting) taxes, the demand for child care services will have to be met by hours of labour and changes in the supply of educated and uneducated workers affect relative wages.

The remainder of this section describes the behaviour of households, firms and the government. This ultimately leads to a description of the macroeconomic equilibrium.

5.2.1 Households

The population consists of an equal number of males (indicated with superscript $j = m$) and females ($j = f$). Every individual lives for four periods or 'life-cycle stages', see Figure 5.1. The first of these (stage 0) is spent passively in the parental household. The remaining three constitute the life span of an adult individual. In stage 1 he or she decides whether to obtain a college degree ($E = 1$, 'educated') or not ($E = 0$, 'uneducated'). At the start of stage 2 everyone gets married and couples jointly decide about fertility and child care. Their children stay with them for exactly one period. The final stage is divided between work and retirement.

Figure 5.1: Life-cycle stages



In every period young adults and married couples make consumption and savings decisions. We assume that there are no bequests from parents, so that each individual enters adulthood with zero financial assets and leaves nothing behind after death.[1] The share of the unit time endowment not spent in college, retirement or child care is sold on the labour market. Preferences over consumption c and the number of children b can be represented by the following felicity function:

$$u(c, b) = \frac{\left[c^{\varepsilon}(1+b)^{1-\varepsilon}\right]^{1-1/\sigma} - 1}{1 - 1/\sigma}, \quad 0 < \varepsilon < 1, \quad 0 < \sigma < 1, \tag{5.1}$$

where ε represents the weight of consumption and σ is the intertemporal substitution elasticity of the consumption-fertility composite. Preferences are non-separable in the sense that the marginal felicity of consumption depends on the number of children and vice versa. Note that children are not a necessary 'good' as utility is well defined when b is equal to zero.

The optimal household allocations during each stage of life can be derived using backward induction, starting from the marriage phase.

[1]In the absence of perfect capital markets intergenerational transfers might be important. If it is not possible to borrow against future human capital then parental income becomes a vital source of education funding. See for example Fernández and Rogerson (2001), who study the effect of increased marital sorting on income inequality.

Marriage and fertility

Married couples jointly decide how many children to raise and how much to consume during life-cycle stages 2 and 3, under the restriction that the level of consumption is the same for all household members. Put differently, we assume that individuals want (or are morally obliged) to provide their spouse and children with a standard of living similar to their own.[2] There are economies of scale for a multi-person household (think of sharing a house, kitchen equipment, etcetera) so that the total consumption expenditures of a household are proportional to the number of 'adult equivalents' that it consists of and not the number of family members. The assumption of equal consumption is then equivalent to assuming that total consumption expenditures are a public good with congestion, with the extent of congestion captured by the adult equivalence scale.

As children are a public good within the household and consumption is equal it follows that felicity during marriage is the same for husband and wife. This means that there is no conflict of interest between married partners and the objective of the household is to maximize individual welfare. A married couple in the last stage of life with b grown-up children faces the following problem:

$$\begin{aligned}\mathcal{M}_{3,t}\big(E^f, E^m, a_3, b\big) &= \max_{c_3}\ u(c_3, b) \\ \text{s.t.}\quad 0 &= (1+r_t)a_3 + (1-\tau_t)w_t(E^f)[1-\bar{R}] + (1-\tau_t)w_t(E^m)[1-\bar{R}] \\ &\quad - 2\bar{\tau}_t - Q^a c_3, \qquad (5.2)\end{aligned}$$

where $\bar{R}$ is the (exogenous) fraction of time spent in retirement and $Q_a \leq 2$ is the equivalence scale for two adults. Household resources in old age consist of savings from the previous period a_3 inclusive of interest accrued at rate r_t, plus education-dependent wages $w_t(E)$ of husband and wife net of labour taxes τ_t, minus lump-sum taxes $\bar{\tau}_t$ and expenditures on consumption c_3. The (trivial) solution to this problem is that the couple consumes all its resources. This gives the optimal level of consumption in stage 3 (the 'policy function') as a function of the predetermined education levels, financial assets and number of children (the 'state variables') which we denote by $\mathbf{c}_{3,t}(E^f, E^m, a_3, b)$.

One period before, in stage 2, couples decide how many children to raise. The cost of having a child is threefold. First, consumption expenditures increase as the adult

[2] In a comment on Becker (1960) published in the same chapter, James S. Duesenberry argues that: "[...] there is no area in which the sociological limitations of freedom of choice apply more strongly than to behavior in regard to bringing up children [...] in many respects the standard of living of the children is mechanically linked to that of the parents" (p. 233-234). Similarly, according to Bernard Okun: "It is almost impossible to conceive of a child who is raised at a much lower level of living than that of his parents" (p. 236).

equivalence scale goes up with $Q_b \leq 1$. Second, childbearing requires a fraction T_b of the time endowment of the mother. Finally, there is a fixed amount of (basic) child care N^b that has to be provided. This care can be created using time inputs of the parents n^p and professional caregivers o:[3]

$$\Omega(n^p, o) = \big[n^p\big]^{\psi}\big[n^p + o\big]^{1-\psi}, \quad 0 < \psi \leq 1, \tag{5.3}$$

where n^p is an combination of mother time n^f and father time n^m:[4]

$$n^p = \Gamma(n^f, n^m) = \Big[\big[n^f\big]^{1-1/\xi} + \big[n^m\big]^{1-1/\xi}\Big]^{\frac{1}{1-1/\xi}}, \quad \xi > 1. \tag{5.4}$$

From (5.4) it follows that parents are equally productive but imperfect substitutes for each other as long as the substitution elasticity ξ is finite. The time input of a professional caregiver, on the other hand, can be perfectly replaced by parental time according to (5.3). The opposite is *not* true: as long as $\psi > 0$ the contribution of parents is a necessary input in the production of child care. This captures the idea that some basic care for children might be outsourced, but it cannot completely replace the attention of a parent.

The decision problem of a couple from the perspective of stage 2 can be written as:

$$\begin{aligned}
\mathcal{M}_{2,t}\big(E^f, E^m, a_2\big) &= \max_{c_2, a_3, b, n^f, n^m, o} \left\{u(c_2, b) + \frac{1}{1+\rho}\mathcal{M}_{3,t+1}(E^f, E^m, a_3, b)\right\} \\
\text{s.t.} \quad a_3 &= (1 + r_t)a_2 + (1 - \tau_t)w_t(E^f)[1 - n^f - T_b b] \\
&\quad + (1 - \tau_t)w_t(E^m)[1 - n^m] - 2\bar{\tau}_t - \big[Q^a + Q^b b\big]c_2 \\
&\quad - (1 - s_t)p_t o + \bar{s}_t b, \\
\Omega\big(\Gamma(n^f, n^m), o\big) &= N^b b, \quad b \geq 0, \quad 0 \leq n^f \leq 1 - T^b b, \quad 0 \leq n^m \leq 1, \quad o \geq 0,
\end{aligned} \tag{5.5}$$

where ρ is the rate of time preference. The household decides how many children to have, in what way to arrange their care and how to split resources between consuming in the current period and saving for the next. From the budget constraint it follows that the cost of child care consists of the foregone wages of the parents and the bill for formal care services, with p_t its relative price and s_t the corresponding ad valorem subsidy rate. There is a specific subsidy of $\bar{s}_t$ per child while it is living in the parental household.

[3] These professional caregivers include any person that provides formal child care such as nannies, au pairs, child minders, employees of day care centres, etcetera. For an overview of the different child care arrangements in the United States, see Laughlin (2013).

[4] For an overview of the properties of the two 'production functions' employed here, see Appendix 5.A.

Since parents do not get any direct utility from caring for their children it follows that the optimal allocation of total child care over parents and professional caregivers for a given number of children b is the one which minimizes the total associated costs. As the child care production functions (5.3) and (5.4) both feature constant returns to scale this equals the lowest possible cost per unit of child care times the total amount of child care required $N^b b$. The unit cost function can be derived in two steps. First, let w_t^p denote the minimum before-tax cost of a unit of parent time at time t. That is:

$$\begin{aligned} w_t^p(E^f, E^m) &\equiv \left\{ \min_{n^f, n^m} \left[w_t(E^f) n^f + w_t(E^m) n^m \right] \text{ s.t. } \Gamma(n^f, n^m) = 1 \right\} \\ &= \left[w_t(E^f)^{1-\xi} + w_t(E^m)^{1-\xi} \right]^{\frac{1}{1-\xi}}. \end{aligned} \tag{5.6}$$

Because the Constant Elasticity of Substitution (CES) production function (5.4) is self-dual, the price of parent time is a CES aggregate of parental wage levels with substitution elasticity $1/\xi$. For a given total production of parent time n^p the optimal input quantities of father and mother time are:

$$n^j = n^p \left[\frac{w_t(E^j)}{w_t^p(E^f, E^m)} \right]^{-\xi}, \quad j \in \{f, m\}. \tag{5.7}$$

As long as parents are imperfect substitutes both will contribute a positive amount of time, but the parent with the lower wage does most. In the next step we define ω_t to be the minimum cost of a unit of child care:

$$\omega_t(E^f, E^m) \equiv \left\{ \min_{n^p, o} \left[(1 - \tau_t) w_t^p(E^f, E^m) n^p + (1 - s_t) p_t o \right] \text{ s.t. } \Omega(n^p, o) = 1 \right\}. \tag{5.8}$$

There is a trade-off between the (after-tax) value of parental time and the (after-subsidy) price of formal child care. Two cases can be distinguished, depending on whether there is an interior solution to the problem expressed in (5.8) or not. First, if child care services are relatively cheap in the sense that $(1 - s_t)p_t \leq (1 - \psi)(1 - \tau_t)w_t^p$ then the care burden is shared between parents and professional caregivers. The optimal allocation is given by:

$$n^p = N^b b \frac{\psi \omega_t(E^f, E^m)}{(1 - \tau_t) w_t^p(E^f, E^m) - (1 - s_t) p_t}, \tag{5.9}$$

$$o = N^b b \frac{(1 - \psi) \omega_t(E^f, E^m)}{(1 - s_t) p_t} - n^p, \tag{5.10}$$

with the following minimum cost function:

$$\omega_t(E^f, E^m) = \left[\frac{(1-s_t)p_t}{1-\psi}\right]^{1-\psi}\left[\frac{(1-\tau_t)w_t^p(E^f, E^m)-(1-s_t)p_t}{\psi}\right]^{\psi}. \tag{5.11}$$

Note that it is not sufficient that the net price of an hour of child care services is lower than the after-tax parental cost index. It has to be even lower than that in order to compensate for the fact that professional caregivers do not provide the same type of care as a parent. If not, then all care is performed by the parents ($o = 0$ and $n^p = N^b b$) and the unit cost is $\omega_t(E^f, E^m) = (1-\tau_t)w_t^p(E^f, E^m)$. These two cases are illustrated in panel (a) of Figure 5.2. The upper diagram in panel (a) depicts the minimum unit cost of child care as a function of the price of child care services net of subsidies, keeping the tax rate fixed. It is increasing up to the point where it becomes optimal to leave all care to the parents, after that it stays flat. By Shephard's Lemma the demand for child care services equals the slope of the minimum cost function which is positive in region (i) but zero in (ii), see the lower diagram. If the ad valorem subsidy on child care goes up then, ceteris paribus, the demand for care providers weakly increases (a movement along the curve to the left). However, if the subsidy is paid for by higher taxes on wage income then the opportunity cost of parental time decreases as well. This is illustrated in panel (b). Both the minimum cost schedule and the demand function shift down and the line of demarkation between region (i) and (ii) moves to the left.

To save on notation, let Υ_t^b denote the total (minimum) time cost of a child:

$$\Upsilon_t^b(E^f, E^m) = N^b\omega_t(E^f, E^m) + T^b(1-\tau_t)w_t(E^f). \tag{5.12}$$

It follows that Υ_t^b depends on the wages of the parents, the price of child care services, the marginal tax rate and the ad valorem child care subsidy. Assuming that the number of children need not be an integer[5] and that an interior solution exists, the remaining first-order conditions of the household problem in (5.5) can be written as:

$$\frac{Q^a c_3}{[Q^a+Q^b b]c_2} = \left(\frac{1+r_{t+1}}{1+\rho}\right)^{\sigma^*}\left(\frac{Q^a}{Q^a+Q^b b}\right)^{1-\sigma^*}, \tag{5.13}$$

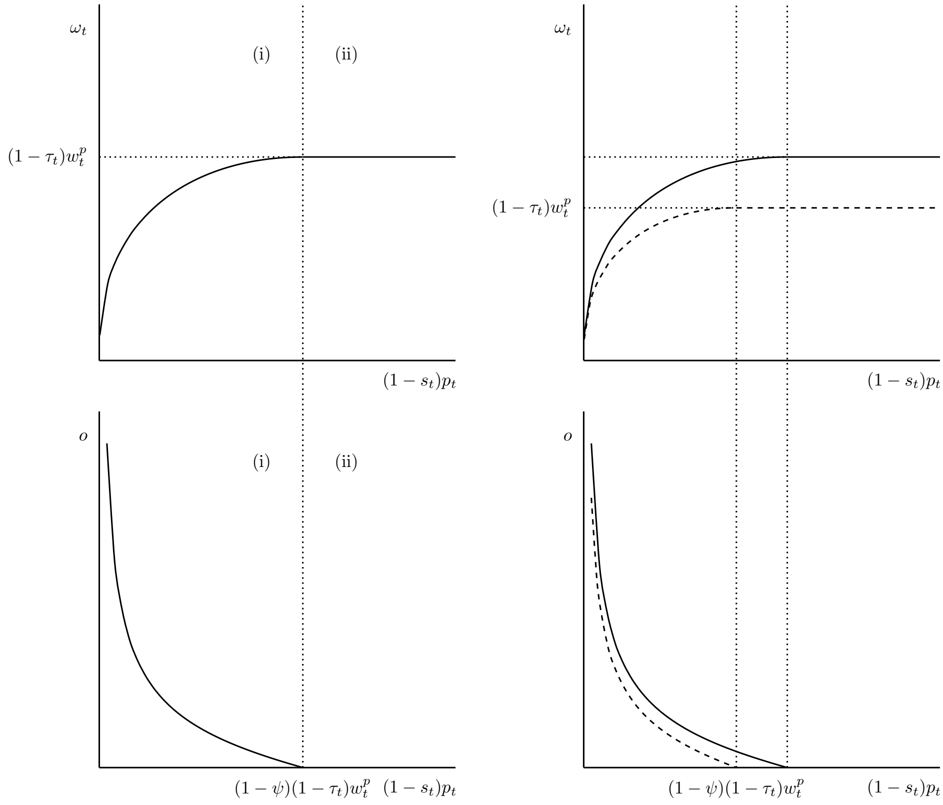
$$\varepsilon\frac{\Upsilon_t^b(E^f, E^m)-\bar{s}_t+Q^b c_2}{[Q^a+Q^b b]c_2} = \left[1+\frac{1}{1+\rho}\left(\frac{c_3}{c_2}\right)^{-\frac{1-\sigma^*}{\sigma^*}}\right]\frac{1-\varepsilon}{1+b}, \tag{5.14}$$

$$W_{2,t}(E^f, E^m, a_2) = Q^a c_2 + \frac{Q^a c_3}{1+r_{t+1}} + \left[\Upsilon_t^b(E^f, E^m)-\bar{s}_t+Q^b c_2\right]b, \tag{5.15}$$

[5]Instead b can be interpreted as the average birth rate across households that are similar in terms of the education and wealth of the spouses.

Figure 5.2: The cost of child care

(a) Minimum cost and demand (b) Increase in labour taxes



where $W_{2,t}$ is household wealth at the start of stage 2, which consists of asset income and discounted wages net of taxes:

$$W_{2,t}(E^f, E^m, a_2) = (1 + r_t)a_2 + (1 - \tau_t)\big[w_t(E^f) + w_t(E^m)\big] - 2\bar{\tau}_t + \frac{(1 - \tau_{t+1})\big[w_{t+1}(E^f) + w_{t+1}(E^m)\big][1 - \bar{R}] - 2\bar{\tau}_{t+1}}{1 + r_{t+1}}. \quad (5.16)$$

Equation (5.13) is the couple's Euler equation for total consumption expenditures with σ^* the corresponding intertemporal substitution elasticity.[6] Under the assumption of equal consumption, the number of adult equivalents present in the household in a given life-cycle stage can be interpreted as the corresponding 'price' of individual consumption. With $\sigma^* < 1$ parents are reluctant to shift consumption towards the period in which its price is lower, that is when the children have left the parental household in stage 3. As a consequence the relative level of total consumption expenditures in each stage depends positively on the number of adult equivalents present in that period. The second equation is the optimality condition for the number of children. The left-hand side represents the marginal cost of an additional child in terms of foregone consumption, while the right-hand side captures the marginal benefit. Equation (5.15) restates the household budget constraint with the minimum time cost of child care substituted in. There is no analytical solution to this system of equations, so they have to be solved numerically. This yields the policy functions for stage 2, for example $\mathbf{b}_t(E^f, E^m, a_2)$ denotes the optimal number of children given the educational attainment and the financial wealth of the spouses and $\mathbf{c}_{2,t}(E^f, E^m, a_2)$ the chosen consumption level. Some comparative static effects on optimal fertility choices are discussed in Section 5.4.

Education decision

Moving back yet another stage in life, the felicity from marriage is perfectly anticipated by a young adult in life-cycle stage 1 and he or she makes decisions accordingly. For example, consider a female with education E who has just left the parental household at the start of period t. She chooses consumption c_1 and savings a_2, taking as given the probability of meeting a future spouse with education level E^m and corresponding

[6] In terms of the underlying preference parameters this substitution elasticity is given by:

$$\sigma^* \equiv -\frac{u_c(c, b)}{u_{cc}(c, b)c} = \frac{1}{1 - \varepsilon(1 - 1/\sigma)}.$$

Empirical estimates suggest that $0 < \sigma^* < 1$ (see Attanasio and Weber (1995)) which requires $0 < \sigma < 1$ as assumed above.

financial assets $\mathbf{a}^m_{2,t}(E^m)$. Her expected lifetime utility is given by:

$$\mathcal{S}^f_{1,t}(E) = \max_{c_1,a_2} \Big\{ u(c_1,0) + \frac{1}{1+\rho}\Big[\pi^{m|f}_{2,t+1}(0|E)\mathcal{M}_{2,t+1}\big(E,0,a_2+\mathbf{a}^m_{2,t}(0)\big) + \pi^{m|f}_{2,t+1}(1|E)\mathcal{M}_{2,t+1}\big(E,1,a_2+\mathbf{a}^m_{2,t}(1)\big)\Big]\Big\}$$

$$\text{s.t.} \quad a_2 = (1-\tau_t)w_t(E)[1-\bar{e}E] - \bar{\tau}_t - \bar{f}_t E - c_1, \tag{5.17}$$

The first term in curly brackets is the immediate felicity from consumption. The remaining ones capture the expected discounted utility from stage 2 onward. There is a probability $\pi^{m|f}_{2,t+1}(0|E)$ of being matched to an uneducated male and a probability $\pi^{m|f}_{2,t+1}(1|E)$ of finding an educated husband in the next period. Importantly, these probabilities are conditional on her own educational attainment. The amount of savings she brings to the marriage consists of labour income net of taxes and consumption expenditures, with $\bar{e}$ the time cost of a college degree and $\bar{f}_t$ the tuition fee.

We assume that upon entering adulthood at the start of life-cycle stage 1 each individual learns his or her utility cost of education θ which is drawn from a time-invariant distribution F_θ (assumed to be the same for men and women).[7] As the cost of education is monotonically increasing in θ while the benefit is independent of it there is a critical level $\bar{\theta}^j_t$ such that the optimal choice $\mathbf{E}^j_t(\theta)$ is characterized by a threshold rule:

$$\mathbf{E}^j_t(\theta) = \begin{cases} 1 & \text{if } \theta \leq \bar{\theta}^j_t \\ 0 & \text{if } \theta > \bar{\theta}^j_t \end{cases} \tag{5.18}$$

By defining $\pi^j_{s,t}(E)$ to be the fraction of individuals of gender j in stage s at time t who have education E, it follows from the policy function above that $\pi^j_{1,t}(1) = F_\theta(\bar{\theta}^j_t)$. It is not possible to obtain education at a later life-cycle stage so that $\pi^j_{3,t+2}(E) = \pi^j_{2,t+1}(E) = \pi^j_{1,t}(E)$. These frequencies in turn will determine the conditional marriage probabilities that feature in (5.17), as explained in the next section.

Marriage market

Let $\pi_{s,t}\big(E^f,E^m\big)$ denote the fraction of marriages between individuals that are in stage s at time t in which the female has education E^f and the male education E^m. As individuals can only get married at the start of life-cycle stage 2 they necessarily have

[7]It could be the case that there is a correlation between the learning ability of parents and their children. In the absence of a bequest motive this does not affect the aggregate outcomes as long as the overall distribution F_θ does not vary over time.

a spouse from the same cohort. To allow for the fact that people tend to get married to someone with a similar level of education (a phenomenon known as 'positive assortative matching') we take the following specification from Chapter 4:

$$\pi_{2,t}(1,1) = (1-\lambda)\pi^{f}_{2,t}(1)\pi^{m}_{2,t}(1) + \lambda \min\left\{\pi^{f}_{2,t}(1), \pi^{m}_{2,t}(1)\right\}, \tag{5.19}$$

where λ is an index of the degree of marital sorting. If $\lambda = 0$ then matching is random, while with $\lambda = 1$ it is perfectly positive assortative. The expression for $\pi_{2,t}(0,0)$ is similar and the cross probabilities follow. Assuming that there is no divorce the marriage pattern in stage 2 carries over to stage 3 such that $\pi_{3,t+1}\left(E^f, E^m\right) = \pi_{2,t}\left(E^f, E^m\right)$.

Bayes' Rule implies that the conditional probability that a woman with education E^f is matched to a man with education E^m is given by:

$$\pi^{m|f}_{2,t}\left(E^m \middle| E^f\right) = \frac{\pi_{2,t}\left(E^f, E^m\right)}{\pi^{f}_{2,t}\left(E^f\right)}. \tag{5.20}$$

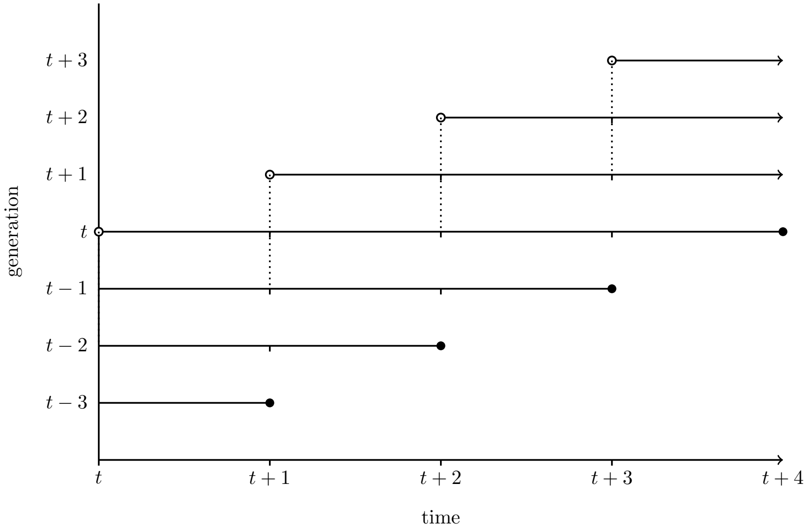
For males the definitions are similar. The marriage market is in equilibrium if the expectations that individuals have about these conditional matching probabilities and the premarital savings of others coincide with the actual education and savings decisions.

Demography and aggregation

At the start of each period a new cohort with an equal number of males and females is born. This means that 4 different cohorts are alive at the same time. These 'overlapping generations' are illustrated in Figure 5.3. The horizontal axis shows the passing of time while the vertical axis records the year in which a generation arrives. An open dot indicates birth, when the dot is closed 4 periods later all individuals belonging to a given cohort pass away. The dotted line represents the parent-child relationship, linking a new generation to one that is currently in life-cycle stage 2. For example, generation $t+2$ are the children of parents who were born in period t.

Let $P_{s,t}$ denote the size of the cohort that is in stage $s \in \{0,1,2,3\}$ at time t and define P_t to be the total population. Since there is no mortality risk it follows that $P_{3,t+3} = P_{2,t+2} = P_{1,t+1} = P_{0,t}$ and that $P_t = \sum_{s=0}^{3} P_{s,t}$. Fertility is a choice variable for married couples and therefore the average number of children $\bar{b}_t$ born to women in stage 2 and the growth rate of the cohorts η_t are endogenous. The dynamic relation

Figure 5.3: Overlapping generations



between the two is:[8]

$$1 + \eta_t = \frac{\bar{b}_t/2}{1 + \eta_{t-1}}. \tag{5.21}$$

This difference equation introduces cyclical dynamics into the model, see Appendix 5.D for a discussion. In the demographic steady state the population growth rate $\bar{\eta}_t$ coincides with that of the cohorts and is given by:

$$1 + \bar{\eta}_t = \sqrt{\bar{b}_t/2}. \tag{5.22}$$

It follows that in order for the population to grow in the long run women should get more than 2 children on average.

Given the demographic structure of the population and its marital composition we can aggregate all household choices to find total financial asset holdings A_t, labour supply by education type $L_t(0)$ and $L_t(1)$, consumption C_t, spending on tuition fees F_t and demand for child care services O_t. The optimal fertility rates of each couple determine the average birth rate $\bar{b}_t$.

[8]By definition of the birth rate, $P_{0,t} = \bar{b}_t P_{2,t}/2$. By definition of the cohort growth rate, $P_{0,t} = (1+\eta_t)(1+\eta_{t-1})P_{2,t}$. Combining yields $\bar{b}_t/2 = (1+\eta_t)(1+\eta_{t-1})$.

5.2.2 Firms

There are two types of commodities in the economy: goods (the numeraire) and child care services. These are produced in separate sectors using two production factors: physical capital and labour. We assume that capital is only used in goods production and that labour is perfectly mobile between the two sectors.

Production sector

The production of a homogeneous good for consumption and investment purposes proceeds according to a constant returns to scale technology:

$$Y_t = \Phi\Big[K_t\Big]^{\phi}\Big[N_t^Y\Big]^{1-\phi}, \quad \Phi > 0, \quad 0 < \phi < 1, \tag{5.23}$$

where K_t is the capital stock at the start of period t and N_t^Y is a composite of labour supplied by educated and uneducated workers:[9]

$$N_t^Y = \Big[N_t^Y(1)\Big]^{\nu}\Big[N_t^Y(0) + N_t^Y(1)\Big]^{1-\nu}, \quad 0 < \nu < 1. \tag{5.24}$$

It follows that educated labour $N_t^Y(1)$ is a perfect substitute for uneducated labour $N_t^Y(0)$ but not the other way around. For example, both can use their 'brawn' on the work floor (the second term in brackets) but educated workers also add a 'brain' component in the form of management tasks and research and development (the first term). The latter is a necessary input for production.[10] Let w_t^Y denote the minimum unit cost of the labour composite. Assuming an interior solution:

$$\begin{aligned} w_t^Y &\equiv \left\{ \min_{N_t^Y(0), N_t^Y(1)} \Big[w_t(0)N_t^Y(0) + w_t(1)N_t^Y(1)\Big] \text{ s.t. } N_t^Y = 1 \right\} \\ &= \left[\frac{w_t(0)}{1-\nu}\right]^{1-\nu}\left[\frac{w_t(1) - w_t(0)}{\nu}\right]^{\nu}. \end{aligned} \tag{5.25}$$

A necessary condition for both types of labour to be employed in the production of consumption goods is that uneducated labour is relatively cheap, in that $w_t(0) \leq (1-\nu)w_t(1)$.

[9] This definition of the labour composite is similar to the child care production function which combines time inputs of parents and professional caregivers. See Appendix 5.A for some of its properties. Alternatively we could have used the more standard CES function to aggregate the two types of labour, see the discussion in Section 5.2.4 below.

[10] This specification is a special case of the one used by Caucutt and Kumar (2003) under the assumptions that (i) uneducated and educated individuals have equal 'brawn', (ii) uneducated individuals provide no 'brain' and (iii) the substitution elasticity between 'brawn' and 'brain' is equal to 1.

The stock of capital increases with firm investment I_t and depreciates at rate δ_K such that:

$$K_{t+1} = (1 - \delta_K)K_t + I_t. \tag{5.26}$$

The representative profit-maximizing firm chooses capital and total labour services in such a way that the following conditions are satisfied:

$$r_t + \delta_K = \frac{\partial Y_t}{\partial K_t} = \phi\Phi\left[\frac{K_t}{N_t^Y}\right]^{-(1-\phi)}, \tag{5.27}$$

$$w_t^Y = \frac{\partial Y_t}{\partial N_t^Y} = (1-\phi)\Phi\left[\frac{K_t}{N_t^Y}\right]^{\phi}. \tag{5.28}$$

That is, each factor of production earns exactly its marginal product. The linear homogeneity of the production function implies that firms in the production sector do not earn a profit. The corresponding demand for uneducated and educated workers follows from:

$$w_t(0) = w_t^Y \frac{\partial N_t^Y}{\partial N_t^Y(0)} = w_t^Y(1-\nu)\mu_t^{\nu}, \tag{5.29}$$

$$w_t(1) = w_t^Y \frac{\partial N_t^Y}{\partial N_t^Y(1)} = w_t^Y\left[(1-\nu)\mu_t^{\nu} + \nu\mu_t^{-(1-\nu)}\right], \tag{5.30}$$

where $\mu_t \equiv N_t^Y(1)/[N_t^Y(0) + N_t^Y(1)]$ is the number of educated workers as a share of total employment in the production sector.

Service sector

The provision of child care requires only labour:

$$Z_t = \Psi N_t^Z, \quad \Psi \geq 1, \tag{5.31}$$

where N_t^Z is given by:

$$N_t^Z = N_t^Z(0) + N_t^Z(1). \tag{5.32}$$

If $\Psi > 1$ then there are economies of scale for professional caregivers compared to parents as each hour of labour results in more than one unit of child care services. This could be the case, for example, if formal care providers can work more efficiently by combining care for children of the same age. In contrast to the production sector, educated and uneducated workers are assumed to be perfect substitutes in creating (basic) child care. As a consequence only uneducated labour will be hired since it is less expensive. Perfect labour mobility between sectors implies that the wage rate is equal to that earned in the production sector so that the minimum unit cost is $w_t^Z = w_t(0)$ as determined in (5.29) above. The competitive price of child care services relative to the consumption good is $p_t = w_t^Z/\Psi$.

5.2.3 Government

The government levies taxes and distributes subsidies. There is no other form of government spending. We assume that there is no debt financing so that in every period the government has to maintain a balanced budget:

$$T_t \equiv \tau_t\big[w_t(0)L_t(0) + w_t(1)L_t(1)\big] + \bar{\tau}_t\big[P_t - P_{0,t}\big] = s_t p_t O_t + \bar{s}_t P_{0,t} \equiv S_t. \quad (5.33)$$

The left-hand side is the total revenue from the proportional labour income tax plus the proceeds from lump-sum taxation of the adult population. The right-hand side consists of ad valorem subsidies paid out for every hour of professional child care demanded by households and specific subsidies for each child.

5.2.4 Macroeconomic equilibrium

A macroeconomic equilibrium is a sequence of prices and allocations such that in every period:

(i) Each young adult and every couple maximizes utility subject to a budget constraint taking prices and the behaviour of everyone else as given.

(ii) Firms maximize profits taking prices as given.

(iii) The government budget is balanced.

(iv) All markets clear.

- Capital market:

$$K_t = A_t$$

- Labour market:

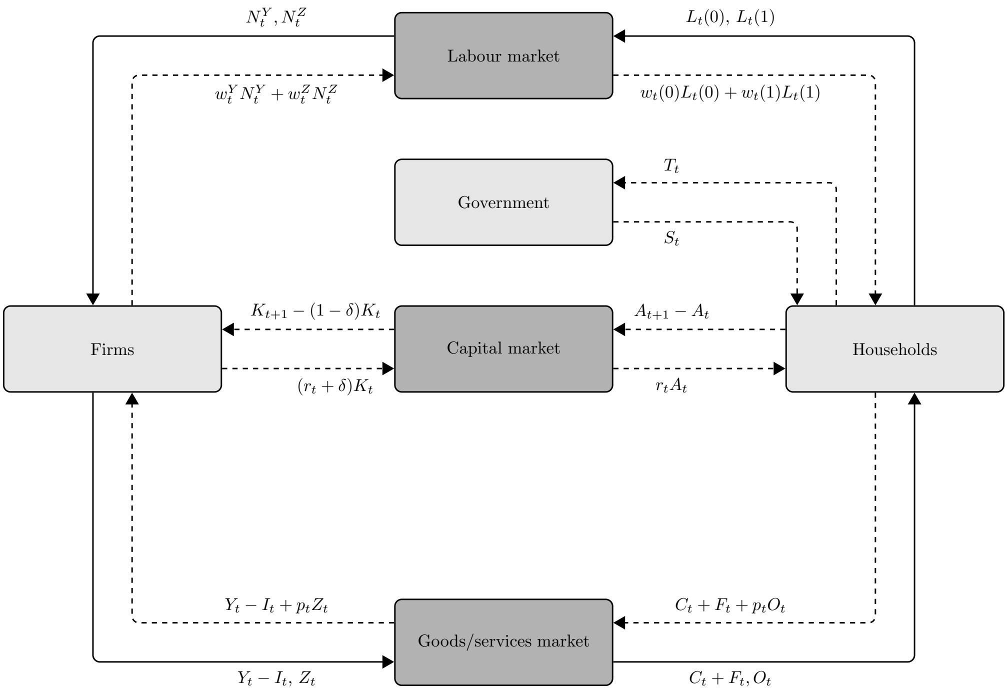
$$N_t^Y(1) = L_t(1), \quad N_t^Y(0) + N_t^Z(0) = L_t(0)$$

– Goods and services market:

$$Y_t = C_t + I_t + F_t, \quad Z_t = O_t$$

The circular flow diagram in Figure 5.4 provides a graphic visualization of the macroeconomic equilibrium. It shows how the three actors in the economy (households, firms and the government) interact on three markets (capital, labour and goods and services). The solid lines capture the flow of goods, the dashed lines represent payments in terms of the numeraire commodity. The gross domestic product (GDP) of this economy is equal to the total value of goods and services, that is $GDP_t = Y_t + p_t Z_t$.

Figure 5.4: Circular flow diagram



Under the assumption that both types of labour are employed in the production sector the relative wage rate for educated versus uneducated workers (the 'college wage premium') is given by:

$$\frac{w_t(1)}{w_t(0)} = 1 + \frac{\nu}{(1-\nu)\mu_t} > 1, \tag{5.34}$$

which follows from (5.29) and (5.30). The college wage premium depends negatively on the relative abundance of educated workers in the production sector (recall that $\mu_t = N_t^Y(1)/[N_t^Y(0) + N_t^Y(1)]$). There are two margins along which μ_t can change. First, an individual's choice of education affects the time left for work in life-cycle stage 1 and the skill composition of the labour force. Second, the number of children and the allocation of child care determine the fraction of time that parents can supply in life-cycle stage 2 and the demand for uneducated workers from the service sector.

As the definition of the labour composite in (5.24) and the corresponding college wage premium (5.34) are non-standard it is useful to contrast them to a more commonly used specification. Suppose (5.24) is replaced by an aggregator function that features a constant elasticity of substitution α:

$$N_t^Y = \left[N_t^Y(0)^{1-1/\alpha} + \beta N_t^Y(1)^{1-1/\alpha}\right]^{\frac{1}{1-1/\alpha}}, \quad \alpha > 0, \quad \beta > 1.$$

In this case uneducated labour is 'as good' a substitute for educated labour as the other way around and the marginal product of uneducated labour becomes infinite as $N_t^Y(0) \to 0$. The college wage premium is given by $w_t(1)/w_t(0) = \beta[N_t^Y(1)/N_t^Y(0)]^{-1/\alpha}$. If the two types of labour are imperfect substitutes, such that α is finite, then the premium depends negatively on the ratio of educated to uneducated workers. The premium could be less than unity when educated workers are relatively abundant (however, this will never hold in an equilibrium with an endogenous choice of education). In contrast, according to (5.34) the more productive workers always earn more.[11] If educated and uneducated labour are perfect substitutes such that $\alpha \to \infty$ then $w_t(1)/w_t(0) = \beta$ is constant. It is important to stress here that the results below would still hold with this functional form as long as α is finite so that the college wage premium depends on relative labour supplies.

5.3 The cross-sectional fertility pattern

Three assumptions in the model form the key for understanding the implied optimal fertility choices. First, parents derive utility from the quantity of children only. There is no quantity-quality trade-off à la Becker (1960). Second, the consumption expenditures on children are proportional to parental consumption. This prevents these costs from becoming negligible as the wages of parents increase and automatically ensures that parents with more income provide their offspring with a higher standard of living. Third,

[11] Caucutt and Kumar (2003) give a sufficient condition that ensures that the college wage premium is positive. That condition is satisfied for the specification in (5.24).

the level of consumption and the number of children enter into the utility function in a non-separable way.

Recall from Section 5.2.1 that the model does not yield a closed-form solution for the optimal fertility choice. Still it is possible to derive some comparative static effects.

Proposition 5.1. *The optimal number of children is increasing in household wealth* $W_{2,t}$ *and the specific child subsidy* $\bar{s}_t$ *and decreasing in the time cost* Υ_t^b.

Proof. See Appendix 5.B. □

It immediately follows from Proposition 5.1 that, ceteris paribus, a higher ad valorem subsidy on child care services will increase fertility rates by lowering the time cost of a child. If the wage of one of the parents goes up then this increases both household wealth (income effect) and the opportunity cost of time (substitution effect), which means that the overall effect on fertility is ambiguous in general. Only under specific assumptions is it possible to derive analytically how fertility choices vary with parental wages in the context of the model.

Proposition 5.2. *Assume that there are no premarital savings, no child care services, no time costs of child birth and no lump-sum taxes or specific subsidies. Then the optimal number of children depends on the relative wages of husband and wife and attains a minimum when wages are equal.*

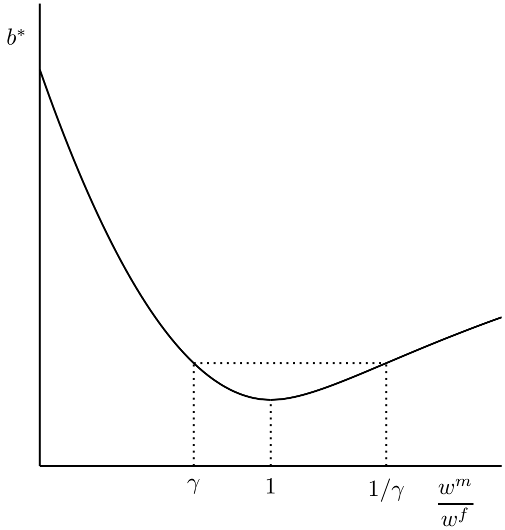
Proof. See Appendix 5.B. □

The optimal number of children as a function of the ratio of parental wages is illustrated in Figure 5.5. The fact that only relative wages matter for the fertility decision is a consequence of the specification of the preferences in (5.1). If both the female and male wage grow at the same rate then income and substitution effects exactly cancel out so that the optimal number of children remains constant.[12] The fact that fertility is lowest when wages are equal hinges crucially on the existence of time costs for which parents are substitutes. It only holds if $N^b > 0$ and $\xi > 0$. Intuitively, couples for which the spouses have the same wage rate miss out on the gains from specialization that can be reaped when a low-wage parent takes on most of the child care while the high-wage spouse can spend more time on market work. In the absence of a time cost of child birth

[12]This result is well known in the context of endogenous labour supply. In order to ensure the existence of a balanced growth path with increasing wages but a stationary hours decision it is necessary to impose restrictions on the utility function, see King et al. (2002). The specification in (5.1) satisfies these restrictions but replaces leisure by the number of children (plus one).

it is irrelevant whether the wife earns more than her husband or vice versa. In terms of Figure 5.5, the optimal number of children is the same for w^m/w^f equal to γ and $1/\gamma$ for any $0 < \gamma \leq 1$.

Figure 5.5: Optimal number of children



In the model we only distinguish two wage rates, $w_t(0)$ and $w_t(1)$, which depend on an individual's education level. Write $b_t^*(E^f, E^m)$ for the optimal fertility choice of a couple where the wife has education E^f and the husband E^m, taking into account the education and gender-specific level of premarital savings. In terms of the policy functions derived above:

$$b_t^*\big(E^f, E^m\big) = \mathbf{b}_t\big(E^f, E^m, \mathbf{a}_{2,t-1}^f(E^f) + \mathbf{a}_{2,t-1}^m(E^m)\big). \tag{5.35}$$

Proposition 5.2 then implies that if there are no premarital savings, child care services, time costs of child birth and lump-sum taxes or specific subsidies then the fertility pattern satisfies:

$$b_t^*(0,0) = b_t^*(1,1) < b_t^*(1,0) = b_t^*(0,1). \tag{5.36}$$

This stark prediction about cross-sectional fertility choices will no longer hold if one of the premises of Proposition 5.2 is changed. We will discuss the relaxation of each assumption in turn. First, if there is a time cost of child birth for women then the symmetry between the sexes is broken and $b_t^*(1,0) < b_t^*(0,1)$ as it is more costly to have an educated wife. Second, the introduction of professional caregivers reduces the time cost of children in relation to the value of parental time more for educated than for uneducated parents such that $b_t^*(0,0) < b_t^*(1,1)$ by Proposition 5.1. Lump-sum taxes, on the other hand, have a greater impact on the wealth of low-wage individuals. If these are negative (a transfer) then $b_t^*(0,0) > b_t^*(1,1)$. A similar argument can be made for specific subsidies: they lower the cost of having a child relatively more for uneducated couples. Finally, the stock of household savings is predetermined at the date of marriage and therefore has a pure income effect on fertility. It is likely that educated men and women save less or borrow more before marriage because (i) they forego wages by going to school, (ii) they have to pay a tuition fee and (iii) they face a better prospect of marrying a high-wage spouse. If so then $b_t^*(0,0) > b_t^*(1,1)$.

As alluded to in the introduction, the main empirical 'stylized fact' about fertility is the negative relationship between the number of children and income. Due to data limitations income is usually captured by a proxy for the male wage, see for example Jones et al. (2010). In the context of the model this would correspond to a comparison between $\bar{b}_t^m(0)$ and $\bar{b}_t^m(1)$ with $\bar{b}_t^m(E^m)$ the average number of births to men with education E^m:

$$\bar{b}_t^m(E^m) = \frac{\pi_{2,t}(0,E^m)b_t^*(0,E^m) + \pi_{2,t}(1,E^m)b_t^*(1,E^m)}{\pi_{2,t}^m(E^m)}. \tag{5.37}$$

It is clear that the correlation between fertility and male education (or wages) will depend both on cross-sectional fertility choices and the marital sorting process. If there is a high degree of positive assortative matching then this correlation is mainly driven by the fertility of couples with the same level of education.

5.4 Child care subsidies in general equilibrium

What happens to education and fertility if child care is subsidized? How does this alter the sectoral distribution of labour and the college wage premium? To answer these questions we compare the long-run equilibrium that arises when $s_t = \bar{s}_t = 0$ to a scenario in which either one is positive. First we parameterize the model and then show the results of the numerical simulations.

5.4.1 Parameterization

The aim is to parameterize the model in order to solve for the equilibrium numerically. This is not intended as a calibration exercise for a specific economy, as the model is by construction much too stylized for that purpose. Nevertheless, we use ‘reasonable’ parameter values that are as much as possible in line with empirical observation.

The length of each period is 18 years. Obtaining a college degree requires a fraction $\bar{e} = 0.25$ of the time available in life-cycle stage 1, while the retirement phase is a share $\bar{R} = 0.4$ of the final stage. The impatience discount factor is set at 1% per annum which translates into $\rho = (1.01)^{18} - 1$. The curvature parameter of the felicity function is $\sigma = 0.7$. We assume that child care requires $N^b = 0.2$ and that the time cost of child birth for the mother is $T^b = 0.02$ (which corresponds to about 4.3 months of unpaid leave). Using the Oxford scale to translate household members into adult equivalents gives $Q^a = 1.7$ and $Q^b = 0.5$. In line with the data for the United States there is a positive degree of marital sorting equal to $\lambda = 0.55$ (see Chapter 4). The tuition fee equals $\bar{f}_t = 0.05$ which corresponds to approximately one year of wages for a full-time employed unskilled worker.

The initial steady state does not have taxes or subsidies ($\tau_t = \bar{\tau}_t = s_t = \bar{s}_t = 0$). The parameterization targets at the macroeconomic level are (i) a normalized wage $w_t(0) = 1$ for uneducated workers with a college wage premium of $w_t(1) = 1.7$, (ii) a net return to capital of 5% per annum or $r_t = (1.05)^{18} - 1$ per period and (iii) a fraction $\pi_t^f(1) = 0.255$ of educated women and a fraction $\pi_t^m(1) = 0.260$ of educated men in each cohort. The targeted proportion of college educated women is a bit less than that of men because, everything else equal between the sexes, the positive time cost of child birth implies that they have less incentive to become educated. This is contrary to the current situation in most developed countries, where women tend to graduate in larger numbers. The model can potentially account for this fact by introducing more asymmetries between the sexes (in particular a gender wage gap or differences in the distribution of utility costs, see Chapter 4) but we will abstract from this here. The child care parameter $\psi = 0.259$ is set in such a way that the couple with the highest unit cost of parental time is just indifferent between employing a professional caregiver or not ($p_t = (1 - \psi)w_t^p$) given that the latter cannot exploit economies of scale ($\Psi = 1$). This means that there will be no demand for child care services in the benchmark steady state.

We aim at an average birth rate of 2 so that the population growth rate is initially zero. Under the assumption that the elasticity of substitution between father and mother time is $\xi = 4$ this can be achieved by choice of $\varepsilon = 0.649$ (the relative preference for consumption). Targets (i) and (ii) are satisfied with $\nu = 0.146$ (the comparative advantage of educated labour in production), $\phi = 0.244$ (the share of capital in production) and $\Phi = 2.798$ (the production constant). The corresponding educational thresholds are $\bar{\theta}_t^f = 0.420$ and $\bar{\theta}_t^m = 0.424$. These can generate the desired proportions of college educated individuals in (iii) by assuming that the utility cost of education follows a lognormal distribution with location parameter -0.550 and scale parameter 0.481 for both sexes.

The chosen parameter values imply that almost 66% of marriages are between two uneducated individuals. In only 8.5% of the cases is an educated woman matched to an uneducated man, the probability of observing the opposite match is 0.090. In the absence of taxes the minimum cost of parent time ranges between 0.794 for uneducated and 1.349 for educated couples, for mixed households it is 0.940. In all cases it is less than the lowest wage rate among husband and wife because they are assumed to be imperfect substitutes in the production of child care.

5.4.2 Simulation results

For the policy analysis we will focus on the steady state or balanced growth path along which all variables grow at a constant rate. In particular, prices (such as p_t) remain fixed, macroeconomic aggregates (such as K_t) grow at the same rate as the population while per capita measures (such as K_t/P_t) are stationary. See Appendix 5.D for some comments on the transitional dynamics between steady states.

Table 5.1 reports the results of the numerical simulation.[13] Column (a) is the benchmark case without subsidies. There is no demand for child care services as its price is higher than the productivity-adjusted parental wage index for all couples. The only differences relative to the premises of Proposition 5.2 above are the positive time cost of child birth and the possibility to borrow or save before marriage. The marriage market equilibrium is such that a couple with an uneducated wife and an educated husband has the most children ($b_t^*(0,1) = 2.130$), while parents who are both educated have the least ($b_t^*(1,1) = 1.783$). Educated men and women save less than uneducated individuals, which explains why $b_t^*(1,1) < b_t^*(0,0)$ (see Section 5.3). There is a negative correlation between male wages and fertility as uneducated fathers have on average more children ($\bar{b}_t^m(0) = 2.034$) than educated ones ($\bar{b}_t^m(1) = 1.903$).

Consider now an economy which provides an ad valorem subsidy of $s_t = 0.5$ or half of the price of child care services, financed by a proportional tax on labour income. In order to highlight the separate mechanisms present in the model the comparison between the two steady states is split up into several steps, each representing a partial equilibrium effect. In part (i) the marriage market equilibrium is kept fixed, which means that the education frequencies and the premarital savings choices remain the

[13] See Appendix 5.C for details on how the macroeconomic equilibrium can be calculated.

Table 5.1: Long-run equilibrium under different subsidization schemes

	(a)	(b)			(c)		
		(i)	(ii)	(iii)	(i)	(ii)	(iii)
Subsidies and taxes							
s_t	0.000	0.500	0.500	0.500			0.000
$\bar{s}_t$	0.000			0.000	0.060	0.049	0.048
τ_t	0.000	0.023	0.023	0.022	0.022	0.022	0.022
$\bar{\tau}_t$	0.000			0.000			0.000
Prices							
$w_t(0)$	1.000			1.002			0.980
$w_t(1)/w_t(0)$	1.700			1.676			1.716
$(1-s_t)p_t$	1.000	0.500	0.500	0.501			0.980
$(1+r_t)^{1/18}-1$	0.050			0.051			0.053
Education frequencies							
$\pi^f_{1,t}(1)$	0.255		0.282	0.250		0.239	0.248
$\pi^m_{1,t}(1)$	0.260		0.288	0.255		0.245	0.255
Premarital savings							
$\mathbf{a}^f_{2,t}(0)$	0.115		0.111	0.115		0.107	0.115
$\mathbf{a}^f_{2,t}(1)$	0.042		0.035	0.039		0.037	0.056
$\mathbf{a}^m_{2,t}(0)$	0.116		0.112	0.116		0.108	0.117
$\mathbf{a}^m_{2,t}(1)$	0.044		0.037	0.040		0.038	0.057
Fertility choices							
$b^*_t(0,0)$	2.036	2.068	2.065	2.074	2.788	2.611	2.633
$b^*_t(1,0)$	2.018	2.132	2.130	2.127	2.586	2.459	2.481
$b^*_t(0,1)$	2.130	2.256	2.253	2.246	2.761	2.618	2.644
$b^*_t(1,1)$	1.783	2.171	2.168	2.159	2.150	2.072	2.085
$\bar{b}^m_t(0)$	2.034			2.080			2.616
$\bar{b}^m_t(1)$	1.903			2.189			2.281
$\bar{b}_t$	2.000			2.108			2.531
Labour supply							
$L_t(0)/P_t$	0.445			0.455			0.420
$L_t(1)/P_t$	0.143			0.143			0.131
μ_t	0.243			0.252			0.238

Notes: Column (a) is the benchmark without taxes or subsidies. In column (b) $s_t > 0$ while τ_t is such that the government budget is balanced. In column (c) τ_t is the same as in column (b) while $\bar{s}_t$ is such that the government budget is balanced. Part (i) is the direct effect on fertility, part (ii) the new marriage market equilibrium and part (iii) is the general equilibrium.

same. It is clear that the lower price of child care increases fertility, in particular for educated (high-wage) parents. The birth rate for couples that are both educated goes up from 1.783 to 2.171. The next step is to allow individuals to optimally adjust their education choice and the level of premarital savings, keeping the college wage premium at its benchmark level of 1.700. In the new marriage market equilibrium (part (ii)) the fraction of individuals that decides to obtain an education increases strongly for both sexes. The subsidization of child care has made having children much cheaper for them, thereby raising the return to education. Optimal fertility is a little lower than in part (i) for all couples because they consume more in the first stage (as evidenced by lower savings) which reduces household wealth upon marriage. Finally, in moving to part (iii) the wage rates adjust in response to the relative supply of educated and uneducated labour in the production sector. The share of educated workers goes up for three reasons. First, more individuals have a college education. Second, educated parents work more hours despite having a larger number of children as a substantial share of the child care is outsourced. Third, the demand for child care services has drawn uneducated workers into the service sector. As a consequence the college wage premium drops, which lowers the return to education. In the final equilibrium the premium is 1.676 and the proportion of college-educated individuals is below that in the benchmark. Compared to part (ii) fertility rates increased slightly for uneducated couples but decreased for all others.

In column (c) we consider an alternative subsidization scheme. The tax rate is held fixed at the same level as in scenario (b) (which is 2.2%), but the ad valorem subsidy on child care is replaced by a specific subsidy per child. This type of scheme is for example used in the Netherlands under the name of "kinderbijslag"[14] but is also present in the tax system of the United States in the form of a personal exemption for dependents and a child tax credit (Crump et al. (2011)). Keeping the marriage market equilibrium fixed we see that the desired number of children increases dramatically, especially for uneducated couples. This is in line with the predictions of Section 5.3. As this subsidization scheme favours low-wage individuals, the return to education drops and in part (ii) the college graduation rates are reduced. Since parents choose not to outsource child care all workers are employed in the production sector. The share of educated workers decreases which results in a rise in the college wage premium. As a consequence the education frequencies partly recover in the general equilibrium results reported in part (iii). Note that the supply of labour goes down for both educated and uneducated workers due to the increase in fertility. Especially married women work less as the time burden of child birth goes up and they perform the most hours of child care (since they are more likely

[14]The "kinderbijslag" is a subsidy per live-in child under the age of 18 which is independent of the income of the parents. It increases with the age of the child but that is of no importance here because of the time aggregation.

to be the low-wage spouse in a couple). There is a clear negative relationship between male wages and fertility, as in the benchmark.

Robustness checks

To the best of our knowledge there are no estimates of the degree of substitution between fathers, mothers and formal caregivers in the production of child care which makes it difficult to assess what is realistic. However, the results are robust to changes in the parameters ψ, Ψ and ξ. An increase in ψ, the weight of non-substitutable parent time in total care (5.3), will reduce the extent to which parents can substitute their own time for formal child care provision. This will dampen the effect that a subsidy has on the macroeconomic outcome, but does not alter the qualitative predictions. If child care providers are able to exploit economies of scale that parents cannot ($\Psi > 1$) then the optimal fertility choice of all couples increases but the effect of a subsidy on education frequencies and labour allocations are similar. Finally, a change in the substitution elasticity ξ affects the returns to specialization in child care versus market work and thereby the optimal fertility choice of unequal-education couples relative to those with equal wages as explained in Section 5.3 above.

A more crucial assumption is the one implicit in the child care subsidization scheme. Thus far we have assumed that every household is eligible for government support, while in reality there might be a 'means test'. As a short-cut to this, suppose that only couples where both parents are uneducated (and thus get low wages) receive subsidies for child care. Even in a partial equilibrium context this is detrimental to the return to education and the proportion of college educated individuals decreases relative to the benchmark. Including general equilibrium effects leads to an increase in the college wage premium such that education frequencies partly recover. The number of children born to uneducated parents rises relative to that of other couples.

One potentially important mechanism that we have ignored in this chapter is the effect of child care arrangements on child development outcomes. There is some empirical evidence that formal child care improves cognitive and non-cognitive skills and thereby school performance, see Brilli et al. (2013) for a literature overview. This is especially true for children from a disadvantaged background. Hence, a more affordable provision of child care services could potentially raise the educational achievement of children from uneducated parents.

5.5 Conclusion

In this chapter we have studied the long-run effects of child care subsidies on education, fertility and the sectoral allocation of the labour force. In the absence of taxes and subsidies the optimal choice of financial assets early in life (taking marriage market conditions into account) is such that a couple with an uneducated wife and an educated husband has the most children, while parents who are both educated have the least. Introducing an ad valorem subsidy on child care financed by a proportional tax on income leads to an increase in fertility for all households. As more uneducated workers are employed in the service sector the college wage premium goes down and college graduation rates drop. If the aim of the subsidy is to stimulate fertility, then this can be more effectively done by providing a specific subsidy per child. However, this reduces the supply of labour, especially by uneducated married women.

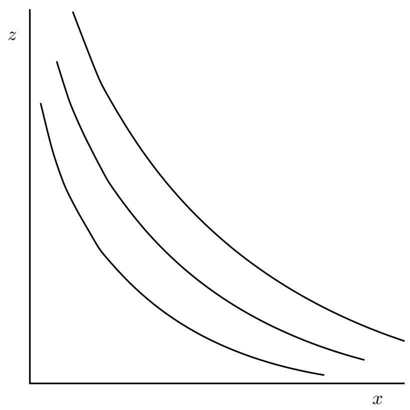
Appendix

5.A Properties of production functions

Table 5.A.1 lists some properties of the two production functions used in this chapter and Figure 5.A.1 shows the corresponding isoquants. Both technologies feature constant returns to scale. The most important difference between them concerns the degree of substitutability between the inputs. In case of the CES production function the substitution elasticity is constant and symmetric: input x is 'as good' a substitute for z as the other way around. Under one-directional substitution, on the other hand, x is a perfect substitute for z but z can only partly replace x. In addition, whereas the marginal product of each production factor becomes infinite at zero for the CES this will not happen for the 'inferior' input z under one-directional substitution. Corner solutions are therefore more likely to occur.

Figure 5.A.1: Isoquants of production functions

(a) Constant Elasticity of Substitution



(b) One-directional substitution

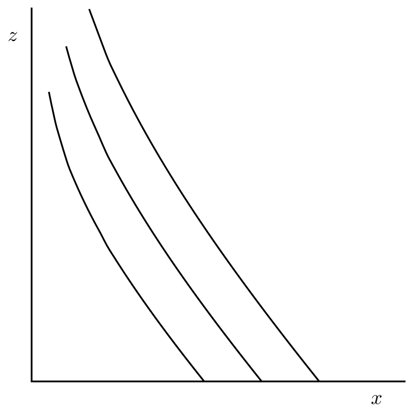


Table 5.A.1: Properties of production functions

	Constant Elasticity of Substitution	One-directional substitution
Definition	$y = \left[x^{1-1/\alpha} + z^{1-1/\alpha}\right]^{\frac{1}{1-1/\alpha}}$, $\alpha > 1$	$y = x^{\beta}\left[x + z\right]^{1-\beta}$, $0 < \beta < 1$
Marginal products	$\dfrac{\partial y}{\partial x} = \left[1 + \left(\dfrac{z}{x}\right)^{1-1/\alpha}\right]^{\frac{1/\alpha}{1-1/\alpha}}$	$\dfrac{\partial y}{\partial x} = \left[1 - \beta + \beta\dfrac{x+z}{x}\right]\left(\dfrac{x}{x+z}\right)^{\beta}$
	$\dfrac{\partial y}{\partial z} = \left[1 + \left(\dfrac{z}{x}\right)^{-(1-1/\alpha)}\right]^{\frac{1/\alpha}{1-1/\alpha}}$	$\dfrac{\partial y}{\partial z} = (1-\beta)\left(\dfrac{x}{x+z}\right)^{\beta}$
Interior solution	$\alpha \ll \infty$	$w_z \leq (1-\beta)w_x$
Minimum cost function	$C(w,y) = y\left[w_x^{-(\alpha-1)} + w_z^{-(\alpha-1)}\right]^{-\frac{1}{\alpha-1}}$	$C(w,y) = y\left[\dfrac{w_z}{1-\beta}\right]^{1-\beta}\left[\dfrac{w_x - w_z}{\beta}\right]^{\beta}$
Conditional factor demands	$x(w,y) = y\left[\dfrac{w_x}{C(w,y)}\right]^{-\alpha}$	$x(w,y) = y\dfrac{\beta C(w,y)}{w_x - w_z}$
	$z(w,y) = y\left[\dfrac{w_z}{C(w,y)}\right]^{-\alpha}$	$z(w,y) = y\dfrac{(1-\beta)C(w,y)}{w_z} - x(w,y)$
Corner solution	$\alpha \to \infty$	$w_z > (1-\beta)w_x$
Minimum cost function	$C(w,y) = \min\{w_x, w_z\}$	$C(w,y) = w_x$
Conditional factor demands	$x(w,y) = \begin{cases} y & \text{if } w_x < w_z \\ \in [0,y] & \text{if } w_x = w_z \\ 0 & \text{if } w_x > w_z \end{cases}$	$x(w,y) = y$
	$z(w,y) = \begin{cases} 0 & \text{if } w_x < w_z \\ y - x(w,y) & \text{if } w_x = w_z \\ y & \text{if } w_x > w_z \end{cases}$	$z(w,y) = 0$

5.B Proofs

The system of first-order conditions for the couple as given in (5.13)-(5.15) can be reduced to a single equation in the number of children b only by substituting out for c_2 and c_3. This yields:

$$g_{1,t}(b)W_{2,t} - g_{2,t}(b)\left[\Upsilon_t^b - \bar{s}_t\right] = 0,$$

where the functions $g_{1,t}(b)$ and $g_{2,t}(b)$ are defined as:

$$g_{1,t}(b) = 1 - \frac{\varepsilon}{1-\varepsilon}\frac{Q^b(1+b)}{Q^a+Q^b b}\left[1 + \frac{1}{1+r_{t+1}}\left(\frac{1+r_{t+1}}{1+\rho}\right)^{\sigma^*}\left(\frac{Q^a}{Q^a+Q^b b}\right)^{1-\sigma^*}\right]^{-1},$$

$$g_{2,t}(b) = \frac{b+\varepsilon}{1-\varepsilon} - \frac{\varepsilon b}{1-\varepsilon}\frac{Q^b(1+b)}{Q^a+Q^b b}\left[1 + \frac{1}{1+r_{t+1}}\left(\frac{1+r_{t+1}}{1+\rho}\right)^{\sigma^*}\left(\frac{Q^a}{Q^a+Q^b b}\right)^{1-\sigma^*}\right]^{-1}.$$

Both are strictly positive because for any $b > 0$ satisfying the first-order condition it holds:

$$\frac{\varepsilon}{1-\varepsilon}\frac{Q^b(1+b)}{Q^a+Q^b b}\left[1 + \frac{1}{1+r_{t+1}}\left(\frac{1+r_{t+1}}{1+\rho}\right)^{\sigma^*}\left(\frac{Q^a}{Q^a+Q^b b}\right)^{1-\sigma^*}\right]^{-1}$$
$$= \frac{W_{2,t} - [\Upsilon_t^b - \bar{s}_t]\frac{b+\varepsilon}{1-\varepsilon}}{W_{2,t} - [\Upsilon_t^b - \bar{s}_t]b} < 1.$$

under the assumption that $\Upsilon_t^b - \bar{s}_t > 0$.

Proposition 5.1. *The optimal number of children is increasing in household wealth $W_{2,t}$ and the specific child subsidy $\bar{s}_t$ and decreasing in the time cost Υ_t^b.*

Proof. Define:

$$G_t\big(b, W_{2,t}, \Upsilon_t^b, \bar{s}_t\big) = g_{1,t}(b)W_{2,t} - g_{2,t}(b)\left[\Upsilon_t^b - \bar{s}_t\right].$$

The optimal choice b^* is such that $G_t\big(b^*, W_{2,t}, \Upsilon_t^b, \bar{s}_t\big) = 0$. This implicitly defines b^* as a function of $\big(W_{2,t}, \Upsilon_t^b, \bar{s}_t\big)$. By the Implicit Function Theorem:

$$\frac{\partial b^*}{\partial W_{2,t}} = -\frac{\partial G_t\big(b^*, W_{2,t}, \Upsilon_t^b, \bar{s}_t\big)/\partial W_{2,t}}{\partial G_t\big(b^*, W_{2,t}, \Upsilon_t^b, \bar{s}_t\big)/\partial b} > 0,$$

$$\frac{\partial b^*}{\partial \Upsilon_t^b} = -\frac{\partial G_t\big(b_t^*, W_{2,t}, \Upsilon_t^b, \bar{s}_t\big)/\partial \Upsilon_t^b}{\partial G_t\big(b_t^*, W_{2,t}, \Upsilon_t^b, \bar{s}_t\big)/\partial b} < 0,$$

$$\frac{\partial b^*}{\partial \bar{s}_t} = -\frac{\partial G_t\big(b_t^*, W_{2,t}, \Upsilon_t^b, \bar{s}_t\big)/\partial \bar{s}_t}{\partial G_t\big(b_t^*, W_{2,t}, \Upsilon_t^b, \bar{s}_t\big)/\partial b} > 0,$$

as the second-order condition for a maximum implies $\partial G_t\big(b_t^*, W_{2,t}, \Upsilon_t^b, \bar{s}_t\big)/\partial b < 0$. □

Proposition 5.2. *Assume there are no premarital savings, no child care services, no time costs of child birth and no lump-sum taxes or specific subsidies. Then the optimal number of children depends on the relative wages of husband and wife and attains a minimum when wages are equal.*

Proof. Let $a_2 = 0$, $\psi = 1$, $T^b = 0$, $\bar{\tau}_t = 0$ and $\bar{s}_t = 0$. Write $X_t = w_t^m / w_t^f$ where w_t^j is the wage rate of spouse $j \in \{f, m\}$. Define:

$$\tilde{G}_t(b, X_t) = g_{1,t}(b)\tilde{W}_{2,t}(X_t) - g_{2,t}(b)\tilde{\Upsilon}_t^b(X_t),$$

where:

$$\tilde{W}_{2,t}(X_t) \equiv \frac{W_{2,t}}{w_t^f} = (1-\tau_t)\big[1 + X_t\big]\left[1 + \frac{1-\bar{R}}{1+r_{t+1}}\right],$$
$$\tilde{\Upsilon}_t^b(X_t) \equiv \frac{\Upsilon_t^b}{w_t^f} = \big[1 + X_t^{1-\xi}\big]^{\frac{1}{1-\xi}} N^b.$$

The optimal choice b^* is such that $\tilde{G}_t(b_t^*, X_t) = 0$. This implicitly defines b^* as a function of X_t. The partial derivatives of $\tilde{G}_t$ satisfy:

$$\frac{\partial \tilde{G}(b^*, X_t)}{\partial b} < 0,$$
$$\frac{\partial \tilde{G}(b^*, X_t)}{\partial X_t} = g_{1,t}(b)\frac{\tilde{W}_{2,t}(X_t)}{1 + X_t} - g_{2,t}(b)\frac{X_t^{-\xi}\tilde{\Upsilon}_t^b(X_t)}{1 + X_t^{1-\xi}},$$
$$\frac{\partial^2 \tilde{G}_t(b^*, X_t)}{\partial X_t^2} = g_{2,t}(b)\frac{\xi X_t^{-(1+\xi)}\tilde{\Upsilon}_t^b(X_t)}{\big[1 + X_t^{1-\xi}\big]^2} > 0,$$

where the sign of the first follows from the second-order condition for a maximum while the second and third expression use:

$$\frac{\partial \tilde{W}_{2,t}(X_t)}{\partial X_t} = \frac{\tilde{W}_{2,t}(X_t)}{1 + X_t}, \qquad \frac{\partial^2 \tilde{W}_{2,t}(X_t)}{\partial X_t^2} = 0,$$
$$\frac{\partial \tilde{\Upsilon}_t^b(X_t)}{\partial X_t} = \frac{X_t^{-\xi}\tilde{\Upsilon}_t^b(X_t)}{1 + X_t^{1-\xi}}, \qquad \frac{\partial^2 \tilde{\Upsilon}_t^b(X_t)}{\partial X_t^2} = -\frac{\xi X_t^{-(1+\xi)}\tilde{\Upsilon}_t^b(X_t)}{\big[1 + X_t^{1-\xi}\big]^2}.$$

In particular, when evaluated at $X_t = 1$ (equal wages for husband and wife):

$$\frac{\partial \tilde{G}_t(b^*, 1)}{\partial X_t} = \frac{1}{2}\tilde{G}_t(b^*, 1) = 0,$$
$$\frac{\partial^2 \tilde{G}_t(b^*, 1)}{\partial X_t^2} = \frac{2^{\frac{1}{1-\xi}}}{4} g_{2,t}(b^*)\xi N^b > 0.$$

The Implicit Function Theorem then implies that b^* attains a minimum at $X_t = 1$:

$$\left.\frac{\partial b^*}{\partial X_t}\right|_{X_t=1} = -\frac{\partial \tilde{G}_t(b^*, 1)/\partial X_t}{\partial \tilde{G}_t(b^*, 1)/\partial b} = 0,$$
$$\left.\frac{\partial^2 b^*}{\partial X_t^2}\right|_{X_t=1} = -\frac{\partial^2 \tilde{G}_t(b^*, 1)/\partial X_t^2}{\partial \tilde{G}_t(b^*, 1)/\partial b} > 0.$$

□

5.C Computational details

The macroeconomic equilibrium is obtained by the following iterative procedure.

(1) Make a guess for the per capita capital stock K_t/P_t and employment levels $N_t^Y(0)/P_t$, $N_t^Y(1)/P_t$ and $N_t^Z(0)/P_t$.

(2) Calculate the implied factor prices from the marginal productivity conditions of the firms.

– The interest rate r_t and the return to effective labour in the production sector w_t^Y:

$$r_t + \delta_K = \phi\Phi\left[\frac{K_t/P_t}{N_t^Y/P_t}\right]^{-(1-\phi)}, \quad w_t^Y = (1-\phi)\Phi\left[\frac{K_t/P_t}{N_t^Y/P_t}\right]^{\phi},$$

where:

$$\frac{N_t^Y}{P_t} = \left[\frac{N_t^Y(1)}{P_t}\right]^{\nu}\left[\frac{N_t^Y(0)}{P_t} + \frac{N_t^Y(1)}{P_t}\right]^{1-\nu}.$$

– The wage rates of uneducated workers $w_t(0)$ and educated workers $w_t(1)$:

$$w_t(0) = w_t^Y(1-\nu)\mu_t^{\nu}, \quad w_t(1) = \left[1 + \frac{\nu}{(1-\nu)\mu_t}\right]w_t(0),$$

where:

$$\mu_t = \frac{N_t^Y(1)/P_t}{N_t^Y(0)/P_t + N_t^Y(1)/P_t}.$$

– The price of child care services p_t:

$$p_t = \frac{w_t(0)}{\Psi}.$$

(3) Solve for the optimal household decisions given the factor prices using backward induction.

– For each level of education of both spouses and stock of joint savings the household allocation and corresponding value function are obtained by numerically solving the first-order conditions of a married couple.

– Conditional on matching probabilities the optimal level of savings in the first stage can be found for each type of individual (male or female, educated or uneducated) given the choice of the other types. The intersection of these best responses determines the actual savings decisions. The education thresholds follow.

– The overall marriage market equilibrium is obtained as a fixed point for the education frequencies and implied matching probabilities.

(4) Aggregate the household decisions.

– The average fertility rate $\bar{b}_t$ and the population growth rate $\bar{\eta}_t$.

– Per capita financial assets A_t/P_t, labour supply by education $L_t(0)/P_t$ and $L_t(1)/P_t$, consumption C_t/P_t and demand for child care services O_t/P_t.

(5) Set either the marginal tax rate τ_t (scenario (b) and (c)) or the specific subsidy $\bar{s}_t$ (scenario (iv)) so that the government budget is balanced:

$$\tau_t \left[w_t(0) \frac{L_t(0)}{P_t} + w_t(1) \frac{L_t(1)}{P_t} \right] - \bar{s}_t \frac{P_{0,t}}{P_t} = s_t p_t \frac{O_t}{P_t} - \bar{\tau}_t \left[1 - \frac{P_{0,t}}{P_t} \right].$$

(6) Calculate the remaining macroeconomic variables.

– Investment I_t/P_t:

$$\frac{I_t}{P_t} = (1 + \bar{\eta}_t) \frac{K_{t+1}}{P_{t+1}} - (1 - \delta_K) \frac{K_t}{P_t}.$$

– Output Y_t/P_t:

$$\frac{Y_t}{P_t} = \Phi \left[\frac{K_t}{P_t} \right]^{\phi} \left[\frac{N_t^Y}{P_t} \right]^{1-\phi}.$$

(7) Check whether the goods market is in equilibrium.

$$\frac{Y_t}{P_t} = \frac{C_t}{P_t} + \frac{I_t}{P_t} + \frac{F_t}{P_t}.$$

If not, then partially update the guess for the per capita capital stock and employment levels and start again from (1).

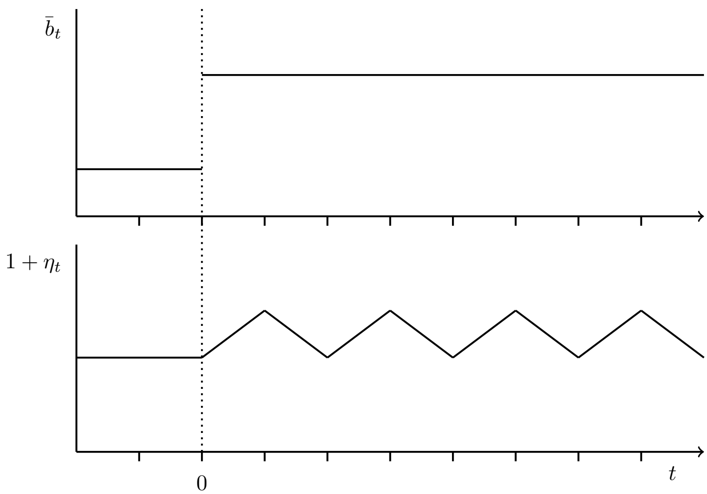
5.D Transitional dynamics

In the main text we have focused exclusively on the steady state of the model under different child care subsidization schemes. The reason that we do not display any of the transitional dynamics between the steady states is that they will be driven to a large extent by (unrealistic) cohort size effects. To see this, note that it follows from (5.21) that the dynamic relation between the cohort growth rate η_t and the average fertility rate $\bar{b}_t$ is given by:

$$1 + \eta_t = \frac{\bar{b}_t/2}{1 + \eta_{t-1}}. \tag{5.38}$$

If fertility is exogenous ($\bar{b}_t$ is determined outside the model) then this demographic structure features a negative unit root. Following an unexpected shock to $\bar{b}_t$ the cohort growth rate will be stuck on a cyclical path, see Figure 5.D.1. As a consequence the (per capita) economic variables will also display cyclical behaviour.

Figure 5.D.1: No convergence



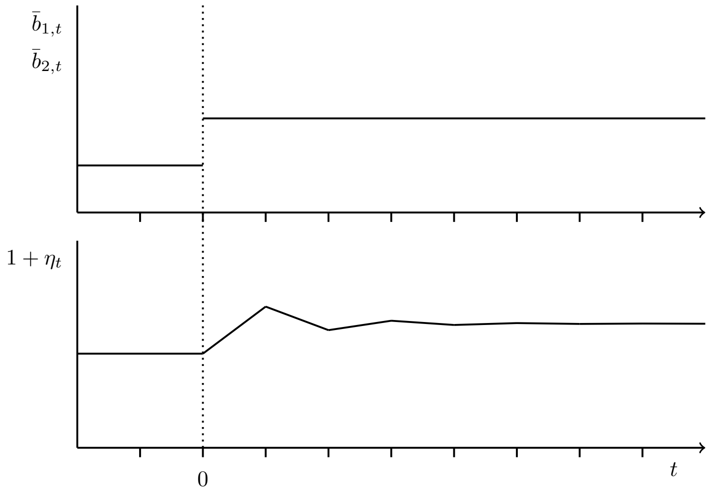
Notes: The initial demographic steady state features $\bar{b}_t = 2$ and $\eta_t = 0$. At time $t = 0$ there is an exogenous increase in $\bar{b}_t$.

This result is the consequence of two assumptions in the model. First, the timing of births is restricted to one specific period of the life cycle. Second, there is more than one period in between birth and parenthood. In the real world children are born to parents of different ages, which ensures that the perpetuation of cohort size differences occurs to a much smaller extent (see Auerbach and Kotlikoff (1987, Chapter 11)). In macroeconomic models with a more stylized demography it is usually assumed that a given birth rate applies to a whole age section of the population, which also serves to circumvent this issue. Suppose that couples have $\bar{b}_{1,t}$ children in stage 1 and $\bar{b}_{2,t}$ in stage 2. Then the population dynamics in (5.38) would be replaced by:

$$1 + \eta_t = \bar{b}_{1,t}/2 + \frac{\bar{b}_{2,t}/2}{1 + \eta_{t-1}}. \tag{5.39}$$

As long as $\bar{b}_{1,t} > 0$ (births are spread over two periods) or $\bar{b}_{2,t} = 0$ (there is only one period in between birth and parenthood) the unit root disappears and the system converges to a new steady state following a shock to fertility, see Figure 5.D.2.

Figure 5.D.2: Convergence



Notes: The initial demographic steady state features $\bar{b}_{1,t} = \bar{b}_{2,t} = 1$ and $\eta_t = 0$. At time $t = 0$ there is an exogenous increase in $\bar{b}_{1,t}$ and $\bar{b}_{2,t}$.

One of the crucial elements of the model employed in this chapter is that adults make their education decision alone before marriage and fertility take place. Hence there is necessarily a period in between birth and parenthood. In addition, given the time aggregation it would be unrealistic to allow couples to have children in their final life-cycle stage. Fortunately, with endogenous fertility even (5.38) can be stable provided that there is sufficient feedback from η_t to $\bar{b}_t$ through factor prices. Still the transitional dynamics will be dominated by the fluctuations in relative cohort size and are therefore not shown.

References Part II

Attanasio, O., H. Low and V. Sánchez-Marcos (2008). "Explaining changes in female labor supply in a life-cycle model". *American Economic Review* 98 (4), pp. 1517–1552.

Attanasio, O. and G. Weber (1995). "Is consumption growth consistent with intertemporal optimization? Evidence from the Consumer Expenditure Survey". *Journal of Political Economy* 103 (6), pp. 1121–1157.

Auerbach, A. J. and L. J. Kotlikoff (1987). *Dynamic Fiscal Policy.* Camebridge University Press.

Becker, G. S. (1960). "An economic analysis of fertility". *Demographic and Economic Change in Developed Countries.* Ed. by Universities-National Bureau. Columbia University Press. Chap. 7, pp. 225–256.

Becker, G. S., W. H. Hubbard and K. M. Murphy (2010). "Explaining the worldwide boom in higher education of women". *Journal of Human Capital* 4 (3), pp. 203–241.

Blau, D. M. and P. K. Robins (1988). "Child-care costs and family labor supply". *Review of Economics and Statistics* 70 (3), pp. 374–381.

Brilli, Y., D. Del Boca and C. Monfardini (2013). *Child care arrangements: Determinants and consequences.* CHILD Working Paper Series 18.

Browning, M., P.-A. Chiappori and Y. Weiss (2014). *Economics of the Family.* Camebridge University Press.

Caucutt, E. M., N. Guner and J. Knowles (2002). "Why do women wait? Matching, wage inequality, and the incentives for fertility delay". *Review of Economic Dynamics* 5, pp. 815–855.

Caucutt, E. M. and K. B. Kumar (2003). “Higher education subsidies and heterogeneity: A dynamic analysis”. *Journal of Economic Dynamics & Control* 27, pp. 1459–1502.

Chiappori, P.-A. (1992). “Collective labor supply and welfare”. *Journal of Political Economy* 100 (3), pp. 437–467.

Chiappori, P.-A., M. Iyigun and Y. Weiss (2009). “Investment in schooling and the marriage market”. *American Economic Review* 99 (5), pp. 1689–1713.

Chiappori, P.-A., B. Salanié and Y. Weiss (2011). *Partner choice and the marital college premium*. Discussion Paper No. 1011-04. Columbia University.

Crump, R., G. S. Goda and K. J. Mumford (2011). “Fertility and the personal exemption: Comment”. *American Economic Review* 101 (4), pp. 1616–1628.

DiPrete, T. A. and C. Buchmann (2006). “Gender-specific trends in the value of education and the emerging gender gap in college completion”. *Demography* 43 (1), pp. 1–24.

Domeij, D. and P. Klein (2013). “Should day care be subsidized?” *Review of Economic Studies* 80, pp. 568–595.

Dougherty, C. (2005). “Why are the returns to schooling higher for women than for men?” *Journal of Human Resources* 40 (4), pp. 969–988.

Echevarria, C. and A. Merlo (1999). “Gender differences in education in a dynamic household bargaining model”. *International Economic Review* 40 (2), pp. 265–286.

Fernández, R., N. Guner and J. Knowles (2005). “Love and money: A theoretical and empirical analysis of household sorting and inequality”. *Quarterly Journal of Economics* 120 (1), pp. 273–344.

Fernández, R. and R. Rogerson (2001). “Sorting and long-run inequality”. *Quarterly Journal of Economics* 116 (4), pp. 1305–1341.

Fernández, R. and J. C. Wong (2011). *The disappearing gender gap: The impact of divorce, wages, and preferences on education choices and women's work*. Working Paper 17508. NBER.

Goldin, C., L. F. Katz and I. Kuziemko (2006). "The homecoming of American college women: The reversal of the college gender gap". *Journal of Economics Perspectives* 20 (4), pp. 133–156.

Guner, N., R. Kaygusuz and G. Ventura (2013). *Childcare subsidies and household labor supply*. Barcelona GSE Working Paper 738.

Guvenen, F. and M. Rendall (2013). *Women's emancipation through education: A macroeconomic analysis*. Working Paper 18979. NBER.

Hotz, V. J., J. A. Klerman and R. J. Willis (1997). "The economics of fertility in developed countries". *Handbook of Population and Family Economics*. Elsevier Science. Chap. 7, pp. 275–347.

Hubbard, W. H. (2011). "The phantom gender difference in the college wage premium". *The Journal of Human Resources* 46 (3), pp. 568–586.

Iyigun, M. and R. P. Walsh (2007a). "Building the family nest: Premarital investments, marriage markets, and spousal allocations". *Review of Economic Studies* 74 (2), pp. 507–535.

– (2007b). "Endogenous gender power, household labor supply and the demographic transition". *Journal of Development Economics* 82, pp. 138–155.

Jacob, B. A. (2002). "Where the boys aren't: Non-cognitive skills, returns to school and the gender gap in higher education". *Economics of Education Review* 21, pp. 589–598.

Jarrell, S. B. and T. Stanley (2004). "Declining bias and gender wage discrimination? A meta-regression analysis". *Journal of Human Resources* 39 (3), pp. 828–838.

Jones, L. E., A. Schoonbroodt and M. Tertilt (2010). "Fertility theories: Can they explain the negative fertility-income relationship?" *Demography and the Economy*. Ed. by J. B. Shoven. University of Chicago Press, pp. 43–100.

King, R. G., C. I. Plosser and S. T. Rebelo (2002). "Production, growth and business cycles: Technical Appendix". *Computational Economics* 20, pp. 87–116.

Laughlin, L. (2013). *Who's minding the kids? Child care arrangements: Spring 2011*. Current Population Reports P70-135. U.S. Census Bureau.

Reijnders, L. S. M. (2014a). *The college gender gap reversal: Insights from a life-cycle perspective.* SOM Research Report 14006-EEF. University of Groningen.

– (2014b). *Child care subsidies with endogenous education and fertility.* SOM Research Report 14007-EEF. University of Groningen.

Ríos-Rull, V. and V. Sánchez-Marcos (2002). "College attainment of women". *Review of Economic Dynamics* 5 (4), pp. 965–998.

Ruggles, S., J. T. Alexander, K. Genadek, R. Goeken, M. B. Schroeder and M. Sobek (2010). *Integrated Public Use Microdata Series: Version 5.0.* machine-readable database. Minneapolis: University of Minnesota.

Sánchez-Marcos, V. (2007). "What accounts for the increase in college attainment for women?" *Applied Economics Letters* 15 (1), pp. 41–44.

CHAPTER 6

Samenvatting (Summary in Dutch)

Economen gaan er doorgaans van uit dat wanneer individuen een beslissing maken over het al dan niet volgen van onderwijs ze rationeel de kosten en baten hiervan tegen elkaar afwegen. Deze zijn niet uitsluitend in geld uit te drukken. De gebruikelijke aanname dat mensen een verhoging van hun eigen welvaart nastreven betekent niet noodzakelijkerwijs dat ze egoïstisch zijn of alleen om materiële zaken geven: dat hangt af van hoe welvaart is gedefinieerd. Bijvoorbeeld, de baten van een extra jaar onderwijs zijn onder andere een beter loon en meer baanzekerheid in de toekomst, maar ook een grotere kans om te gaan trouwen en een hogere levensverwachting. Naast lesgeld en gederfde arbeidsinkomsten heeft studeren ook mentale kosten, hoe hoog deze zijn hangt af van de cognitieve en niet-cognitieve vaardigheden die een persoon bezit.

In dit proefschrift bestuderen we het effect van veranderingen in de economische, demografische en sociale context op de beslissing van een individu om al dan niet te investeren in hoger onderwijs. We houden hierbij rekening met de manier waarop deze beslissing samenhangt met andere keuzes die een persoon maakt gedurende zijn of haar leven, zoals participatie in de arbeidsmarkt, gezinsplanning en de zorg voor kinderen. De modellen die we gebruiken zijn dynamisch van aard en nemen als uitgangspunt dat individuen in staat zijn tot het maken van consistente plannen voor de toekomst. In de meeste hoofdstukken hanteren we een algemeen evenwichtsperspectief, wat betekent dat we in ogenschouw nemen dat individuele beslissingen gezamenlijk leiden tot uitkomsten op macro niveau en hoe deze op hun beurt de afwegingen van huishoudens en bedrijven beïnvloeden.

Het proefschrift bestaat uit twee delen. In het eerste deel kijken we naar onderwijs in de bredere context van het opbouwen van menselijk kapitaal. Dit kapitaal bestaat uit alle vaardigheden en kennis die een individu bezit en die aangewend kunnen worden in de productie van economische waarde. Aan het begin van het leven omvat dit met name 'aangeboren talent', maar gedurende de levenscyclus breidt het zich uit door middel van investeringen in onderwijs en als gevolg van leren op de werkvloer. Tegelijkertijd zal een deel van de vaardigheden verloren gaan en zal kennis worden vergeten. Wanneer de mate waarin deze depreciatie van het menselijke kapitaal plaats vindt stijgt met de leeftijd dan kan dit ertoe leiden dat oudere mensen besluiten zich terug te trekken uit de arbeidsmarkt en met pensioen te gaan.

In Hoofdstuk 2 bestuderen we de rol van levensverwachting in de onderwijsbeslissing. Er is een horizon effect: wanneer mensen langer leven kunnen zij potentieel een groter profijt halen uit hun investeringen in menselijk kapitaal en dit zal hen stimuleren om langer onderwijs te blijven volgen. Echter, het betekent niet automatisch dat ze ook langer door blijven werken. We laten zien dat een verbetering van overlevingskansen in het algemeen een ambigu effect heeft op de optimale pensioenleeftijd. In onze numerieke simulaties besluiten individuen iets langer te werken. Desalniettemin verwachten ze meer jaren dan voorheen met pensioen te zijn en zullen ze hun besparingen daar op aan passen. Dit leidt tot een toename van de kapitaalintensiteit in het productieproces en daarmee een daling in de rentevoet en een stijging van de betaling aan arbeid. Een gevolg hiervan is dat gepensioneerden die al hun vermogen in de vorm van spaargeld hebben slechter af zijn terwijl jonge generaties van werkende individuen er op vooruit gaan. Aangezien het aantal werkende mensen per gepensioneerde naar beneden gaat wordt het moeilijker om een ongedekt pensioenstelsel in stand te houden. Significante aanpassingen zijn noodzakelijk, bijvoorbeeld een verhoging van de belasting op arbeid, een verlaging van de pensioenen of een hogere pensioengerechtigde leeftijd.

We onderzoeken ook een alternatief scenario, waarin de verbeteringen in gezondheid die leidden tot een hogere levensverwachting ook de duurzaamheid van het menselijk kapitaal beïnvloeden. Mensen verliezen dan hun kennis en vaardigheden op een lager tempo. Dit leidt tot een stijging van zowel de prijs van tijd als totale rijkdom, welke een tegenovergesteld effect hebben op de arbeidsaanbodbeslissing. Het is wederom niet eenduidig vanuit een theoretisch perspectief of de optimale pensioenleeftijd omhoog of omlaag gaat, maar in de simulaties stijgt deze aanzienlijk. Menselijk kapitaal wordt relatief overvloedig in het productieproces zodat de prijs voor arbeid en kapitaal zich in de tegenovergestelde richting bewegen als onder het vorige scenario. Doordat individuen langer werken vermindert de druk op het pensioenstelsel, wat betekent dat kleine aanpassingen voldoende zijn om het voortbestaan ervan te garanderen.

In Hoofdstuk 3 breiden we dit raamwerk uit met twee belangrijke componenten. De eerste is arbeidsmarktrisico. We nemen aan dat individuen voordat ze beginnen met werken niet precies weten in welke mate ze in staat zijn om te leren op de werkvloer.

Bovendien krijgen ze jaarlijks te maken met individuele productiviteitsschokken, waaronder het risico om werkeloos te raken. De tweede extensie is het expliciet modelleren van studiefinanciering. De manier waarop dit vormgegeven is bepaalt hoe risicovol het is om in onderwijs te investeren. Bijvoorbeeld, onze uitgangssituatie is een systeem van studieleningen die studenten geacht worden in vaste termijnen terug te betalen. Dit betekent dat ze jaarlijks een gegeven aflossing moeten betalen, ongeacht hoeveel inkomen ze verdienen. Gedurende periodes van lage arbeidsproductiviteit of werkeloosheid heeft dit tot gevolg dat ze weinig middelen overhouden om vrij te besteden. Dit kan sommige individuen ervan weerhouden om een opleiding te volgen.

We bestuderen vervolgens twee mogelijke beleidsalternatieven. De eerste is een belasting op hoger opgeleiden. In plaats van een expliciete studieschuld op te bouwen krijgt iedere student een toelage van de overheid om collegegeld en levensonderhoud uit te bekostigen. Deze worden betaald door middel van een belasting op het arbeidsinkomen van afgestudeerden. Het gevolg is dat in periodes met lage inkomsten de vereiste bijdrage ook klein zal zijn. Dit type risicodeling tussen hoger opgeleiden heeft met name een effect op de intensieve marge van de onderwijsbeslissing: het aantal afgestudeerden verandert nauwelijks maar ieder van hen investeert meer jaren in scholing. Daarnaast is er een positief effect op de algemene welvaart, mits de generaties die er op vooruit gaan door deze beleidswijzing de verliezers adequaat compenseren.

De tweede optie is een systeem van algemene inkomstenbelastingen. Ook lager opgeleiden betalen mee aan de studiebeurzen, wat een groot effect heeft op de extensieve marge van de onderwijsbeslissing: individuen die eerder niet een vervolgopleiding zouden volgen besluiten dat nu wel te doen. Er is niet alleen herverdeling van fortuinlijke personen (met een hoge arbeidsproductiviteit) naar minder goed bedeelden (zoals de werkelozen), maar ook van ongeschoolde naar geschoolde individuen. Dit leidt tot een algemeen welvaartsverlies.

In het tweede deel van het proefschrift besteden we aandacht aan de sociale omgeving van een individu en hoe deze de onderwijsbeslissing beïnvloedt. Ten eerste onderkennen we dat een huishouden doorgaans bestaat uit meerdere personen die met elkaar interacteren in het maken van keuzes. Bijvoorbeeld, een echtpaar zal samen besluiten hoeveel kinderen ze willen en op welke manier ze de zorg voor hen verdelen. Ten tweede, op het moment dat een individu kiest om al dan niet hoger onderwijs te gaan volgen is hij of zij doorgaans nog vrijgezel, maar verwachtingen aangaande de kans om te gaan trouwen en de eigenschappen van een toekomstige partner spelen in deze beslissing een belangrijke rol.

In Hoofdstuk 4 bestuderen we het fenomeen dat tegenwoordig in veel ontwikkelde landen meer vrouwen dan mannen een universitaire graad behalen. Op het eerste gezicht lijkt dit moeilijk te verklaren, aangezien vrouwen over het algemeen een lager uurloon krijgen dan mannen met vergelijkbare kwalificaties (de loonkloof) en ze doorgaans minder uren werken als gevolg van zwangerschap en de zorg voor kinderen. Om inzicht te krijgen in de prikkels om in onderwijs te investeren splitsen we de baten van een vervolgopleiding op in twee componenten. De eerste is een arbeidsmarktvoordeel, wat correspondeert met het profijt van een opleiding voor iemand die met zekerheid de rest van zijn of haar leven vrijgezel blijft (zoals in de modellen van Hoofdstuk 5 en 3). We laten zien dat wanneer het relatieve loon van hoger opgeleiden gelijk is voor beide geslachten maar er ook sprake is van een loonkloof tussen mannen en vrouwen, het arbeidsmarktvoordeel van een opleiding groter is voor vrouwen. Met een sterk afnemend marginaal welvaartseffect van extra rijkdom hebben zij een grotere prikkel om hun inkomen te verhogen door middel van een investering in onderwijs. Dit resultaat wordt versterkt wanneer er vaste lasten zijn om een huishouden draaiende te houden, aangezien deze het zwaarst wegen voor ongeschoolde vrouwen (gegeven dat hun loonvoet het laagst is). Onderwijs biedt deze vrouwen een mogelijkheid om armoede te voorkomen.

Het resterende deel van de onderwijsbaten kan worden toegewezen aan de rol van de huwelijksmarkt. In hoeverre deze de onderwijsbeslissing verstoort hangt af van de mate waarin het volgen van extra onderwijs de kans vergroot om een hoger opgeleide partner te vinden en de manier waarop het onderwijsniveau van echtgenoten de verdeling van tijd en middelen binnen het huishouden beïnvloedt. Het is waarschijnlijk dat de huwelijksmarktverstoring de baten van onderwijs verlaagt voor vrouwen relatief tot mannen. Wanneer er een loonkloof is verwacht een vrouw een rijkere echtgenoot te trouwen, wat haar minder prikkels geeft om te investeren in het vergroten van haar eigen arbeidsinkomsten. Bovendien is de prijs van tijd van een vrouw belangrijker in de beslissing aangaande het aantal kinderen dat een echtpaar heeft dan dat van de man, gezien het feit dat alleen zij de zwangerschap kan volbrengen.

Met behulp van een numerieke simulatie laten we zien welke veranderingen in de economische en sociale context er toe kunnen leiden dat er een omslag plaats vindt in het relatieve aantal hoger opgeleiden van elk geslacht. Een verlaging van de kans om te gaan trouwen zoals recentelijk geobserveerd in de Verenigde Staten zou voldoende zijn. In het nieuwe evenwicht investeren risico-afkerige vrouwen meer in onderwijs dan mannen omdat zij de extra inkomsten harder nodig hebben in het geval ze vrijgezel blijven.

Hoofdstuk 5 plaatst de huishoudens van Hoofdstuk 4 in een algemeen evenwichtsmodel om vervolgens het gezinsbeleid van de overheid te bestuderen. We zijn hierbij met name geïnteresseerd in het effect van subsidies voor kinderopvang op fertiliteitskeuzes en onderwijsbeslissingen. Door professionele kinderopvang betaalbaarder te maken worden de kosten van een kind lager en kunnen ouders meer tijd doorbrengen in de arbeidsmarkt. Dit zou er toe kunnen leiden dat mensen meer kinderen willen en langer naar school gaan. Echter, het subsidieprogramma zal gefinanceerd moeten worden uit belastinginkomsten. De extra belastingen veranderen de prijs van tijd van ouders en dit zal een gevolg hebben voor de keuze om zelf voor kinderen te zorgen of ze naar de opvang te laten gaan. Bovendien zal de grotere vraag naar personeel bij de kinderopvang leiden tot een afname van het aantal ongeschoolde medewerkers in de productiesector. Dit heeft een negatief effect op de relatieve lonen van een hoger opgeleiden en daarmee op de prikkels om in onderwijs te investeren.

In onze numerieke simulatie is in de uitgangssituatie zonder belastingen en subsidies het aantal kinderen het hoogst voor echtparen die bestaan uit een ongeschoolde vrouw en een hoger opgeleide man en het laagst voor huishoudens waar beide ouders geschoold zijn. Er is een negatieve relatie tussen het opleidingsniveau van de vader en het aantal kinderen, wat consistent is met bevindingen uit de empirie. De introductie van een subsidie op de prijs van kinderopvang gefinanceerd door middel van een vlakke belasting op arbeid zorgt voor een toename in het aantal kinderen bij alle type huishoudens. De verdeling van arbeid over sectoren zorgt ervoor dat de relatieve lonen van hoger opgeleiden dalen, met als gevolg dat in het nieuwe evenwicht het aantal geschoolde individuen lager is. Wanneer het beleid tot doel heeft om fertiliteit te stimuleren dan kan dit effectiever worden bereikt door het verstrekken van een vaste subsidie per kind. Echter, in dat geval besteden ouders de kinderopvang niet uit, wat leidt tot een afname in arbeidsparticipatie. Dit is met name het geval voor ongeschoolde getrouwde vrouwen aangezien zij het grootste deel van de zorgtaken op zich nemen.

Al met al omvat dit proefschrift een breed scala aan perspectieven en geeft daarmee een goed beeld van de afwegingen die een rol spelen bij de beslissing al dan niet in onderwijs te investeren.

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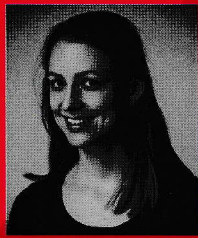

Laurie Sarah May Reijnders

Education Choices in a Changing Economic, Demographic and Social Environment

In this thesis we study the effect of changes in the economic, demographic and social environment on an individual's decision to invest in tertiary education. These changes include adjustments to the system of educational loans, an increase in longevity, the introduction of subsidies on professional child care and a drop in marriage rates. We also consider how the education decision interacts with various other choices that a person makes over the course of his or her life, such as labour market participation and fertility.

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ISBN 978-90-367-7624-0

9 789036 776240 >