Some unconventional properties of New Keynesian DSGE models: Supplementary material*

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Abstract. In the main paper we examine the theoretical and numerical properties of a prototypical New Keynesian DSGE model featuring endogenous capital accumulation and labour supply. This mathematical appendix contains the derivations of all the results mentioned in the main text. In additional it presents some further material, for example money supply shocks and government spending shocks are also studied. Matlab programs are available upon request from the corresponding author.

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A.1 Model derivations

A.1.1 Equation (15)

The price set by a green-light firm (15) is derived as follows. The first-order necessary condition for maximizing $V_t^0(i)$ by choice of $P_t(i)$ is:

$$\frac{dV_t^0(i)}{dP_t(i)} = \mathbb{E}_t \sum_{\tau=0}^{\infty} \zeta^{\tau} \mathcal{N}_{t,t+\tau} \frac{\partial \Phi\left(P_t(i), X_{t+\tau}\right)}{\partial P_t(i)} = 0.$$
(A.1)

It follows from the definition in (12) that:

$$\frac{\partial \Phi\left(P_{t}(i), X_{t+\tau}\right)}{\partial P_{t}(i)} = P_{t}(i)^{-\theta} P_{t+\tau}^{\theta} Y_{t+\tau} \left[1 - \theta \frac{P_{t}(i) - MC_{t+\tau}}{P_{t}(i)}\right]$$
$$= (1 - \theta) P_{t}(i)^{-\theta} P_{t+\tau}^{\theta} Y_{t+\tau} \left[1 - \frac{\theta}{\theta - 1} \frac{MC_{t+\tau}}{P_{t}(i)}\right].$$
(A.2)

By substituting (A.2) into (A.1) (and eliminating $1 - \theta$) we find that the first-order condition can be written as:

$$0 = \mathbb{E}_t \sum_{\tau=0}^{\infty} \zeta^{\tau} \mathcal{N}_{t,t+\tau} P_t(i)^{-\theta} P_{t+\tau}^{\theta} Y_{t+\tau} \left[1 - \frac{\theta}{\theta - 1} \frac{MC_{t+\tau}}{P_t(i)} \right].$$
(A.3)

The expression in (A.3) can be written as:

$$\Xi_t^D P_t(i) = \frac{\theta}{\theta - 1} \Xi_t^N, \tag{A.4}$$

with:

$$\Xi_t^D \equiv \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \zeta^{\tau} \mathcal{N}_{t,t+\tau} P_{t+\tau}^{\theta} Y_{t+\tau} \right], \tag{A.5}$$

$$\Xi_t^N \equiv \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \zeta^{\tau} \mathcal{N}_{t,t+\tau} P_{t+\tau}^{\theta} Y_{t+\tau} M C_{t+\tau} \right].$$
(A.6)

Finally, by substituting (A.5)–(A.6) in (A.4) and simplifying we obtain the expression for $P_t^n(i)$ stated in (15).

A.1.2 Equation (16)

The expression for the aggregate price level P_t , given in (16), can be derived as follows. By using (2)–(3) we find:

$$P_t^{1-\theta} \equiv \int_0^1 P_t(i)^{1-\theta} di.$$
 (A.7)

At time t a fraction $1 - \zeta$ of firms in the intermediate goods sector obtain a green light and set the price equal to P_t^n . Hence, a component of P_t consists of the prices newly set in period t:

$$P_t^{1-\theta} \equiv (1-\zeta) \left(P_t^n\right)^{1-\theta} + \int_{1-\zeta}^1 P_t \left(i\right)^{1-\theta} di.$$
(A.8)

The second expression on the right-hand side represent the component of P_t resulting from prices set in the past (i.e., P_{t-s}^n for s = 1, 2, ...). The law of large numbers says that $(1 - \zeta) \zeta^s$ is the fraction of firms which determined its new price s periods before period t. Hence, we know exactly the weights that should be given to prices set in previous periods: $\zeta(1 - \zeta)$ is the weight for P_{t-1}^n , $\zeta^2(1 - \zeta)$ is the weight for P_{t-2}^n , etcetera. We thus obtain from (A.8) that:

$$P_t^{1-\theta} = (1-\zeta) \left[(P_t^n)^{1-\theta} + \zeta (P_{t-1}^n)^{1-\theta} + \zeta^2 (P_{t-2}^n)^{1-\theta} + \dots \right].$$
(A.9)

It follows from (A.9) that the lagged price level can be written as:

$$\zeta P_{t-1}^{1-\theta} = (1-\zeta) \left[\zeta \left(P_{t-1}^n \right)^{1-\theta} + \zeta^2 \left(P_{t-2}^n \right)^{1-\theta} + \zeta^3 \left(P_{t-3}^n \right)^{1-\theta} + \dots \right].$$
(A.10)

Hence, $P_{t-1}^{1-\theta}$ shares all but one of the terms appearing in $P_t^{1-\theta}$. Indeed, by using (A.10) in (A.9) and taking the exponent to the other side we find:

$$P_t^{1-\theta} = (1-\zeta) \left(P_t^n\right)^{1-\theta} + \zeta P_{t-1}^{1-\theta}.$$
(A.11)

By raising both sides of this expression to the exponent $1/(1-\theta)$ we find equation (16).

A.1.3 Equations (30) and (32)

The relationship between the alternative output measure Y_t^a and aggregate factor supplies in (30) is derived as follows. Recall that for each firm *i* we have:

$$\frac{W_t}{R_t^K} = \frac{(1-\alpha)MC_t \frac{Y_t(i)}{L_t(i)-\bar{L}}}{\alpha MC_t \frac{Y_t(i)}{K_{t-1}(i)}} = \frac{1-\alpha}{\alpha} \frac{K_{t-1}(i)}{L_t(i)-\bar{L}}.$$
(A.12)

Hence, at both firm and aggregate level we have:

$$\frac{K_{t-1}}{L_t - \bar{L}} = \frac{K_{t-1}(i)}{L_t(i) - \bar{L}} = \Gamma_t \quad \left[\equiv \frac{\alpha}{1 - \alpha} \frac{W_t}{R_t^K} \right].$$
(A.13)

The alternative quantity index for aggregate output can now be computed as:

$$Y_t^a \equiv \int_0^1 Y_t(i) di = \int_0^1 K_{t-1}(i)^\alpha \left[Z_t \left(L_t(i) - \bar{L} \right) \right]^{1-\alpha} di$$
$$= Z_t^{1-\alpha} \int_0^1 \left[\Gamma_t \left(L_t(i) - \bar{L} \right) \right]^\alpha \left[L_t(i) - \bar{L} \right]^{1-\alpha} di$$

$$= Z_t^{1-\alpha} \Gamma_t^{\alpha} \int_0^1 (L_t(i) - \bar{L}) di = Z_t^{1-\alpha} \Gamma_t^{\alpha} (L_t - \bar{L}).$$
(A.14)

By using the fact that $\Gamma_t = K_{t-1}/(L_t - \bar{L})$ in (A.14) we obtain (30) in the text. To derive (32) we substitute the demand for variety *i* (stated in equation (4)) into the definition of Y_t^a (given in (30)):

$$Y_t^a \equiv \int_0^1 Y_t(i) \, di = Y_t P_t^\theta \, \int_0^1 P_t(i)^{-\theta} \, di.$$
(A.15)

By using the definition for P_t^a (given in (31)) in (A.15) we find (32).

A.1.4 Household behaviour

In order to find the first-order conditions for the household's decision problem we postulate the Lagrangian:

$$\mathcal{L}_{t}^{H} \equiv \mathbb{E}_{t} \sum_{\tau=t}^{\infty} \left(\frac{1}{1+\rho} \right)^{\tau-t} \left[U(C_{\tau}, L_{\tau}, M_{\tau+1}/P_{\tau}) + \lambda_{\tau} \left(W_{\tau} L_{\tau} + R_{\tau}^{K} K_{\tau-1} + (1+R_{\tau-1}) B_{\tau} + \sum_{s=0}^{\infty} X_{\tau}^{s} S_{\tau}^{s} + M_{\tau} - P_{\tau} T_{\tau} - P_{\tau} \left[C_{\tau} + K_{\tau} - (1-\delta) K_{\tau-1} \right] - M_{\tau+1} - B_{\tau+1} - \sum_{s=0}^{\infty} Q_{\tau}^{s} S_{\tau+1}^{s} \right) \right],$$

where λ_{τ} is the Lagrange multiplier for the budget identity in period τ . Assuming an interior solution the first-order conditions for this problem (for $\tau = t, t + 1, t + 2, ...$) are:

$$\frac{\partial \mathcal{L}_t^H}{\partial C_{\tau}} = \left(\frac{1}{1+\rho}\right)^{\tau-t} \mathbb{E}_t \left[U_C(C_{\tau}, L_{\tau}, M_{\tau+1}/P_{\tau}) - \lambda_{\tau} P_{\tau} \right] = 0, \tag{A.16}$$

$$\frac{\partial \mathcal{L}_t^H}{\partial L_\tau} = \left(\frac{1}{1+\rho}\right)^{\tau-t} \mathbb{E}_t \left[U_L(C_\tau, L_\tau M_{\tau+1}/P_\tau) + \lambda_\tau W_\tau \right] = 0, \tag{A.17}$$

$$\frac{\partial \mathcal{L}_t^H}{\partial K_{\tau}} = \left(\frac{1}{1+\rho}\right)^{\tau-t} \mathbb{E}_t \left[-\lambda_{\tau} P_{\tau} + \frac{\lambda_{\tau+1}}{1+\rho} (R_{\tau+1}^K + (1-\delta)P_{\tau+1})\right] = 0, \tag{A.18}$$

$$\frac{\partial \mathcal{L}_t^H}{\partial B_{\tau+1}} = \left(\frac{1}{1+\rho}\right)^{\tau-t} \mathbb{E}_t \left[-\lambda_\tau + \frac{\lambda_{\tau+1}}{1+\rho}(1+R_\tau)\right] = 0, \tag{A.19}$$

$$\frac{\partial \mathcal{L}_t^H}{\partial S_{\tau+1}^s} = \left(\frac{1}{1+\rho}\right)^{\tau-t} \mathbb{E}_t \left[-\lambda_\tau Q_\tau^s + \frac{\lambda_{\tau+1}}{1+\rho} X_{\tau+1}^s\right] = 0, \tag{A.20}$$

$$\frac{\partial \mathcal{L}_t^H}{\partial M_{\tau+1}} = \left(\frac{1}{1+\rho}\right)^{\tau-t} \mathbb{E}_t \left[\frac{U_{M/P}(C_{\tau}, L_{\tau}, M_{\tau+1}/P_{\tau})}{P_{\tau}} - \lambda_{\tau} + \frac{\lambda_{\tau+1}}{1+\rho}\right] = 0.$$
(A.21)

For the planning period, the expression in (A.16) implies that $\lambda_t P_t = U_C(C_t, L_t, M_{t+1}/P_t)$. But when period t + 1 comes around the household will set $\lambda_{t+1}P_{t+1} = U_C(C_{t+1}, L_{t+1}, M_{t+2}/P_{t+1})$. Using these results in (A.18)-(A.21) we find the expressions in (22)–(24) and (26).

A.1.5 Equation (T7.9)

Equation (T7.9) in Table 7 in the paper is derived as follows. In the first step we linearize equations (T1.9)-(T1.11) to obtain:

$$\begin{split} \tilde{P}_t^n &= \tilde{\Xi}_t^N - \tilde{\Xi}_t^D, \\ \tilde{\Xi}_t^N &= \frac{1+\rho-\zeta}{1+\rho} \left[-\frac{1}{\sigma} \tilde{C}_t + \theta \tilde{P}_t + \tilde{Y}_t + \widetilde{mc}_t \right] + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\tilde{\Xi}_{t+1}^N \right], \\ \tilde{\Xi}_t^D &= \frac{1+\rho-\zeta}{1+\rho} \left[-\frac{1}{\sigma} \tilde{C}_t + (\theta-1) \tilde{P}_t + \tilde{Y}_t \right] + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\tilde{\Xi}_{t+1}^D \right]. \end{split}$$

By combining these expressions we thus find that:

$$\tilde{P}_t^n = \frac{1-\zeta+\rho}{1+\rho} \left[\tilde{P}_t + \widetilde{mc}_t \right] + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\tilde{P}_{t+1}^n \right].$$
(A.22)

By linearizing (T1.12) we obtain:

$$\tilde{P}_t = (1-\zeta)\tilde{P}_t^n + \zeta\tilde{P}_{t-1}.$$
(A.23)

Armed with equations (A.22)–(A.23) we can derive the linearized inflation equation (T7.9) as follows. First we solve (A.23) for P_t^n and \tilde{P}_{t+1}^n :

$$\tilde{P}_t^n = \frac{\tilde{P}_t - \zeta \tilde{P}_{t-1}}{1 - \zeta}, \qquad \tilde{P}_{t+1}^n = \frac{\tilde{P}_{t+1} - \zeta \tilde{P}_t}{1 - \zeta}$$

Next we substitute both expressions into (A.22):

$$\tilde{P}_{t}^{n} = \frac{1-\zeta+\rho}{1+\rho} \left[\tilde{P}_{t} + \widetilde{m}c_{t} \right] + \frac{\zeta}{1+\rho} \mathbb{E}_{t} \left[\tilde{P}_{t+1}^{n} \right]$$

$$\tilde{P}_{t} - \zeta \tilde{P}_{t-1} = \frac{(1-\zeta+\rho)\left(1-\zeta\right)}{1+\rho} \left[\tilde{P}_{t} + \widetilde{m}c_{t} \right] + \frac{\zeta}{1+\rho} \mathbb{E}_{t} \left[\tilde{P}_{t+1} - \zeta \tilde{P}_{t} \right]$$

$$\zeta \left[\tilde{P}_{t} - \tilde{P}_{t-1} \right] = \frac{(1-\zeta+\rho)\left(1-\zeta\right)}{1+\rho} \widetilde{m}c_{t} + \frac{\zeta}{1+\rho} \mathbb{E}_{t} \left[\tilde{P}_{t+1} - \tilde{P}_{t} \right]$$
(A.24)

Finally, we define $\pi_t \equiv \tilde{P}_t - \tilde{P}_{t-1}$ and rewrite (A.24) in terms of (expected) inflation rates:

$$\pi_t = \frac{\phi}{1+\rho} \widetilde{mc}_t + \frac{1}{1+\rho} \mathbb{E}_t \pi_{t+1}, \tag{A.25}$$

where ϕ is a composite parameter:

$$\phi \equiv \frac{(1-\zeta+\rho)(1-\zeta)}{\zeta}.$$
(A.26)

Equation (A.25) is reported as (T7.9) in Table 7 in the paper.

A.2 Parameterization

We postulate that the deterministic steady state of the benchmark model in Table 1 has the following features:

- The money supply and government consumption are both constant: $M_{t+1} = \overline{M}$ and $G_t = \overline{G}$.
- The technology is constant: $\eta_t^z = 0$, and $\tilde{Z}_t = \tilde{Z}_{t-1} = 0$.
- All prices are constant: $P_t = P_{t-1} = P_t^a = P_{t-1}^a = P^*.$
- The output measures coincide: $Y_t = Y_t^a = Y^*$.
- Capital and consumption are constant over time: $K_{t+1} = K_t = K^*$ and $C_{t+1} = C_t = C^*$.
- The remaining variables attain their steady-state values: $I_t = I^*$, $L_t = L^*$, $R_t = R^*$, $w_t = w^*$, $r_t^K = r^{K*}$, $mc_t = mc^*$, $\Xi_t^N = \Xi^{N*}$, and $\Xi_t^D = \Xi^{D*}$.

We parameterize the benchmark model in such a way that the following targets are met for the deterministic steady state:

$$Y^* = 1, \quad L^* = \frac{1}{3}, \quad \frac{\bar{G}}{Y^*} = \frac{2}{10}, \quad \frac{\bar{M}}{P^*Y^*} = \frac{2}{10}, \quad P^* = 1, \quad \epsilon_{MR} \equiv -\frac{d\ln\left(M_{t+1}/P_t\right)}{dR_t} = 0.08.$$

Output and the equilibrium price level are both normalized to unity, employment is one-third of the available time endowment, and the government spending share of output is twenty percent. Finally we set the ratio of the money supply to nominal quarterly GDP equal to twenty percent. For the Netherlands in the Fall of 2020 we find that $\overline{M} \approx 74 \notin 10^9$ and $P^*Y^* \approx 238 \notin 10^9$ yielding a slightly higher fraction of 0.31. Finally, we target the (local) interest semi-elasticity of the demand for real money balances to equal 0.08 in absolute value (this is in the range of values reported by Ball (2012)).

In the top panel of Table 3 in the paper we report the parameters that are set a priori. Felicity is logarithmic in consumption ($\sigma = 1$), the Frisch elasticity of labour supply ($1/\kappa$) equals unity, the rate of time preference is five percent on a annual basis. Chetty et al. (2011) provide evidence on the Frisch elasticity of labour supply. They suggest that "it would be reasonable to calibrate representative-agent macro models to match a Frisch elasticity of aggregate hours of 0.75." (2011, p. 474). See also Chetty el al. (2013, p. 20) where they report a value of 0.86. Capital depreciates at ten percent per annum. In the model $\frac{\theta}{\theta-1}$ is the steady-state markup of price over marginal cost. A reasonable range of values for that markup is between 1.1 and 1.2 (implying some monopoly power but not an outrageous amount). We select a fairly conservative estimate and set $\theta = 9$ yielding a markup of 1.125. We set $\xi = 0.1$ so overhead labour plays a modest role in the steady state. Even though we do not need them to calibrate the steady-state, in the computations of the transition paths values for ζ and ξ_z are required. Following Bernanke et al. (1999) we assume that the probability that a firm does not change its price in a given quarter is 75 percent (i.e. $\zeta = 0.75$). The average period between adjustments is thus four quarters. The value for ξ_z is familiar from the RBC literature featuring technology shocks.

In the bottom panel of Table 3 in the paper we report the calibrated and implied parameters. The calibration parameters are $\Omega_0, \eta, \varepsilon_l$, and ε_m . The calibration proceeds as follows:

• Set:

$$R^* = \rho, \qquad mc^* = \frac{\theta - 1}{\theta}, \qquad \frac{K^*}{Y^*} = \frac{\theta - 1}{\theta} \frac{\alpha}{\rho + \delta} = 5.8069.$$

• Compute:

$$\omega_I^* = \frac{\delta K^*}{Y^*} = 0.1510, \qquad \omega_C^* = 1 - \omega_I^* - \omega_G^* = 0.6490.$$

• Set $\overline{L} = \xi L^*$ and $Y^* = 1$ (so that $K^* = K^*/Y^*$) and compute:

$$\Omega_0 = \frac{(K^*)^{\alpha/(\alpha-1)}}{(1-\xi)L^*} = 1.8545, \qquad w^* = (1-\alpha)\frac{\theta-1}{\theta} \frac{Y^*}{(1-\xi)L^*} = 2.2222.$$

• Since $C^* = \omega_C^*$ we set ε_l equal to:

$$\varepsilon_l = w^* (C^*)^{-1/\sigma} (L^*)^{-\kappa} = 10.2716.$$

• We use η to achieve the target for ϵ_{MR} . Consider the steady-state annual interest rate, $R_0^a = 0.05$, and the alternative interest rate, $R_1^a = 0.06$. In quarterly term we obtain $R_0 = (1 + R_0^a)^{1/4} - 1$ and $R_1 = (1 + R_1^a)^{1/4} - 1$. Next we set:

$$\eta = \frac{\epsilon_{MR}}{\ln\left(\frac{R_1}{1+R_1}\right) - \ln\left(\frac{R_0}{1+R_0}\right)} = 0.4563.$$

We motivate our calibration of the parameter for real money balances by using the empirical results of Ball (2001, 2012). Given our calibration, setting $\sigma = 1$ and $\eta = 0.4563$ results in an interest semi-elasticity of $\epsilon_{MR} = 0.08$ and a consumption elasticity of $\epsilon_{MC} = 0.4543$. Ball (2012, p. 628) finds the estimates $\hat{\epsilon}_{MR} \in \{0.040, 0.082\}$ and $\hat{\epsilon}_{MY} \in \{0.532, 0.467\}$. If consumption and output are proportional in the long run then these results are similar to our calibration.

• Finally, to achieve the target for $\overline{M}/(P^*Y^*) = 0.2$ we set ε_m equal to:

$$\varepsilon_m = \frac{R^*}{1+R^*} (C^*)^{-1/\sigma} \left(\frac{\bar{M}}{P^*}\right)^{1/\eta} = 5.3764 \ 10^{-4}.$$

• The Dynare computations confirm that the rank condition is satisfied as there are 5 eigenvalues larger than 1 in modulus for 5 forward-looking variables.

A.3 Alternative models

In this section we present some of the details on the alternative model specifications that are discussed in the paper. The simulation results for a technology shock have been reported in Tables 5 and 8 in the paper.

A.3.1 Fixed non-depreciating production factor

- Assume that the capital stock is fixed (and does not depreciate). Possible interpretation: K represents a fixed stock of non-depreciating land that is used as a productive input.
- The representative agent can buy or sell units of land.
- Obviously, there is no capital accumulation so that we set $I_t = 0$.
- The household's periodic budget identity (19) is changed to:

$$P_{\tau}C_{\tau} + P_{\tau}^{K}(K_{\tau} - K_{\tau-1}) + M_{\tau+1} + B_{\tau+1} + \sum_{s=0}^{\infty} Q_{\tau}^{s} S_{\tau+1}^{s} = W_{\tau}L_{\tau} + R_{\tau}^{K}K_{\tau-1} + (1 + R_{\tau-1})B_{\tau} + \sum_{s=0}^{\infty} X_{\tau}^{s} S_{\tau}^{s} + M_{\tau} - P_{\tau}T_{\tau},$$

where P_{τ}^{K} and R_{τ}^{K} are, respectively, the purchase price of and rental rate on units of land at time τ .

• Following similar derivations to those given above (in subsection A.1.4) we find that equation (22) is changed to:

$$1 = \mathbb{E}_t \left[\frac{r_{t+1}^K + p_{t+1}^K}{p_t^K} \frac{1}{1+\rho} \left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \right],$$
(A.27)

where $p_t^K \equiv P_t^K / P_t$ is the real price of land.

• Using this expression we find that p_t^K is the present expected value of future rental rates on land:

$$p_t^K = \mathbb{E}_t \sum_{\tau=1}^{\infty} \mathcal{R}_{t,t+\tau} r_{t+\tau}^K, \tag{A.28}$$

where $\mathcal{R}_{t,t+\tau}$ is the real stochastic discount factor:

$$\mathcal{R}_{t,s} \equiv \left(\frac{1}{1+\rho}\right)^{s-t} \frac{U_C(C_s, L_s, M_{s+1}/P_s)}{U_C(C_t, L_t, M_{t+1}/P_t)}.$$
(A.29)

• The system is given in Table A.1. The differences between this table and Table 1 in the paper are: (i) $K_t = K$ is fixed, (ii) $I_t = 0$, (iii) equation (T1.1) is dropped, and equation

(T1.4) is replaced by (TA1.3).

- In the simulations we set the stock of land equal to the steady-state capital stock for the benchmark model ($K^* = 5.8069$). We keep government consumption at its benchmark level ($\bar{G} = 0.2$) so that in the absence of investment spending we obtain $C^* = 0.8$. We recalibrate the model (by changing ε_l and ε_m) such that it features the same (deterministic) steady-state solutions for the remaining endogenous variables (except Ξ^{N*} and Ξ^{D*}).
- The simulation results have been reported in Table 5(c) in the paper.
- The impulse response function for the persistent technology shock are presented in Figure A.1. The main features are:
 - The odd responses of output and employment are not observed (panels (c) and (b)).
 - Employment and the real wage rate both fall at impact (panels (c) and (f)). In the spot market for labour, at impact labour demand shifts to the left (because marginal cost has fallen) and the supply curve shifts to the left (because consumption has gone up). The demand effect dominates the supply effect leading to a fall in both the wage and employment.
 - At impact the rental rate on units of land falls (panel (g)). In the rental market for land, at impact the demand curve shifts to the left (because employment has fallen) and the supply curve stays put (as it is perfectly inelastic at $K_t = K$). To clear the rental market the rental rate on land falls.
 - The impact reduction in both factor prices leads to a sharp decrease in real marginal cost (panel (i)) and a sharp reduction in the price set by green-light firms (see the open dots in panel (h)).
 - The purchase price of land rises at impact (panel (d)). This is because over time the rental rate on land overshoots its ultimate long-run equilibrium level (panel (g)).

Table A.1: The New Keynesian DSGE model with a fixed non-depreciating factor

$$Y_t = C_t + \bar{G}$$

$$1 = \mathbb{E}_t \left[\frac{1 + R_t}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \frac{P_t}{P_{t+1}} \right]$$
(TA1.1)
(TA1.2)

$$1 = \mathbb{E}_t \left[\frac{r_{t+1}^K + p_{t+1}^K}{(1+\rho)p_t^K} \left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \right]$$
(TA1.3)

$$\frac{M_{t+1}}{P_t} = C_t^{\eta/\sigma} \left(\frac{R_t}{\varepsilon_m(1+R_t)}\right)^{-\eta}$$
(TA1.4)
$$\varepsilon_l L_t^{\kappa} = w_t C_t^{-1/\sigma}$$
(TA1.5)

$$w_t = (1 - \alpha) mc_t \frac{Y_t^a}{L_t - \bar{L}}$$
(TA1.6)

$$r_t^K = \alpha \, mc_t \, \frac{Y_t^a}{K} \tag{TA1.7}$$

$$P_t^n = \frac{\theta}{\theta - 1} \frac{\Xi_t^N}{\Xi_t^D} \tag{TA1.8}$$

$$\Xi_t^N = C_t^{-1/\sigma} P_t^{\theta} Y_t m c_t + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^N \right]$$
(TA1.9)

$$\Xi_t^D = C_t^{-1/\sigma} P_t^{\theta-1} Y_t + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^D \right]$$
(TA1.10)

$$P_{t} = \left[(1 - \zeta) (P_{t}^{n})^{1-\theta} + \zeta P_{t-1}^{1-\theta} \right]^{1/(1-\theta)}$$
(TA1a.11)

$$Y_t^a = K^\alpha \left[\Omega_0 e^{Z_t} (L_t - \bar{L}) \right]^{1-\alpha}$$
(TA1a.12)

$$P_t^a = \left[\left(1 - \zeta\right) \left(P_t^n\right)^{-\theta} + \zeta \left(P_{t-1}^a\right)^{-\theta} \right]^{-1/\theta}$$
(TA1a.13)

$$Y_t = \left(\frac{P_t^a}{P_t}\right)^* Y_t^a \tag{TA1a.14}$$

$$\tilde{Z}_t = \xi_z \tilde{Z}_{t-1} + \eta_t^z \tag{TA1a.15}$$

Definitions: Y_t is output, C_t is private consumption, L_t is employment, K is the fixed stock of land, $w_t \equiv W_t/P_t$ is the real wage rate, $r_t^K \equiv R_t^K/P_t$ is the real rental rate on land, $mc_t \equiv MC_t/P_t$ is real marginal cost, P_t is the price level, P_t^n is the price set by green-light firms, Y_t^a is an alternative output measure, P_t^a is an alternative price index, R_t is the rate of interest on risk-free bonds, and p_t^K is the real market price of land. The exogenous variables are the nominal money supply M_{t+1} , government consumption \bar{G} , and the innovation term in the technology process η_t^z . The structural parameters are σ , η , κ , ε_l , ε_m , ρ , ζ , θ , \bar{L} , Ω_0 , ξ_z , and α .



Figure A.1: Transitory productivity shock: fixed non-depreciating factor



Figure A.1, Continued

A.3.2 No capital

- The model without capital is obtained by setting $\alpha = 0$ and $\delta = 0$. For this model we thus have that $K_t = I_t = 0$. The resulting model is listed in Table A.2.
- Equations (T1.1), (T1.4), (T1.8) from the benchmark model in Table 1 are dropped.
- In the simulations we keep government consumption at its benchmark level ($\bar{G} = 0.2$) so that in the absence of investment spending we obtain $C^* = 0.8$. We recalibrate the model (by changing Ω_0 , ε_l , and ε_m) such that it features the same (deterministic) steady-state solutions for the remaining endogenous variables (except w^* , Ξ^{N*} , and Ξ^{D*}).
- The simulation results have been reported in Table 5(d) in the paper.
- The impulse response function for the persistent technology shock are presented in Figure A.2. The main features are:
 - The odd responses of output and employment are not observed (panels (b) and (c)).
 - Employment and the real wage rate both fall at impact (panels (c) and (e)). In the spot market for labour, at impact the horizontal labour demand equation shifts down (because marginal cost has fallen) and the supply curve shifts to the left (because consumption has gone up). This results in a fall in both the wage rate and employment.
 - With a constant level of government consumption both consumption and output paths are hump-shaped (panel (b) and (d)).
 - The impact reduction in the wage rate leads to a sharp decrease in real marginal cost (panel (h)) and a sharp reduction in the price set by green-light firms (see the open dots in panel (g)).

Table A.2: The New Keynesian DSGE model without capital	L
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$$Y_t = C_t + \bar{G}$$
(TA2.1)
$$1 = \mathbb{E}_t \left[\frac{1 + R_t}{C_t} \left(\frac{C_t}{C_t} \right)^{1/\sigma} \frac{P_t}{C_t} \right]$$
(TA2.2)

$$\frac{M_{t+1}}{P_t} = C_t^{\eta/\sigma} \left(\frac{R_t}{\varepsilon_m(1+R_t)}\right)^{-\eta}$$
(TA2.3)

$$\varepsilon_l L_t^{\kappa} = w_t C_t^{-1/\sigma} \tag{TA2.4}$$

$$w_t = mc_t \Omega_0 e^{Z_t} \tag{TA2.5}$$

$$P_t^n = \frac{\theta}{\theta - 1} \frac{\Xi_t^N}{\Xi_t^D} \tag{TA2.6}$$

$$\Xi_t^N = C_t^{-1/\sigma} P_t^{\theta} Y_t m c_t + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^N \right]$$
(TA2.7)

$$\Xi_t^D = C_t^{-1/\sigma} P_t^{\theta-1} Y_t + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^D \right]$$
(TA2.8)

$$P_{t} = \left[(1 - \zeta) \left(P_{t}^{n} \right)^{1-\theta} + \zeta P_{t-1}^{1-\theta} \right]^{1/(1-\theta)}$$
(TA2.9)

$$Y_t^a = \Omega_0 e^{Z_t} (L_t - L)$$

$$TA2.10$$

$$P_t^a = \left[(1 - \epsilon) (D^n)^{-\theta} + \epsilon (D^a)^{-\theta} \right]^{-1/\theta}$$

$$TA2.11$$

$$P_t^{\alpha} = \left[(1 - \zeta) \left(P_t^{\alpha} \right)^{-\beta} + \zeta \left(P_{t-1}^{\alpha} \right) \right]$$

$$(1A2.11)$$

$$(P_t^{\alpha})^{-\theta}$$

$$Y_t = \left(\frac{T}{P_t}\right) Y_t^a \tag{TA2.12}$$

$$\tilde{Z} = \left(\tilde{Z} + z^{\tilde{Z}}\right) \tag{TA2.12}$$

$$Z_t = \xi_z Z_{t-1} + \eta_t^z \tag{TA2.13}$$

Definitions: Y_t is output, C_t is private consumption, L_t is employment, $w_t \equiv W_t/P_t$ is the real wage rate, $mc_t \equiv MC_t/P_t$ is real marginal cost, P_t is the price level, P_t^n is the price set by green-light firms, Y_t^a is an alternative output measure, P_t^a is an alternative price index, and R_t is the rate of interest on risk-free bonds. The exogenous variables are the nominal money supply M_{t+1} , government consumption \bar{G} , and the innovation term in the technology process η_t^z . The structural parameters are σ , η , κ , ε_l , ε_m , ρ , ζ , θ , Ω_0 , ξ_z , and \bar{L} .



Figure A.2: Transitory productivity shock: no capital



Figure A.2, Continued

A.3.3 Quadratic price adjustment costs

- We drop the Calvo (1977) pricing friction and instead adopt Rotemberg's (1982) model featuring quadratic price adjustment costs. See also Ascari and Rossi (2012) for a model without capital.
- Firm *i* in the intermediate goods sector chooses its price path $P_{t+\tau}(i)$ in order to maximize the present value of nominal profits:

$$V_{t}(i) \equiv \mathbb{E}_{t} \sum_{\tau=0}^{\infty} \mathcal{N}_{t,t+\tau} \left[\left(P_{t+\tau}(i) - MC_{t+\tau} \right) Y_{t+\tau}(i) - W_{t+\tau} \bar{L} - \frac{\chi}{2} \left(\frac{P_{t+\tau}(i)}{P_{t+\tau-1}(i)} - 1 \right)^{2} P_{t+\tau} Y_{t+\tau} \right],$$
(A.30)

where χ is the adjustment cost parameter (such that $\chi > 0$) and where the firm's demand function is given by:

$$Y_{t+\tau}(i) = \left(\frac{P_{t+\tau}(i)}{P_{t+\tau}}\right)^{-\theta} Y_{t+\tau}.$$
(A.31)

• Recall that the nominal and real stochastic discount factors are related according to $\mathcal{N}_{t,t+\tau} = \mathcal{R}_{t,t+\tau} P_t / P_{t+\tau}$. Using this relationship in (A.30) we find:

$$V_{t}(i) \equiv \mathbb{E}_{t} \sum_{\tau=0}^{\infty} P_{t} \mathcal{R}_{t,t+\tau} \left[\left(\frac{P_{t+\tau}(i) - MC_{t+\tau}}{P_{t+\tau}} \right) \left(\frac{P_{t+\tau}(i)}{P_{t+\tau}} \right)^{-\theta} Y_{t+\tau} - \frac{W_{t+\tau}}{P_{t+\tau}} \bar{L} - \frac{\chi}{2} \left(\frac{P_{t+\tau}(i)}{P_{t+\tau-1}(i)} - 1 \right)^{2} Y_{t+\tau} \right],$$
(A.32)

or:

$$v_t(i) \equiv \mathbb{E}_t \sum_{\tau=0}^{\infty} \mathcal{R}_{t,t+\tau} \left[\left(\frac{P_{t+\tau}(i)}{P_{t+\tau}} - mc_{t+\tau} \right) \left(\frac{P_{t+\tau}(i)}{P_{t+\tau}} \right)^{-\theta} Y_{t+\tau} - w_{t+\tau} \bar{L} - \frac{\chi}{2} \left(\frac{P_{t+\tau}(i)}{P_{t+\tau-1}(i)} - 1 \right)^2 Y_{t+\tau} \right],$$
(A.33)

where $v_t(i) \equiv V_t(i)/P_t$, $w_{t+\tau}$ is the real wage rate and $mc_{t+\tau}$ is real marginal cost.

- The price path $\{P_{t+\tau}(i)\}_{\tau=0}^{\infty}$ is set such that $v_t(i)$ is maximized. The firm takes as given the paths for $\mathcal{R}_{t,t+\tau}$, $P_{t+\tau}$, $Y_{t+\tau}$, $mc_{t+\tau}$, and $w_{t+\tau}$. Furthermore, the price charged in the previous period, $P_{t-1}(i)$, is taken as given also.
- The first-order condition for $P_t(i)$ is given by:

$$\frac{\partial v_t(i)}{\partial P_t(i)} = \frac{\partial}{\partial P_t(i)} \left[\left(\frac{P_t(i)}{P_t} - mc_t \right) \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t - w_t \bar{L} - \frac{\chi}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 Y_t \right]$$

$$+ \frac{\partial}{\partial P_{t}(i)} \mathcal{R}_{t,t+1} \left[\left(\frac{P_{t+1}(i)}{P_{t+1}} - mc_{t+1} \right) \left(\frac{P_{t+1}(i)}{P_{t+1}} \right)^{-\theta} Y_{t+1} - w_{t+1} \bar{L} - \frac{\chi}{2} \left(\frac{P_{t+1}(i)}{P_{t}(i)} - 1 \right)^{2} Y_{t+1} \right] = 0,$$
(A.34)

where we have used the fact that $\mathcal{R}_{t,t} = 1$.

• By taking the derivatives on the right-hand side of (A.34) we obtain:

$$0 = \frac{Y_t}{P_t} \left(1 - \theta \frac{P_t(i) - MC_t}{P_t(i)} \right) \left(\frac{P_t(i)}{P_t} \right)^{-\theta} - \chi \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right) \frac{Y_t}{P_{t-1}(i)} + \chi \mathcal{R}_{t,t+1} \left(\frac{P_{t+1}(i)}{P_t(i)} - 1 \right) Y_{t+1} \frac{P_{t+1}(i)}{P_t(i)^2}.$$
(A.35)

• We assume that all firms set the same price in period t-1, i.e. $P_{t-1}(i) = P_{t-1}$. It follows that $P_{t+\tau}(i) = P_{t+\tau}$ and $Y_{t+\tau}(i) = Y_{t+\tau}(i)$ for all *i*. By incorporating the symmetry results in (A.35) we find:

$$0 = \frac{1 - \theta \left[1 - mc_t\right]}{P_t} Y_t - \chi \left(\frac{P_t}{P_{t-1}} - 1\right) \frac{Y_t}{P_{t-1}} + \mathcal{R}_{t,t+1} \chi \left(\frac{P_{t+1}}{P_t} - 1\right) Y_{t+1} \frac{P_{t+1}}{P_t^2}.$$
(A.36)

• By multiplying the expression in (A.36) by P_t/Y_t , incorporating the definition of the real stochastic discount factor (given in (A.29)), and putting the expectations operator back in we obtain:

$$\theta \left[1 - mc_t \right] = 1 - \chi \left(\frac{P_t}{P_{t-1}} - 1 \right) \frac{P_t}{P_{t-1}} + \frac{\chi}{1+\rho} \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \left(\frac{P_{t+1}}{P_t} - 1 \right) \frac{Y_{t+1}}{Y_t} \frac{P_{t+1}}{P_t} \right].$$
(A.37)

• The model is listed in Table A.3. The model features the same deterministic steady state as the benchmark model. In order to make the dynamics of the linearized model observationally equivalent (to a first order) with the dynamics of the linearized benchmark model we set the adjustment cost parameter as follows (Ascari and Rossi, 2012, p. 1122):

$$\chi = \frac{\zeta(\theta - 1) (1 + \rho)}{(1 - \zeta) (1 + \rho - \zeta)}.$$
(A.38)

- The simulation results have been reported in Table 8(b) in the paper.
- The impulse-response functions for the persistent technology shock are presented in Figure A.3. The main features are very similar to those of the benchmark NK-FMS model. At

impact there are huge decreases in investment, employment and the wage rate and a moderate drop in output.

Table A.3: The New Keynesian DSGE model with quadratic price adjustment costs

$$K_t = I_t + (1 - \delta)K_{t-1}$$
(TA3.1)

$$Y_t = C_t + I_t + \bar{G} + \frac{\chi}{2} \left(\frac{P_t}{P_{t-1}} - 1\right)^2 Y_t$$
(TA3.2)

$$1 = \mathbb{E}_t \left[\frac{1 + R_t}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \frac{P_t}{P_{t+1}} \right]$$
(TA3.3)

$$1 = \mathbb{E}_t \left[\frac{1 + r_{t+1}^K - \delta}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \right]$$
(TA3.4)

$$\frac{M_{t+1}}{P_t} = C_t^{\eta/\sigma} \left(\frac{R_t}{\varepsilon_m(1+R_t)}\right)^{-\eta} \tag{TA3.5}$$

$$\theta \left[1 - mc_{t}\right] = 1 - \chi \left(\frac{P_{t}}{P_{t-1}} - 1\right) \frac{P_{t}}{P_{t-1}} + \frac{\chi}{1 + \rho} \mathbb{E}_{t} \left[\left(\frac{C_{t}}{C_{t+1}}\right)^{1/\sigma} \left(\frac{P_{t+1}}{P_{t}} - 1\right) \frac{Y_{t+1}}{Y_{t}} \frac{P_{t+1}}{P_{t}} \right]$$
(TA3.6)

$$\varepsilon_l L_t^{\kappa} = w_t C_t^{-1/\sigma} \tag{TA3.7}$$

$$w_t = (1 - \alpha) mc_t \frac{Y_t}{L_t - \bar{L}}$$
(TA3.8)

$$r_t^K = \alpha \, mc_t \, \frac{Y_t}{K_{t-1}} \tag{TA3.9}$$

$$Y_t = K_{t-1}^{\alpha} \left[\Omega_0 e^{\tilde{Z}_t} (L_t - \bar{L}) \right]^{1-\alpha}$$
(TA3.10)

$$\tilde{Z}_t = \xi_z \tilde{Z}_{t-1} + \eta_t^z \tag{TA3.11}$$

Definitions: Y_t is output, C_t is private consumption, L_t is employment, K_t is the capital stock at the end of period t, $w_t \equiv W_t/P_t$ is the real wage rate, $r_t^K \equiv R_t^K/P_t$ is the real rental rate on capital, $mc_t \equiv MC_t/P_t$ is real marginal cost, P_t is the price level, R_t is the rate of interest on risk-free bonds, and I_t is gross investment. The exogenous variables are the nominal money supply M_{t+1} , government consumption \bar{G} , and the innovation term in the technology process η_t^z . The structural parameters are χ , σ , η , κ , ε_l , ε_m , ρ , δ , θ , \bar{L} , Ω_0 , ξ_z , and α .



Figure A.3: Transitory productivity shock: quadratic price adjustment costs



Figure A.3, Continued

A.3.4 Perfectly flexible prices

- The model featuring perfectly flexible prices is listed in Table A.4. Since the price level satisfies the familiar markup equation, $P_t = [\theta/(\theta 1)]MC_t$, it follows that real marginal cost is constant, i.e. $mc_t = (\theta 1)/\theta$. Factor demand are unaffected by fluctuations in real marginal cost (see equations (TA4.7) and (TA4.8)).
- The simulation results have been reported in Table 5(f) in the paper.
- The impulse response functions for the persistent technology shock are presented in Figure A.4. The main features are:
 - Lack of propagation: the output path is very similar in shape as the productivity path (see panels (a) and (b)).
 - There is a hump-shaped response in the capital stock (panel (d)) and employment undershoots its long-run steady-state value (panel (c)).
 - At impact both employment and the real wage rate increase (panels (c) and (g)).
 In the spot market for labour, at impact labour demand shifts to the right (because productivity has gone up) and the supply curve shifts to the left (because consumption has gone up). The demand effect dominates the supply effect leading to the observed result.
 - At impact the rental rate on capital increases (panel (h)). In the rental market for capital, at impact the demand curve shifts to the right (because productivity and employment both increase) and the supply curve stays put (as the available capital stock is predetermined at time t = 0). To clear the rental market the rental rate on capital increases.
 - With a given nominal money supply there is an impact reduction in the price level (see panel (i)).

Table A.4: The New Classical DSGE model with perfectly flexible prices

$$K_t = I_t + (1 - \delta)K_{t-1} \tag{TA4.1}$$

$$Y_t = C_t + I_t + \bar{G} \tag{TA4.2}$$

$$Y_t = C_t + I_t + G \tag{1A4.2}$$

$$1 = \mathbb{E}_t \left[\frac{1 + R_t}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \frac{P_t}{P_{t+1}} \right]$$
(TA4.3)
$$\left[1 + r^K - \delta \left(C \right)^{1/\sigma} \right]$$

$$1 = \mathbb{E}_t \left[\frac{1 + r_{t+1}^{T} - \delta}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1/\delta} \right]$$
(TA4.4)

$$\frac{M_{t+1}}{P_t} = C_t^{\eta/\sigma} \left(\frac{R_t}{\varepsilon_m(1+R_t)}\right)^{-\eta}$$
(TA4.5)
$$\varepsilon_t L^{\kappa} = w_t C^{-1/\sigma}$$
(TA4.6)

$$\varepsilon_l L_t = w_t C_t \tag{TA4.0}$$
$$w_t = (1 - \alpha) \frac{\theta - 1}{\theta} \frac{Y_t}{L_t - \bar{L}} \tag{TA4.7}$$

$$r_t^K = \alpha \frac{\theta - 1}{\theta} \frac{Y_t}{K_{t-1}}$$
(TA4.8)

$$Y_t = K_{t-1}^{\alpha} \left[\Omega_0 e^{\tilde{Z}_t} (L_t - \bar{L}) \right]^{1-\alpha}$$
(TA4.9)

$$Z_t = \xi_z Z_{t-1} + \eta_t^z \tag{TA4.10}$$

Definitions: Y_t is output, C_t is private consumption, L_t is employment, K_t is the capital stock at the end of period t, $w_t \equiv W_t/P_t$ is the real wage rate, $r_t^K \equiv R_t^K/P_t$ is the real rental rate on capital, P_t is the price level, R_t is the rate of interest on risk-free bonds, and I_t is gross investment. The exogenous variables are the nominal money supply M_{t+1} , government consumption \overline{G} , and the the innovation term in the technology process η_t^z . The structural parameters are σ , η , κ , ε_l , ε_m , ρ , δ , θ , \overline{L} , Ω_0 , ξ_z , and α .



Figure A.4: Transitory productivity shock: perfectly flexible prices



Figure A.4, Continued

A.3.5 Predetermined price level

In this subsection we assume that a firm which gets a green light in period t sets its price to be charged from the next period onward. There is thus an implementation lag of one period. We first briefly comment on the model changes that occur as a result of this assumption.

• The objective function of the green-light firm at time t is:

$$\begin{aligned} V_t^0(i) &\equiv \left[P_{t-1}(i) - MC_t \right] Y_t \left(\frac{P_{t-1}(i)}{P_t} \right)^{-\theta} - W_t \bar{L} \\ &+ \left[\left[P_t(i) - MC_{t+1} \right] Y_{t+1} \left(\frac{P_t(i)}{P_{t+1}} \right)^{-\theta} - W_{t+1} \bar{L} \right] \mathcal{N}_{t,t+1} \\ &+ \left[\left[P_t(i) - MC_{t+2} \right] Y_{t+2} \left(\frac{P_t(i)}{P_{t+2}} \right)^{-\theta} - W_{t+2} \bar{L} \right] \zeta \mathcal{N}_{t,t+2} \\ &+ \left[\left[P_t(i) - MC_{t+3} \right] Y_{t+3} \left(\frac{P_t(i)}{P_{t+3}} \right)^{-\theta} - W_{t+3} \bar{L} \right] \zeta^2 \mathcal{N}_{t,t+3} \\ &+ \dots \end{aligned}$$

• Redoing the derivations we find that (T1.10)–(T1.11) in Table 1 in the paper are replaced by:

$$\begin{split} \Xi_{t}^{N} &= \frac{\zeta}{1+\rho} \mathbb{E}_{t} C_{t+1}^{-1/\sigma} P_{t+1}^{\theta} Y_{t+1} m c_{t+1} + \frac{\zeta}{1+\rho} \mathbb{E}_{t} \Xi_{t+1}^{N}, \\ \Xi_{t}^{D} &= \frac{\zeta}{1+\rho} \mathbb{E}_{t} C_{t+1}^{-1/\sigma} P_{t+1}^{\theta-1} Y_{t+1} + \frac{\zeta}{1+\rho} \mathbb{E}_{t} \Xi_{t+1}^{D}. \end{split}$$

• Similarly, (T1.12) and (T1.14) in Table 1 are replaced by:

$$P_{t} = \left[(1-\zeta) \left(P_{t-1}^{n} \right)^{1-\theta} + \zeta P_{t-1}^{1-\theta} \right]^{1/(1-\theta)},$$

$$P_{t}^{a} = \left[(1-\zeta) \left(P_{t-1}^{n} \right)^{-\theta} + \zeta P_{t-1}^{-\theta} \right]^{-1/\theta}.$$

Since both P_{t-1}^n and P_{t-1} are predetermined at time t, both P_t and P_t^a are also predetermined at that time.

- The model featuring this type of lagged pricing is listed in Table A.5.
- The simulation results have been reported in Table 8(c) in the paper.
- The impulse-response functions for the persistent technology shock are presented in Figure A.5. The main features are similar as those of the benchmark model:
 - At impact there is a huge drop in output, employment, and investment (panels (b), (c), and (f)).

- One period after the shock there are huge increases in these variables and over time the effect of the shock dies down.
- Quantitatively the effects are even more implausible than those of the benchmark NK-FMS model.
- We close by noting that after redoing the derivations leading to equation (T7.9) in Table 7 we find:

$$\pi_t = \frac{\phi}{1+\rho} \mathbb{E}_t \widetilde{mc}_{t+1} + \frac{1}{1+\rho} \mathbb{E}_t \pi_{t+1},$$

where ϕ is defined in (A.26) above.

$$K_{t} = I_{t} + (1 - \delta)K_{t-1}$$
(TA5.1)

$$Y_{t} = C_{t} + I_{t} + \bar{G}$$
(TA5.2)

$$1 = \mathbb{E}_t \left[\frac{1 + R_t}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \frac{P_t}{P_{t+1}} \right]$$
(TA5.3)

$$1 = \mathbb{E}_t \left[\frac{1 + r_{t+1}^K - \delta}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \right]$$
(TA5.4)

$$\frac{M_{t+1}}{P_t} = C_t^{\eta/\sigma} \left(\frac{R_t}{\varepsilon_m(1+R_t)}\right)^{-\eta} \tag{TA5.5}$$

$$\varepsilon_l L_t^{\kappa} = w_t C_t^{-1/\sigma} \tag{TA5.6}$$

$$w_t = (1 - \alpha) mc_t \frac{Y_t^a}{L_t - \bar{L}}$$
(TA5.7)

$$r_t^K = \alpha \, mc_t \, \frac{r_t}{K_{t-1}} \tag{TA5.8}$$

$$P_t^n = \frac{\theta}{\theta - 1} \frac{\Xi_t^N}{\Xi_t^D} \tag{TA5.9}$$

$$\Xi_t^N = \frac{\zeta}{1+\rho} \mathbb{E}_t C_{t+1}^{-1/\sigma} P_{t+1}^{\theta} Y_{t+1} m c_{t+1} + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^N \right]$$
(TA5.10)

$$\Xi_t^D = \frac{\zeta}{1+\rho} \mathbb{E}_t C_{t+1}^{-1/\sigma} P_{t+1}^{\theta-1} Y_{t+1} + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^D \right]$$
(TA5.11)

$$P_{t} = \left[(1 - \zeta) \left(P_{t-1}^{n} \right)^{1-\theta} + \zeta P_{t-1}^{1-\theta} \right]^{1/(1-\theta)}$$
(TA5.12)

$$Y_t^a = K_{t-1}^{\alpha} \left[\Omega_0 e^{\tilde{Z}_t} (L_t - \bar{L}) \right]^{1-\alpha}$$
(TA5.13)

$$P_t^a = \left[(1 - \zeta) \left(P_{t-1}^n \right)^{-\theta} + \zeta \left(P_{t-1}^a \right)^{-\theta} \right]^{-1/\theta}$$
(TA5.14)

$$Y_t = \left(\frac{P_t^a}{P_t}\right)^* Y_t^a \tag{TA5.15}$$

$$\tilde{Z}_t = \xi_z \tilde{Z}_{t-1} + \eta_t^z \tag{TA5.16}$$

Definitions: Y_t is output, C_t is private consumption, L_t is employment, K_t is the capital stock at the end of period t, $w_t \equiv W_t/P_t$ is the real wage rate, $r_t^K \equiv R_t^K/P_t$ is the real rental rate on capital, $mc_t \equiv MC_t/P_t$ is real marginal cost, P_t is the price level, P_t^n is the price set by green-light firms, Y_t^a is an alternative output measure, P_t^a is an alternative price index, R_t is the rate of interest on risk-free bonds, and I_t is gross investment. The exogenous variables are the nominal money supply M_{t+1} , government consumption \bar{G} , and the innovation term in the technology process η_t^z . The structural parameters are σ , η , κ , ε_l , ε_m , ρ , δ , ζ , θ , \bar{L} , Ω_0 , ξ_z , and α .



Figure A.5: Transitory productivity shock: lagged price setting



Figure A.5, Continued

A.3.6 Countercyclical money supply ruled

In this subsection we assume that the monetary authority sets the nominal money supply in a countercyclical fashion. In particular, we assume that:

$$\frac{M_t - \bar{M}}{\bar{M}} = -\mu_y \frac{Y_t - Y^*}{Y^*} - \mu_\pi \frac{P_t - P_{t-1}}{P_{t-1}},$$

where \overline{M} is a constant, Y^* is the (deterministic) steady-state output level, and we assume that $\mu_y \ge 0$ and $\mu_\pi \ge 0$.

- The model featuring the money supply rule is listed in Table A.6.
- We consider two extreme cases:
 - A traditional (output-based) rule which reacts to output only ($\mu_y = 0.5$ and $\mu_{\pi} = 0$) in Table 8(d)
 - An inflation-fighting rule which reacts to price inflation only ($\mu_y = 0$ and $\mu_{\pi} = 100$) in Table 8(e)
- The impulse-response functions under the output-based rule are presented in Figure A.6. The main features are similar as those of the benchmark model:
 - At impact there is a huge drop in output, employment, and investment (panels (b), (c), and (f)).
 - One period after the shock there are huge increases in these variables and over time the effect of the shock dies down.
 - Quantitatively the effects are similar but even more implausible than those of the benchmark model.
- The impulse-response functions for inflation-fighting rule are presented in Figure A.7. The main features are as follows:
 - The Implausible Result is not obtained.
 - The impulse-response functions look very similar to those of the NC-TR model.

$$K_{t} = I_{t} + (1 - \delta)K_{t-1}$$
(TA6.1)

$$Y_{t} = C_{t} + I_{t} + \bar{G}$$
(TA6.2)

$$1 = \mathbb{E}_t \left[\frac{1 + R_t}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \frac{P_t}{P_{t+1}} \right]$$
(TA6.3)

$$1 = \mathbb{E}_t \left[\frac{1 + r_{t+1}^K - \delta}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \right]$$
(TA6.4)

$$\frac{M_{t+1}}{P_t} = C_t^{\eta/\sigma} \left(\frac{R_t}{\varepsilon_m(1+R_t)}\right)^{-\eta}$$
(TA6.5)

$$\varepsilon_l L_t^{\kappa} = w_t C_t^{1/\ell} \tag{TA6.6}$$

$$w_t = (1 - \alpha) mc_t \frac{T_t}{L_t - \bar{L}}$$
(TA6.7)

$$r_t^K = \alpha \, mc_t \, \frac{r_t}{K_{t-1}} \tag{TA6.8}$$
$$\theta \quad \Xi^N$$

$$P_t^n = \frac{\theta}{\theta - 1} \frac{\Xi_t^n}{\Xi_t^D} \tag{TA6.9}$$

$$\Xi_t^N = C_t^{-1/\sigma} P_t^{\theta} Y_t m c_t + \frac{\varsigma}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^N \right]$$
(TA6.10)
$$\Xi_t^D = C_t^{-1/\sigma} P_t^{\theta-1} W_t + \frac{\varsigma}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^N \right]$$
(TA6.11)

$$\Xi_t^D = C_t^{-1/\delta} P_t^{\theta-1} Y_t + \frac{\varsigma}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^D \right]$$
(TA6.11)

$$P_{t} = \left[(1 - \zeta) (P_{t}^{n})^{1-\theta} + \zeta P_{t-1}^{1-\theta} \right]^{1/(1-\theta)}$$
(TA6.12)
$$V_{t}^{n} = V_{t}^{0} - \left[Q_{t} - \tilde{\chi}_{t}^{1} \right]^{1-\alpha}$$
(TA6.12)

$$Y_t^a = K_{t-1}^{\alpha} \left[\Omega_0 e^{\mathcal{Z}_t} (L_t - L) \right]$$
(TA6.13)

$$P_t^a = \left[(1-\zeta) \left(P_t^n \right)^{-\theta} + \zeta \left(P_{t-1}^a \right)^{-\theta} \right]^{-1/\delta}$$
(TA6.14)

$$Y_t = \left(\frac{P_t^a}{P_t}\right)^{\circ} Y_t^a \tag{TA6.15}$$

$$\tilde{Z}_t = \xi_z \tilde{Z}_{t-1} + \eta_t^z \tag{TA6.16}$$

$$U_t = \bar{M} \qquad Y_t - Y^* \qquad P_t - P_{t-1}$$

$$\frac{M_t - M}{\bar{M}} = -\mu_y \frac{Y_t - Y^*}{Y^*} - \mu_\pi \frac{P_t - P_{t-1}}{P_{t-1}}$$
(TA6.17)

Definitions: Y_t is output, C_t is private consumption, L_t is employment, K_t is the capital stock at the end of period t, $w_t \equiv W_t/P_t$ is the real wage rate, $r_t^K \equiv R_t^K/P_t$ is the real rental rate on capital, $mc_t \equiv MC_t/P_t$ is real marginal cost, P_t is the price level, P_t^n is the price set by green-light firms, Y_t^a is an alternative output measure, P_t^a is an alternative price index, R_t is the rate of interest on risk-free bonds, I_t is gross investment, M_{t+1} is the nominal money supply. The exogenous variables are government consumption \bar{G} , and the the innovation term in the technology process η_t^z . The structural parameters are σ , η , κ , ε_l , ε_m , ρ , δ , ζ , θ , \bar{L} , μ_y , μ_{π} , Ω_0 , ξ_z , and α .



Figure A.6: Transitory productivity shock: output-based money supply rule







Figure A.7: Transitory productivity shock: inflation-based money supply rule




A.3.7 Capital adjustment costs

In this subsection we assume that capital accumulation is subject to adjustment costs. We first briefly comment on the model changes that occur as a result of this assumption.

• We postulate the existence of a representative investment firm which constructs the macroeconomic capital stock and rents out units of capital to firms in the intermediate goods sector. The investment firm's objective function is given by:

$$V_t \equiv \mathbb{E}_t \sum_{\tau=t}^{\infty} \left[R_{\tau}^K K_{\tau-1} - P_{\tau} I_{\tau} \right] \mathcal{N}_{t,\tau},$$

where $\mathcal{N}_{t,\tau}$ is the nominal stochastic discount factor.

• The net capital accumulation function is given by:

$$\frac{K_{\tau} - K_{\tau-1}}{K_{\tau-1}} = \Phi\left(\frac{I_{\tau}}{K_{\tau-1}}\right), \qquad \Phi(\delta) = 0, \quad \Phi'(\delta) = 1, \quad \Phi''(\cdot) < 0, \tag{A.39}$$

where $\delta > 0$ is the constant rate of capital depreciation.

• In order to derive the first-order necessary conditions we formulate the Lagrangian:

$$\mathcal{L}_t \equiv \mathbb{E}_t \sum_{\tau=t}^{\infty} \left[R_{\tau}^K K_{\tau-1} - P_{\tau} I_{\tau} - \lambda_{\tau} \left[K_{\tau} - K_{\tau-1} - \Phi \left(\frac{I_{\tau}}{K_{\tau-1}} \right) K_{\tau-1} \right] \right] \mathcal{N}_{t,\tau}.$$

• In period τ the immediate choices are about I_{τ} and K_{τ} (since $K_{\tau-1}$ is predetermined):

$$\frac{\partial \mathcal{L}_t}{\partial I_\tau} = \left[-P_\tau + \lambda_\tau \Phi' \left(\frac{I_\tau}{K_{\tau-1}} \right) \right] \mathcal{N}_{t,\tau} = 0, \tag{A.40}$$

$$\frac{\partial \mathcal{L}_t}{\partial K_\tau} = \left[-\lambda_\tau + \frac{\mathcal{N}_{t,\tau+1}}{\mathcal{N}_{t,\tau}} \left(R_{\tau+1}^K + \lambda_{\tau+1} \left[1 + \Phi \left(\frac{I_{\tau+1}}{K_\tau} \right) - \Phi' \left(\frac{I_{\tau+1}}{K_\tau} \right) \frac{I_{\tau+1}}{K_\tau} \right] \right) \right] \mathcal{N}_{t,\tau} = 0. \tag{A.41}$$

• Recall that $\mathcal{N}_{t,\tau}$ is defined as:

$$\mathcal{N}_{t,\tau} \equiv \left(\frac{1}{1+\rho}\right)^{\tau} \frac{C_t^{1/\sigma}}{C_{\tau}^{1/\sigma}} \frac{P_t}{P_{\tau}}.$$
(A.42)

• We define Tobin's q as:

$$q_{\tau} \equiv \frac{\lambda_{\tau}}{P_{\tau}}.\tag{A.43}$$

• Using (A.42)-(A.43) in (A.40) we find:

$$1 = q_{\tau} \Phi' \left(\frac{I_{\tau}}{K_{\tau-1}} \right) \tag{A.44}$$

• Similarly, by using (A.42)–(A.43) in (A.41) we find:

$$q_{\tau} = \frac{R_{\tau+1}^{K}}{P_{\tau+1}} \frac{P_{\tau+1}}{P_{\tau}} \frac{\mathcal{N}_{t,\tau+1}}{\mathcal{N}_{t,\tau}} + \frac{\lambda_{\tau+1}\mathcal{N}_{t,\tau+1}}{P_{\tau}\mathcal{N}_{t,\tau}} \left[1 + \Phi\left(\frac{I_{\tau+1}}{K_{\tau}}\right) - \Phi'\left(\frac{I_{\tau+1}}{K_{\tau}}\right) \frac{I_{\tau+1}}{K_{\tau}} \right] \\ = r_{\tau+1}^{K} \frac{P_{\tau+1}}{P_{\tau}} \frac{\mathcal{N}_{t,\tau+1}}{\mathcal{N}_{t,\tau}} + q_{\tau+1} \frac{P_{\tau+1}\mathcal{N}_{t,\tau+1}}{P_{\tau}\mathcal{N}_{t,\tau}} \left(1 + \Phi\left(\frac{I_{\tau+1}}{K_{\tau}}\right) - \Phi'\left(\frac{I_{\tau+1}}{K_{\tau}}\right) \frac{I_{\tau+1}}{K_{\tau}} \right) \\ = \frac{1}{1+\rho} \frac{C_{\tau}^{1/\sigma}}{C_{\tau+1}^{1/\sigma}} \left(r_{\tau+1}^{K} + q_{\tau+1} \left(1 + \Phi\left(\frac{I_{\tau+1}}{K_{\tau}}\right) \right) - \frac{I_{\tau+1}}{K_{\tau}} \right).$$
(A.45)

• To implement the adjustment cost model we assume that the installation function takes the following form:

$$\Phi(x) \equiv \frac{\bar{z}}{1 - \sigma_x} \left[\left[\frac{\bar{z} + x - \delta}{\bar{z}} \right]^{1 - \sigma_x} - 1 \right], \tag{A.46}$$

with $\sigma_x > 1$ and $\bar{z} > 0$. See below for further comments on this functional form.

• By using (A.46) we find that (A.39), (A.44), and (A.45) are given by:

$$K_t = \left[1 + \frac{\bar{z}}{1 - \sigma_x} \left[q_t^{(1 - \sigma_x)/\sigma_x} - 1\right]\right] K_{t-1}$$
(A.47)

$$I_t = \left[\delta + \bar{z} \left(q_t^{1/\sigma_x} - 1\right)\right] K_{t-1} \tag{A.48}$$

$$q_{t} = \frac{1}{1+\rho} \left(\frac{C_{t}}{C_{t+1}}\right)^{1/\sigma} \left[r_{t+1}^{K} + q_{t+1} \left(1 + \frac{\bar{z}}{1-\sigma_{x}} \left[q_{t+1}^{(1-\sigma_{x})/\sigma_{x}} - 1 \right] \right) - \frac{I_{t+1}}{K_{t}} \right]$$
(A.49)

• In the deterministic steady state we find:

$$q^* = 1, \qquad \left(\frac{I}{K}\right)^* = \delta, \qquad \left(r^K\right)^* = \rho + \delta$$

• The subsystem stated in (A.47)–(A.49) replaces the following equations in the benchmark model:

$$K_{t} = I_{t} + (1 - \delta)K_{t-1},$$

$$1 = \frac{1 + r_{t+1}^{K} - \delta}{1 + \rho} \left(\frac{C_{t}}{C_{t+1}}\right)^{1/\sigma}$$

- The capital adjustment cost model is listed in Table A.7.
- The simulation results have been reported in Table 8(f) in the paper.
- The impulse response function for the persistent technology shock are presented in Figure A.9. The main features are:
 - The odd responses of output and employment are not observed (panels (b) and (c)).

- Employment and the real wage rate both fall at impact (panels (c) and (g)). In the spot market for labour, at impact the labour demand equation shifts to the left (because marginal cost has fallen) and the supply curve shifts to the left (because consumption has gone up). The demand effect dominates the supply effect resulting in a simultaneous drop in employment and the real wage.
- At impact the rental rate on capital falls (panel (h)). In the rental market for capital, the demand curve shifts to the left at impact (because marginal cost has fallen) and the supply curve stays put (as K_{t-1} is predetermined). To clear the rental market the rental rate on existing units of capital falls.
- The impact reduction in both factor prices leads to a sharp decrease in real marginal cost (panel (j)) and a sharp reduction in the price set by green-light firms (see the open dots in panel (i)).
- The paths for output, consumption, and investment are all hump-shaped (panel (b), (e), and (f)).
- With respect to the installation function given in (A.46) we note that:

$$\frac{K_{\tau} - K_{\tau-1}}{K_{\tau-1}} = \Phi(x) \equiv \frac{\bar{z}}{1 - \sigma_x} \left[\left(\frac{x - \delta + \bar{z}}{\bar{z}} \right)^{1 - \sigma_x} - 1 \right], \text{ for } x = \frac{I_{\tau}}{K_{\tau-1}}, \\ \frac{I_{\tau}}{K_{\tau-1}} = \Psi(x) \equiv \delta + \bar{z} \left[\left(\frac{\bar{z} + (1 - \sigma_x)x}{\bar{z}} \right)^{1/(1 - \sigma_x)} - 1 \right], \text{ for } x = \frac{K_{\tau} - K_{\tau-1}}{K_{\tau-1}}.$$

• In Figure A.8(a) we illustrate the logarithmic accumulation function (obtained by setting $\sigma_x = 1$):

$$\Phi\left(x\right) = \bar{z} \ln\left(\frac{x - \delta + \bar{z}}{\bar{z}}\right)$$

The 45° line is the accumulation function in the absence of adjustment costs.

• In Figure A.8(b) we depict the implied required-investment function (the inverse of the installation function):

$$\Psi\left(x\right) = \delta + \bar{z} \left[e^{x/\bar{z}} - 1\right]$$





(a) installation function

$$K_{t} = \left[1 + \frac{\bar{z}}{1 - \sigma_{x}} \left[q_{t}^{(1 - \sigma_{x})/\sigma_{x}} - 1\right]\right] K_{t-1}$$
(TA7.1)

$$q_{t} = \frac{1}{1+\rho} \left(\frac{C_{t}}{C_{t+1}}\right)^{1/\sigma} \left[r_{t+1}^{K} + q_{t+1} \left(1 + \frac{\bar{z}}{1-\sigma_{x}} \left[q_{t+1}^{(1-\sigma_{x})/\sigma_{x}} - 1 \right] \right) - \frac{I_{t+1}}{K_{t}} \right]$$
(TA7.2)

$$I_{t} = \begin{bmatrix} o + z \left(q_{t}^{T} - 1 \right) \end{bmatrix} K_{t-1}$$

$$Y_{t} = C_{t} + I_{t} + \bar{G}$$
(TA7.4)

$$1 = \mathbb{E}_t \left[\frac{1 + R_t}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \frac{P_t}{P_{t+1}} \right]$$
(TA7.5)

$$\frac{M_{t+1}}{P_t} = C_t^{\eta/\sigma} \left(\frac{R_t}{\varepsilon_m(1+R_t)}\right)^{-\eta} \tag{TA7.6}$$

$$\varepsilon_l L_t^{\kappa} = w_t C_t^{-1/\delta} \tag{TA7.7}$$

$$w_t = (1 - \alpha) mc_t \frac{T_t}{L_t - \bar{L}}$$
(TA7.8)

$$r_t^K = \alpha \, mc_t \, \frac{r_t}{K_{t-1}} \tag{TA7.9}$$

$$P_t^n = \frac{\theta}{\theta - 1} \frac{\Xi_t^n}{\Xi_t^D} \tag{TA7.10}$$

$$\Xi_t^N = C_t^{-1/\sigma} P_t^{\theta} Y_t m c_t + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^N \right]$$
(TA7.11)

$$\Xi_t^D = C_t^{-1/\sigma} P_t^{\theta-1} Y_t + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^D \right]$$
(TA7.12)

$$P_{t} = \left[(1 - \zeta) \left(P_{t}^{n} \right)^{1-\theta} + \zeta P_{t-1}^{1-\theta} \right]^{1/(1-\theta)}$$
(TA7.13)

$$Y_t^a = K_{t-1}^{\alpha} \left[\Omega_0 e^{\bar{Z}_t} (L_t - \bar{L}) \right]^{1-\alpha}$$
(TA7.14)

$$P_t^a = \left[(1-\zeta) \left(P_t^n \right)^{-\theta} + \zeta \left(P_{t-1}^a \right)^{-\theta} \right]^{-1/\theta}$$
(TA7.15)

$$Y_t = \left(\frac{P_t^a}{P_t}\right)^{\circ} Y_t^a \tag{TA7.16}$$

$$\tilde{Z}_t = \xi_z \tilde{Z}_{t-1} + \eta_t^z \tag{TA7.17}$$

Definitions: Y_t is output, C_t is private consumption, L_t is employment, K_t is the capital stock at the end of period t, $w_t \equiv W_t/P_t$ is the real wage rate, $r_t^K \equiv R_t^K/P_t$ is the real rental rate on capital, $mc_t \equiv MC_t/P_t$ is real marginal cost, P_t is the price level, P_t^n is the price set by green-light firms, Y_t^a is an alternative output measure, P_t^a is an alternative price index, R_t is the rate of interest on risk-free bonds, and I_t is gross investment. The exogenous variables are the nominal money supply M_{t+1} , government consumption \bar{G} , and the innovation term in the technology process η_t^z . The structural parameters are σ , η , κ , ε_l , ε_m , ρ , δ , ζ , θ , \bar{L} , Ω_0 , ξ_z , and α .



Figure A.9: Transitory productivity shock: capital adjustment costs



Figure A.9, Continued

A.3.8 Taylor rule for interest rates

In this subsection we assume that the monetary authority sets the nominal interest rate according to the following Taylor Rule:

$$R_t = \rho + \phi_{\pi}(1+\rho)\frac{P_t - P_{t-1}}{P_{t-1}} + \phi_y(1+\rho)\frac{Y_t - Y^*}{Y^*},$$

where Y^* is the (deterministic) steady-state output level, and we assume that $\phi_{\pi} > 0$ and $\phi_{y} > 0$.

- The model featuring the Taylor Rule is listed in Table A.8.
- The simulation results (for the case with $\phi_{\pi} = 1.5$ and $\phi_y = 0.5$) have been reported in Table 5(a) in the paper.
- The impulse-response functions for the persistent technology shock are presented in Figure A.10. The main features are:
 - The odd responses of output and employment are not observed (panels (b) and (c)).
 - At impact employment falls slightly and the real wage rises (panels (c) and (g)). In the spot market for labour, at impact the labour demand equation shifts to the right (because marginal cost has fallen) and the supply curve shifts to the left (because consumption has gone up). The demand effect dominates the supply effect resulting in the observed pattern.
 - At impact the rental rate on capital increases (panel (h)). In the rental market for capital, the demand curve shifts to the right at impact (because marginal cost has fallen) and the supply curve stays put (as K_{t-1} is predetermined). To clear the rental market the rental rate on existing units of capital increases.
 - The impact increase in both factor prices ensures that the decrease in real marginal cost is much smaller than in the benchmark NK-FMS model (panel (j)).
 - At impact there is a sharp reduction in the price set by green-light firms (see the open dots in panel (i)).
 - There is a hysteretic effect on nominal prices in the sense that the temporary productivity shock causes a long-run drop in the price level (panel (i)).

$$K_{t} = I_{t} + (1 - \delta)K_{t-1}$$
(TA8.1)

$$Y_{t} = C_{t} + I_{t} + \bar{G}$$
(TA8.2)

$$1 = \mathbb{E}_t \left[\frac{1 + R_t}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \frac{P_t}{P_{t+1}} \right]$$
(TA8.3)

$$R_t = \rho + \phi_\pi (1+\rho) \frac{P_t - P_{t-1}}{P_{t-1}} + \phi_Y (1+\rho) \frac{Y_t - Y^*}{Y^*}$$
(TA8.5)

$$\varepsilon_l L_t^{\kappa} = w_t C_t^{-1/\sigma} \tag{TA8.6}$$

$$w_t = (1 - \alpha) mc_t \frac{Y_t^a}{L_t - \bar{L}}$$
(TA8.7)

$$r_t^K = \alpha \, mc_t \, \frac{Y_t}{K_{t-1}} \tag{TA8.8}$$

$$P_t^n = \frac{\theta}{\theta - 1} \frac{\Xi_t^N}{\Xi_t^D} \tag{TA8.9}$$

$$\Xi_t^N = C_t^{-1/\sigma} P_t^{\theta} Y_t m c_t + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^N \right]$$
(TA8.10)

$$\Xi_{t}^{D} = C_{t}^{-1/\sigma} P_{t}^{\theta-1} Y_{t} + \frac{\zeta}{1+\rho} \mathbb{E}_{t} \left[\Xi_{t+1}^{D} \right]$$
(TA8.11)

$$P_{t} = \left[(1 - \zeta) (P_{t}^{n})^{1-\theta} + \zeta P_{t-1}^{1-\theta} \right]^{1/(1-\theta)}$$
(TA8.12)
$$W_{t}^{a} = V_{t}^{\alpha} \left[Q_{t} \tilde{Z}_{t} (I_{t} - \bar{I}_{t})^{1-\alpha} \right]^{1-\alpha}$$
(TA8.12)

$$Y_t^a = K_{t-1}^a \left[\Omega_0 e^{Z_t} (L_t - L) \right]$$

$$P_t^a = \left[(1 - \zeta) (P_t^n)^{-\theta} + \zeta (P_t^a)^{-\theta} \right]^{-1/\theta}$$
(TA8.13)
(TA8.14)

$$P_t^a = \left[(1-\zeta) \left(P_t^a \right)^{\theta} + \zeta \left(P_{t-1}^a \right)^{-1} \right]$$

$$(TA8.14)$$

$$V = \left(\frac{P_t^a}{t} \right)^{\theta} V_t^a$$

$$(TA8.15)$$

$$Y_t = \left(\frac{T}{P_t}\right) Y_t^a \tag{TA8.15}$$

$$Z_t = \xi_z Z_{t-1} + \eta_t^z \tag{TA8.16}$$

$$I_{t+1} = \pi/\pi \left(R_t \right)^{-\eta}$$

$$\frac{M_{t+1}}{P_t} = C_t^{\eta/\sigma} \left(\frac{R_t}{\varepsilon_m(1+R_t)}\right)^{-\eta}$$
(TA8.17)

Definitions: Y_t is output, C_t is private consumption, L_t is employment, K_t is the capital stock at the end of period t, $w_t \equiv W_t/P_t$ is the real wage rate, $r_t^K \equiv R_t^K/P_t$ is the real rental rate on capital, $mc_t \equiv MC_t/P_t$ is real marginal cost, P_t is the price level, P_t^n is the price set by green-light firms, Y_t^a is an alternative output measure, P_t^a is an alternative price index, R_t is the rate of interest on risk-free bonds, and I_t is gross investment. The exogenous variables are government consumption \bar{G} and the innovation term in the technology process η_t^z . The structural parameters are σ , η , κ , ε_l , ε_m , ρ , δ , ζ , θ , \bar{L} , ϕ_{π} , ϕ_y , Ω_0 , ξ_z , and α .



Figure A.10: Transitory productivity shock: interest rate rule



Figure A.10, Continued

A.3.9 Variable utilization rate of capital

• We assume that the household determines the utilization rate of capital, u_t , and that firms rent capital services, $K_{t-1}^s(i) = u_t K_{t-1}(i)$, in order to produce output. The household's periodic budget identity in real terms is now given by:

$$P_{\tau} \left[C_{\tau} + I_{\tau} \right] + M_{\tau+1} + B_{\tau+1} + \sum_{s=0}^{\infty} Q_{\tau}^{s} S_{\tau+1}^{s} = W_{\tau} L_{\tau} + R_{\tau}^{K} u_{\tau} K_{\tau-1} + (1 + R_{\tau-1}) B_{\tau} + \sum_{s=0}^{\infty} X_{\tau}^{s} S_{\tau}^{s} + M_{\tau} - P_{\tau} T_{\tau}, \qquad (A.50)$$

whilst the capital accumulation identity is changed to:

$$K_{\tau} = I_{\tau} + (1 - \delta(u_{\tau})) K_{\tau-1}, \tag{A.51}$$

where we assume that the depreciation function takes the form suggested by Baxter and Farr (2005, p. 338):

$$\delta(u_t) \equiv \delta_0 + \frac{a_0}{1+a_1} u_t^{1+a_1}, \qquad \delta_0 > 0, \ a_0 > 0, \ \text{and} \ a_1 > 0.$$
(A.52)

• The Lagrangian is given by:

$$\mathcal{L}_{t}^{H} \equiv \mathbb{E}_{t} \sum_{\tau=t}^{\infty} \left(\frac{1}{1+\rho} \right)^{\tau-t} \left[U(C_{\tau}, L_{\tau}, M_{\tau+1}/P_{\tau}) + \lambda_{\tau} \left(W_{\tau} L_{\tau} + R_{\tau}^{K} u_{\tau} K_{\tau-1} + (1+R_{\tau-1}) B_{\tau} + \sum_{s=0}^{\infty} X_{\tau}^{s} S_{\tau}^{s} + M_{\tau} - P_{\tau} T_{\tau} - P_{\tau} \left[C_{\tau} + K_{\tau} - (1-\delta(u_{\tau})) K_{\tau-1} \right] - M_{\tau+1} - B_{\tau+1} - \sum_{s=0}^{\infty} Q_{\tau}^{s} S_{\tau+1}^{s} \right) \right],$$

where λ_{τ} is the Lagrange multiplier for the budget identity in period τ .

• The first-order conditions for C_{τ} , L_{τ} , $B_{\tau+1}$, $S^s_{\tau+1}$, and $M_{\tau+1}$ are the same as before. The ones for K_{τ} and u_{τ} (for $\tau = t, t+1, t+2, ...$) are:

$$\frac{\partial \mathcal{L}_{t}^{H}}{\partial u_{\tau}} = \left(\frac{1}{1+\rho}\right)^{\tau-t} \mathbb{E}_{t} \left[r_{\tau}^{K} - \delta'\left(u_{\tau}\right)\right] \lambda_{\tau} P_{\tau} K_{\tau-1} = 0,$$

$$\frac{\partial \mathcal{L}_{t}^{H}}{\partial K_{\tau}} = \left(\frac{1}{1+\rho}\right)^{\tau-t} \mathbb{E}_{t} \left[-\lambda_{\tau} P_{\tau} + \frac{\lambda_{\tau+1}}{1+\rho} (R_{\tau+1}^{K} u_{\tau+1} + (1-\delta\left(u_{\tau+1}\right))P_{\tau+1})\right] = 0.$$
(A.54)

• For the planning period, these expressions can be written as:

$$r_t^K = \delta'\left(u_t\right),\tag{A.55}$$

$$1 = \mathbb{E}_{t} \left[\frac{r_{t+1}^{K} u_{t+1} + 1 - \delta\left(u_{t+1}\right)}{1 + \rho} \frac{U_{C}(C_{t+1}, L_{t+1}, M_{t+2}/P_{t+1})}{U_{C}(C_{t}, L_{t}, M_{t+1}/P_{t})} \right].$$
(A.56)

Equation (A.55) is new and equation (A.56) replaces (22) in the text.

- The resulting macroeconomic model is reported in Table A.9. Notice that the utilization rate features in equations (TA9.1), (TA9.4), (TA9.8), (TA9.13), and (TA9.17).
- The simulation results have been reported in Table 8(g) in the paper.
- The impulse-response functions for the persistent technology shock are presented in Figure A.11. The main features are the same as for the benchmark NK-FMS model.

Table A.9: The New Keynesian DSGE model with a variable capital utilization rate

$$K_{t} = I_{t} + [1 - \delta(u_{t})] K_{t-1}$$
(TA9.1)

$$Y_t = C_t + I_t + G \tag{1A9.2}$$

$$1 = \mathbb{E}_t \left[\frac{1 + R_t}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{\prime} \frac{P_t}{P_{t+1}} \right]$$
(TA9.3)

$$1 = \mathbb{E}_t \left[\frac{1 + r_{t+1}^K u_{t+1} - \delta(u_{t+1})}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \right]$$
(TA9.4)

$$\frac{M_{t+1}}{P_t} = C_t^{\eta/\sigma} \left(\frac{R_t}{\varepsilon_m(1+R_t)}\right)^{-\eta} \tag{TA9.5}$$

$$\varepsilon_l L_t^{\kappa} = w_t C_t^{-1/\sigma} \tag{TA9.6}$$

$$w_t = (1 - \alpha) mc_t \frac{Y_t^{*}}{L_t - \bar{L}}$$
(TA9.7)

$$r_t^K = \alpha \, mc_t \, \frac{r_t}{u_t K_{t-1}} \tag{TA9.8}$$

$$P_t^n = \frac{\theta}{\theta - 1} \frac{\Xi_t^n}{\Xi_t^D} \tag{TA9.9}$$

$$\Xi_t^N = C_t^{-1/\sigma} P_t^{\theta} Y_t m c_t + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^N \right]$$
(TA9.10)

$$\Xi_t^D = C_t^{-1/\sigma} P_t^{\theta-1} Y_t + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^D \right]$$
(TA9.11)

$$P_{t} = \left[(1 - \zeta) (P_{t}^{n})^{1-\theta} + \zeta P_{t-1}^{1-\theta} \right]^{1/(1-\theta)}$$
(TA9.12)
$$V_{t}^{\theta} = \left[(I - \zeta) (P_{t}^{n})^{1-\theta} + \zeta P_{t-1}^{1-\theta} \right]^{1-\alpha}$$
(TA9.12)

$$Y_{t}^{a} = [u_{t}K_{t-1}]^{\alpha} \left[\Omega_{0}e^{Z_{t}}(L_{t}-L)\right]$$
(TA9.13)
$$P_{t}^{a} = \left[(1-e^{2})\left(D_{t}^{a}\right)^{-\theta} + c\left(D_{t}^{a}\right)^{-\theta}\right]^{-1/\theta}$$
(TA9.14)

$$P_t^a = \left[(1 - \zeta) \left(P_t^n \right)^{-\theta} + \zeta \left(P_{t-1}^a \right)^{-\theta} \right]$$
(TA9.14)
$$\left(P_t^a \right)^{\theta}$$

$$Y_t = \left(\frac{T_t}{P_t}\right) Y_t^a \tag{TA9.15}$$

$$\tilde{Z}_t = \xi_z \tilde{Z}_{t-1} + \eta_t^z \tag{TA9.16}$$

$$\delta'(u_t) = r_t^k \tag{TA9.17}$$

Definitions: Y_t is output, C_t is private consumption, L_t is employment, K_t is the capital stock at the end of period t, u_t is the capital utilization rate, $w_t \equiv W_t/P_t$ is the real wage rate, $r_t^K \equiv R_t^K/P_t$ is the real rental rate on capital, $mc_t \equiv MC_t/P_t$ is real marginal cost, P_t is the price level, P_t^n is the price set by green-light firms, Y_t^a is an alternative output measure, P_t^a is an alternative price index, R_t is the rate of interest on risk-free bonds, and I_t is gross investment. The exogenous variables are the nominal money supply M_{t+1} , government consumption \overline{G} , and the innovation term in the technology process η_t^z . The structural parameters are σ , η , κ , ε_l , ε_m , ρ , δ , ζ , θ , \overline{L} , Ω_0 , ξ_z , and α .



Figure A.11: Transitory productivity shock; variable utilization rate of capital





A.3.10 Flex-price and sticky-price firms

In this subsection we assume that a fraction ψ of firms in the intermediate sector face the Calvo friction whilst the remaining fraction $1 - \psi$ of firms can freely adjust their prices at all times. Here we sketch how the benchmark model is changed as a result of this assumption.

• Flexible-price firms set their price according to the usual markup formula:

$$P_t^f = \frac{\theta}{\theta - 1} P_t m c_t.$$

• Sticky-price firms which get a green light in period t set the new price according to:

$$P_t^{ns} = \frac{\theta}{\theta - 1} \frac{\Xi_t^N}{\Xi_t^D},$$

where Ξ_t^N and Ξ_t^D evolve according to the usual expectational difference equations:

$$\begin{aligned} \Xi_t^N &= C_t^{-1/\sigma} P_t^{\theta} Y_t m c_t + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^N \right], \\ \Xi_t^D &= C_t^{-1/\sigma} P_t^{\theta-1} Y_t + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^D \right]. \end{aligned}$$

• The regular and alternative price indices for all sticky-price firms together are given by:

$$P_t^s = \left[(1 - \zeta) (P_t^{ns})^{1-\theta} + \zeta (P_{t-1}^s)^{1-\theta} \right]^{1/(1-\theta)},$$

$$P_t^{as} = \left[(1 - \zeta) (P_t^{ns})^{-\theta} + \zeta (P_{t-1}^{as})^{-\theta} \right]^{-1/\theta}.$$

• Finally, the regular and alternative aggregate price levels are:

$$P_{t} = \left[(1 - \psi) (P_{t}^{f})^{1-\theta} + \psi (P_{t}^{s})^{1-\theta} \right]^{1/(1-\theta)},$$

$$P_{t}^{a} = \left[(1 - \psi) (P_{t}^{f})^{-\theta} + \psi (P_{t}^{as})^{-\theta} \right]^{-1/\theta}.$$

- The flex-sticky model is listed in Table A.10.
- The simulation results (for $\psi = 0.8$) have been reported in Table 8(h) in the paper.
- The impulse response diagrams for $\psi = 0.8$ are depicted in Figure A.12. The main features are:
 - The Implausible Result is not observed.
 - Hump-shaped responses in output, consumption, investment, and the capital stock.
 - Dampened effect on marginal cost (compared to the NK-FMS model).

$$K_{t} = I_{t} + (1 - \delta)K_{t-1}$$
(TA10.1)

$$Y_{t} = C_{t} + I_{t} + \bar{G}$$
(TA10.2)

$$1 = \mathbb{E}_t \left[\frac{1 + R_t}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \frac{P_t}{P_{t+1}} \right]$$
(TA10.3)
$$\left[1 + \pi K - \delta \left(C_t - \gamma \right)^{1/\sigma} \right]$$

$$1 = \mathbb{E}_t \left[\frac{1 + r_{t+1}^R - \delta}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1/\delta} \right]$$
(TA10.4)

$$\frac{M_{t+1}}{P_t} = C_t^{\eta/\sigma} \left(\frac{R_t}{\varepsilon_m(1+R_t)}\right)^{-\eta} \tag{TA10.5}$$

$$\varepsilon_l L_t = w_t C_t \qquad (1A10.6)$$

$$w_t = (1 - \alpha) m_{ct} \frac{Y_t^a}{Y_t^a} \qquad (TA10.7)$$

$$r_t^K = \alpha mc_t \frac{Y_t^a}{K_{t-1}}$$
(TA10.8)

$$P_t^{ns} = \frac{\theta}{\theta - 1} \frac{\Xi_t^N}{\Xi_t^D} \tag{TA10.9}$$

$$P_t^f = \frac{\theta}{\theta - 1} P_t m c_t \tag{TA10.10}$$

$$\Xi_t^N = C_t^{-1/\sigma} P_t^{\theta} Y_t m c_t + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^N \right]$$
(TA10.11)

$$\Xi_t^D = C_t^{-1/\sigma} P_t^{\theta-1} Y_t + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^D \right]$$
(TA10.12)

$$P_{t}^{s} = \left[(1 - \zeta) (P_{t}^{ns})^{1-\theta} + \zeta (P_{t-1}^{s})^{1-\theta} \right]^{1/(1-\theta)}$$
(TA10.13)

$$P_t^{as} = \left[(1 - \zeta) (P_t^{ns})^{-\theta} + \zeta (P_{t-1}^{as})^{-\theta} \right]^{1/(1-\theta)}$$
(TA10.14)
$$P_t = \left[(1 - \zeta) (P_t^{f_1 - \theta} + \zeta (P_{t-1}^{s})^{1-\theta})^{1/(1-\theta)} \right]^{1/(1-\theta)}$$
(TA10.15)

$$P_{t} = \left[(1 - \psi) (P_{t}^{f})^{1-\theta} + \psi (P_{t}^{s})^{1-\theta} \right]$$
(TA10.15)
$$P_{t}^{a} = \left[(1 - \psi) (P_{t}^{f})^{-\theta} + \psi (P_{t}^{s})^{-\theta} \right]^{-1/\theta}$$
(TA10.16)

$$P_t^{\alpha} = \left[(1 - \psi) \left(P_t^{\alpha} \right)^{\alpha} + \psi \left(P_t^{\alpha} \right)^{\alpha} \right]$$

$$(\text{TA10.16})$$

$$V_t^{\alpha} = K^{\alpha} \left[O_{\alpha} e^{\tilde{Z}_t} \left(I_{\alpha} - \bar{I}_{\alpha} \right)^{1 - \alpha} \right]$$

$$(\text{TA10.17})$$

$$Y_t = K_{t-1} \left[\frac{\Omega_0 e^{-t} (L_t - L)}{D} \right]$$
(TA10.17)
$$Y_t = \left(\frac{P_t^a}{D} \right)^{\theta} Y_t^a$$
(TA10.18)

$$\tilde{Z}_t = \xi_z \tilde{Z}_{t-1} + \eta_t^z \tag{TA10.19}$$

Definitions: Y_t is output, C_t is private consumption, L_t is employment, K_t is the capital stock at the end of period t, $w_t \equiv W_t/P_t$ is the real wage rate, $r_t^K \equiv R_t^K/P_t$ is the real rental rate on capital, $mc_t \equiv MC_t/P_t$ is real marginal cost, P_t is the price level, P_t^n is the price set by green-light firms, Y_t^a is an alternative output measure, P_t^a and P_t^{as} are alternative price indices, R_t is the rate of interest on risk-free bonds, and I_t is gross investment. The exogenous variables are the nominal money supply M_{t+1} , government consumption \overline{G} , and the innovation term in the technology process η_t^z . The structural parameters are σ , η , κ , ε_l , ε_m , ρ , δ , ζ , θ , \overline{L} , ψ , Ω_0 , ξ_z , and α .



Figure A.12: Transitory productivity shock: flex-price and sticky-price firm



Figure A.12, Continued

A.3.11 Non-separable preferences

As a final robustness check we consider preferences that are non-separable in $(C_{\tau}, M_{\tau+1}/P_{\tau})$ and L_{τ} . In order to find the first-order conditions for the household's decision problem we postulate the Lagrangian:

$$\begin{aligned} \mathcal{L}_{t}^{H} &\equiv \mathbb{E}_{t} \sum_{\tau=t}^{\infty} \left(\frac{1}{1+\rho} \right)^{\tau-t} \left[U(C_{\tau}, M_{\tau+1}/P_{\tau}) - \varepsilon_{l} \frac{L_{\tau}^{1+\kappa}}{1+\kappa} \right. \\ &+ \lambda_{\tau} \Big(W_{\tau} L_{\tau} + R_{\tau}^{K} K_{\tau-1} + (1+R_{\tau-1}) B_{\tau} + \sum_{s=0}^{\infty} X_{\tau}^{s} S_{\tau}^{s} + M_{\tau} - P_{\tau} T_{\tau} \\ &- P_{\tau} \left[C_{\tau} + K_{\tau} - (1-\delta) K_{\tau-1} \right] - M_{\tau+1} - B_{\tau+1} - \sum_{s=0}^{\infty} Q_{\tau}^{s} S_{\tau+1}^{s} \Big) \Big], \end{aligned}$$

where λ_{τ} is the Lagrange multiplier for the budget identity in period τ and $U(C_{\tau}, M_{\tau+1}/P_{\tau})$ is the subfelicity function for consumption and real money balances. Assuming an interior solution the first-order conditions for this problem (for $\tau = t, t+1, t+2, ...$) are:

$$\frac{\partial \mathcal{L}_t^H}{\partial C_{\tau}} = \left(\frac{1}{1+\rho}\right)^{\tau-t} \mathbb{E}_t \left[U_C(C_{\tau}, M_{\tau+1}/P_{\tau}) - \lambda_{\tau} P_{\tau} \right] = 0, \tag{A.57}$$

$$\frac{\partial \mathcal{L}_t^H}{\partial L_\tau} = \left(\frac{1}{1+\rho}\right)^{\tau-t} \mathbb{E}_t \left[-\varepsilon_l L_\tau^\kappa + \lambda_\tau W_\tau\right] = 0, \tag{A.58}$$

$$\frac{\partial \mathcal{L}_t^H}{\partial K_\tau} = \left(\frac{1}{1+\rho}\right)^{\tau-t} \mathbb{E}_t \left[-\lambda_\tau P_\tau + \frac{\lambda_{\tau+1}}{1+\rho} (R_{\tau+1}^K + (1-\delta)P_{\tau+1})\right] = 0, \tag{A.59}$$

$$\frac{\partial \mathcal{L}_t^H}{\partial B_{\tau+1}} = \left(\frac{1}{1+\rho}\right)^{\tau-t} \mathbb{E}_t \left[-\lambda_\tau + \frac{\lambda_{\tau+1}}{1+\rho}(1+R_\tau)\right] = 0, \tag{A.60}$$

$$\frac{\partial \mathcal{L}_t^H}{\partial S_{\tau+1}^s} = \left(\frac{1}{1+\rho}\right)^{\tau-t} \mathbb{E}_t \left[-\lambda_\tau Q_\tau^s + \frac{\lambda_{\tau+1}}{1+\rho} X_{\tau+1}^s\right] = 0, \tag{A.61}$$

$$\frac{\partial \mathcal{L}_t^H}{\partial M_{\tau+1}} = \left(\frac{1}{1+\rho}\right)^{\tau-t} \mathbb{E}_t \left[\frac{U_{M/P}(C_{\tau}, M_{\tau+1}/P_{\tau})}{P_{\tau}} - \lambda_{\tau} + \frac{\lambda_{\tau+1}}{1+\rho}\right] = 0.$$
(A.62)

• Following Fischer (1979) we assume that:

$$U(C_{\tau}, M_{\tau+1}/P_{\tau}) \equiv \frac{\left[C_{\tau}^{\gamma}(M_{\tau+1}/P_{\tau})^{1-\gamma}\right]^{1-1/\sigma} - 1}{1 - 1/\sigma}$$

- $-\sigma$ is the intertemporal substitution elasticity ($\sigma > 0$)
- for $\sigma = 1$, we get $U(C, m) = \gamma \ln C + (1 \gamma) \ln m$ (separable case)
- $\ \gamma$ is the subfelicity taste parameter for consumption $(0 < \gamma < 1)$
- Useful things to know:

$$U_m(C,m) = \frac{1-\gamma}{m} \left[C^{\gamma} m^{1-\gamma} \right]^{1-1/\sigma}$$

$$U_C(C,m) = \frac{\gamma}{C} \left[C^{\gamma} m^{1-\gamma} \right]^{1-1/\sigma}$$
$$\frac{U_m(C,m)}{U_C(C,m)} = \frac{1-\gamma}{\gamma} \frac{C}{m}$$

• The Euler equations for consumption become:

$$1 = E_t \left[\frac{1 + R_t}{1 + \rho} \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \frac{\Gamma_{t+1}}{\Gamma_t} \right],$$

$$1 = E_t \left[\frac{1 + r_{t+1}^K - \delta}{1 + \rho} \frac{C_t}{C_{t+1}} \frac{\Gamma_{t+1}}{\Gamma_t} \right],$$

where:

$$\Gamma_t \equiv \left[C_t^{\gamma} (M_{t+1}/P_t)^{1-\gamma} \right]^{1-1/\sigma}.$$

• We find:

$$\frac{\Gamma_{t+1}}{\Gamma_t} = \frac{\left[C_{t+1}^{\gamma} M_{t+2}^{1-\gamma} P_{t+1}^{\gamma-1}\right]^{1-1/\sigma}}{\left[C_t^{\gamma} M_{t+1}^{1-\gamma} P_t^{\gamma-1}\right]^{1-1/\sigma}} = \left(\frac{C_t}{C_{t+1}}\right)^{\gamma(1/\sigma-1)} \left(\frac{P_t}{P_{t+1}}\right)^{-(1-\gamma)(1/\sigma-1)} \left(\frac{M_{t+1}}{M_{t+2}}\right)^{(1-\gamma)(1/\sigma-1)}.$$

• With a constant money supply

$$\begin{split} 1 &= E_t \left[\frac{1 + R_t}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1 + \gamma(1/\sigma - 1)} \left(\frac{P_t}{P_{t+1}} \right)^{1 - (1 - \gamma)(1/\sigma - 1)} \right]. \\ 1 &= E_t \left[\frac{1 + r_{t+1}^K - \delta}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1 + \gamma(1/\sigma - 1)} \left(\frac{P_t}{P_{t+1}} \right)^{-(1 - \gamma)(1/\sigma - 1)} \right]. \end{split}$$

• We also find that the money demand function takes the following form:

$$\frac{M_{t+1}}{P_t} = \frac{1-\gamma}{\gamma} C_t \frac{1+R_t}{R_t}.$$

• After linearization we obtain:

$$\tilde{M}_{t+1} - \tilde{P}_t = \tilde{C}_t - \frac{1}{\rho} \frac{dR_t}{1+\rho}.$$

• The (annual) semi-elasticity for this specification is thus:

$$\epsilon_{MR} = \left| -\frac{1}{\rho_a (1+\rho_a)} \right| = \frac{1}{0.05 \times 1.05} = 19.05$$

which exceeds the value of $\epsilon_{M\!R}=8$ that is used in the calibration.

Sticky prices In Table A.11 we present the non-separable sticky-price model. Comment on its main features compared to the benchmark model.

In Figure A.13 we depict the impulse-response functions for a persistent technology shock. (a) Explain the recalibration; (b) Comment on the main features. Table A.11: The New Keynesian DSGE model with non-separable preferences

$$K_t = I_t + (1 - \delta)K_{t-1}$$
(TA11.1)

$$Y_t = C_t + I_t + G$$
(TA11.2)

$$1 = \mathbb{E}_t \left[\frac{1 + R_t}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1 + \gamma(1/\sigma - 1)} \left(\frac{P_t}{P_{t+1}} \right)^{1 - (1 - \gamma)(1/\sigma - 1)} \left(\frac{M_{t+1}}{M_{t+2}} \right)^{(1 - \gamma)(1/\sigma - 1)} \right]$$
(TA11.3)

$$1 = \mathbb{E}_t \left[\frac{1 + r_{t+1}^K - \delta}{1 + \rho} \left(\frac{C_t}{C_{t+1}} \right)^{1 + \gamma(1/\sigma - 1)} \left(\frac{P_t}{P_{t+1}} \right)^{-(1 - \gamma)(1/\sigma - 1)} \left(\frac{M_{t+1}}{M_{t+2}} \right)^{(1 - \gamma)(1/\sigma - 1)} \right]$$
(TA11.4)

$$\frac{M_{t+1}}{P_t} = \frac{1-\gamma}{\gamma} C_t \frac{1+R_t}{R_t} \tag{TA11.5}$$

$$\varepsilon_l L_t^{\kappa} = \gamma w_t C_t^{-[1+\gamma(1/\sigma-1)]} \left(\frac{M_{t+1}}{P_t}\right)^{-(1-\gamma)(1/\sigma-1)}$$
(TA11.6)

$$w_t = (1 - \alpha) mc_t \frac{Y_t^a}{L_t - \bar{L}}$$
(TA11.7)

$$r_t^K = \alpha \, mc_t \, \frac{Y_t^\alpha}{K_{t-1}} \tag{TA11.8}$$

$$P_t^n = \frac{\theta}{\theta - 1} \frac{\Xi_t^N}{\Xi_t^D} \tag{TA11.9}$$

$$\Xi_t^N = \gamma C_t^{-(1+\gamma(1/\sigma-1))} \left(\frac{M_{t+1}}{P_t}\right)^{-(1-\gamma)(1/\sigma-1)} P_t^{\theta} Y_t m c_t + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^N\right]$$
(TA11.10)

$$\Xi_t^D = \gamma C_t^{-(1+\gamma(1/\sigma-1))} \left(\frac{M_{t+1}}{P_t}\right)^{-(1-\gamma)(1/\sigma-1)} P_t^{\theta-1} Y_t + \frac{\zeta}{1+\rho} \mathbb{E}_t \left[\Xi_{t+1}^D\right]$$
(TA11.11)

$$P_{t} = \left[(1 - \zeta) (P_{t}^{n})^{1-\theta} + \zeta (P_{t-1})^{1-\theta} \right]^{1/(1-\theta)}$$
(TA11.12)

$$Y_t^a = K_{t-1}^{\alpha} \left[\Omega_0 e^{\tilde{Z}_t} (L_t - \bar{L}) \right]^{1-\alpha}$$
(TA11.13)

$$P_t^a = \left[(1-\zeta) \left(P_t^n \right)^{-\theta} + \zeta \left(P_{t-1}^a \right)^{-\theta} \right]^{-1/\theta}$$
(TA11.14)

$$Y_t = \left(\frac{P_t^a}{P_t}\right)^v Y_t^a \tag{TA11.15}$$

$$\tilde{Z}_t = \xi_z \tilde{Z}_{t-1} + \eta_t^z \tag{TA11.16}$$

Definitions: Y_t is output, C_t is private consumption, L_t is employment, K_t is the capital stock at the end of period t, $w_t \equiv W_t/P_t$ is the real wage rate, $r_t^K \equiv R_t^K/P_t$ is the real rental rate on capital, $mc_t \equiv MC_t/P_t$ is real marginal cost, P_t is the price level, P_t^n is the price set by green-light firms, Y_t^a is an alternative output measure, P_t^a and P_t^{as} are alternative price indices, R_t is the rate of interest on risk-free bonds, and I_t is gross investment. The exogenous variables are the nominal money supply M_{t+1} , government consumption \bar{G} , and the innovation term in the technology process η_t^z . The structural parameters are σ , η , κ , ε_l , ε_m , ρ , δ , ζ , θ , γ , \bar{L} , ψ , Ω_0 , ξ_z , and α .



Figure A.13: Transitory productivity shock: nonseparable preferences and sticky prices



Figure A.13, Continued

A.4 Other shocks

In the main text we restrict attention to the dynamic affects of a temporary (but persistent) productivity shock. This does not mean that the Implausible Result is not obtained for other shocks. In this appendix we consider two typical macro shocks, namely a fiscal policy shock and a monetary policy shock.

A.4.1 Government spending shocks

• Form of the government spending process:

$$\tilde{G}_t = \xi_q \tilde{G}_{t-1} + \eta_t^g$$

with $\xi_q = 0.95$.

- Just as in the paper we assume that at shock-time t = 0 the system is in the deterministic steady state and we compute the effects at impact and over time of a single positive innovation term in the government spending process, i.e. we set $\eta_0^g = 0.0099503$ (so that G_t increases by one percent at impact) and $\eta_t^g = 0$ for $t = 1, 2, \ldots$ The implied perturbation in government spending is thus given by $\tilde{G}_t = \xi_g^t \eta_0^g$ for $t = 1, 2, \ldots$
- The salient features for eleven cases are reported in Table A.12
- Table A.12(a): Benchmark NK-FMS model
 - Large positive impact effects on output, investment, employment, and marginal cost: $\tilde{Y}_0 = 0.5325$, $\tilde{I}_0 = 3.9009$, $\tilde{L}_0 = 0.6403$, and $\tilde{mc}_0 = 0.7312$.
 - Impact effects almost entirely eliminated one period after the shock, i.e. $(Y_1 Y_0)/Y^* = \tilde{Y}_1 \tilde{Y}_0 = -0.4478$, $\tilde{I}_1 \tilde{I}_0 = -2.9695$, $\tilde{L}_1 \tilde{L}_0 = -0.5581$, and $\tilde{mc}_1 \tilde{mc}_0 = -0.7292$.
 - The impulse-response functions are depicted in Figure A.14.
 - The pattern of adjustment suggests the Implausible Result for G-shocks summarized in a definition.

Definition A.1 (Implausible Result for G **shocks)** In the New Keynesian DSGE model with a constant nominal money supply and endogenous capital accumulation (NK-FMS), a temporary increase in government consumption results in implausible effects in that: (i) output, employment, and investment increase sharply at impact, (ii) the wage rate and the rental rate on units of capital both increase at impact, (iii) there is a large increase in real marginal cost at impact,(iv) the effects on output, employment, investment, the wage rate, the rental rate, and marginal costs are virtually eliminated one period after the shock.

• Robustness results are reported in columns (b)–(k) of Table A.12.

Table A.12: Impact effects of a persistent government spending shock

variable:	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)
\tilde{Y}_0	0.5325	0.1522	0.1663	0.5312	0.0814	1.4085	0.4310	0.1631	0.0532	0.6708	0.2053
$\tilde{Y}_1 - \tilde{Y}_0$	-0.4478	-0.0271	-0.0223	-0.4462	-0.0046	-1.3081	-0.3294	-0.0285	-0.0031	-0.5887	-0.0673
$ ilde{C}_0$	-0.0868	-0.0598	-0.0421	-0.0870	-0.1248	-0.0119	-0.0957	-0.0650	-0.1259	-0.0768	-0.1160
\tilde{I}_0	3.9009			2.5561	1.0760	9.3814	3.2667	-0.0350	0.8937	4.7737	1.8591
$\tilde{I}_1 - \tilde{I}_0$	-2.9695			-2.8805	-0.0443	-8.6487	-2.1958	-0.0243	-0.0239	-3.9009	-0.4812
\tilde{L}_0	0.6403	0.1829	0.1498	0.6381	0.0977	1.6941	0.5180	0.1960	0.0641	0.6501	0.2472
$\tilde{L}_1 - \tilde{L}_0$	-0.5581	-0.0325	-0.0200	-0.5559	-0.0037	-1.6365	-0.4109	-0.0345	-0.0001	-0.5739	-0.0853
\tilde{P}_0	0.0626	0.0362	0.0245	0.0627	0.0944	0.0000	0.0515	0.0386	-0.0449	0.0538	0.0888
\tilde{P}_0^a	0.0625	0.0361	0.0244			0.0000	0.0514	0.0386	-0.0450	0.0538	0.0887
\tilde{P}_0^n	0.2524	0.1453	0.0982			0.0151	0.2072	0.1552	-0.1786	0.2168	0.1583
$ ilde{w}_0$	0.5529	0.1230	0.1076	0.5505	-0.0271	1.6820	0.4218	0.1309	-0.0619	0.5728	0.1309
\tilde{r}_0^K	0.0485	0.0125		0.0483	0.0031	0.1376	0.0383	0.0134	0.0004	0.0298	0.0155
\tilde{R}_0	0.0006	0.0002	0.0001	0.0006	0.0010	-0.0001	0.0061	0.0002	-0.0413	0.0005	0.0010
\widetilde{mc}_0	0.7312	0.1738	0.1076	0.7283	0.0000	2.1571	0.5560	0.1854	-0.0441	0.6240	0.1996
$\widetilde{mc}_1 - \widetilde{mc}_0$	-0.7292	-0.0629	-0.0353	-0.7262	0.0000	-2.1517	-0.5345	-0.0672	0.0034	-0.6220	-0.1041
\tilde{P}_0^K		-0.0141									
\tilde{M}_1							-0.2155		1.4426		
\widetilde{q}_0								0.0488			
\tilde{u}										0.5177	

Notes: \tilde{x}_0 is the relative change in variable x at impact. Column (a) is the solution for the benchmark New Keynesian model featuring a constant money supply. Column (b): model with a fixed capital stock. Column (c) model without capital. Columns (d) model with Rotemberg pricing with χ set according to $\chi = \zeta(\theta - 1)(1 + \rho)/[(1 - \zeta)(1 + \rho - \zeta)]$. Column (e) model with flexible prices. Column (f) model with a predetermined price level. The entry for P_0^n refers to the new price to be effective one period after the shock. Column (g) model with a Keynesian (countercyclical) money supply rule featuring $\mu_y = 0.5$. Column (h) model with capital adjustment costs. Column (i) model with a Taylor rule featuring $\phi_{\pi} = 1.5$ and $\phi_y = 0.5$. Column (j) is the case with an endogenously chosen utilization rate of capital. Column (k) is the case with a fraction $1 - \psi$ of firms featuring perfectly flexible prices with the rest of the firms facing the Calvo friction (we set $\psi = 0.8$).

- The Implausible Result for G-shocks is also observed in columns (d) [Rotemberg pricing], (f) [predetermined price level], (g) [output-based money supply rule], and (j) [endogenous utilization rate of capital].
- The Implausible Result is not observed in the other cases covered in Table A.12



Figure A.14: Transitory government consumption shock in the NK-FMS model





A.4.2 Money supply shocks

• Form of the nominal money supply process:

$$\tilde{M}_t = \xi_m \tilde{M}_{t-1} + \eta_t^m$$

with $\xi_m = 0.95$.

- Just as in the paper we assume that at shock-time t = 0 the system is in the deterministic steady state and we compute the effects at impact and over time of a single positive innovation term in the nominal money supply process, i.e. we set $\eta_0^m = 0.0099503$ (so that M_t increases by one percent at impact) and $\eta_t^m = 0$ for $t = 1, 2, \ldots$ The implied perturbation in the money supply is thus given by $\tilde{M}_t = \xi_m^t \eta_0^m$ for $t = 1, 2, \ldots$
- The salient features for eleven cases are reported in Table A.13
- Table A.13(a): Benchmark NK-FMS model
 - Large positive impact effects on output, investment, employment, and marginal cost: $\tilde{Y}_0 = 1.7043$, $\tilde{I}_0 = 10.6669$, $\tilde{L}_0 = 2.0596$, and $\tilde{mc}_0 = 2.7873$.
 - Impact effects almost entirely eliminated one period after the shock, i.e. $(Y_1 Y_0)/Y^* = \tilde{Y}_1 \tilde{Y}_0 = -1.6864$, $\tilde{I}_1 \tilde{I}_0 = -11.1295$, $\tilde{L}_1 \tilde{L}_0 = -2.1149$, and $\tilde{mc}_1 \tilde{mc}_0 = -2.7921$.
 - The impulse-response functions are depicted in Figure A.15.
 - The pattern of adjustment suggests the Implausible Result for *M*-shocks summarized in a definition.

Definition A.2 (Implausible Result for M **shocks)** In the New Keynesian DSGE model with an exogenous nominal money supply and endogenous capital accumulation (NK-FMS), a temporary increase in the nominal money supply results in implausible effects in that: (i) output, employment, and investment increase sharply at impact, (ii) the wage rate and the rental rate on units of capital both increase at impact, (iii) there is a large increase in real marginal cost at impact,(iv) the effects on output, employment, investment, the wage rate, the rental rate, and marginal costs are virtually eliminated one period after the shock.

- Robustness results are reported in columns (b)–(k) of Table A.13.
 - The Implausible Result for *M*-shocks is also observed in columns (d) [Rotemberg pricing], (f) [predetermined price level], (g) [output-based money supply rule], and (j) [endogenous utilization rate of capital].
 - The Implausible Result is not observed in the other cases covered in Table A.13

Table A.13	: Impact	effects	of a	persistent	money	supply	shock
					•/		

variable:	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)
$ ilde{Y}_0$	1.7043	0.1957	0.2066	1.6883	0.0000	4.8661	1.5487	0.1920	0.0000	2.6392	0.4699
$\tilde{Y}_1 - \tilde{Y}_0$	-1.6864	-0.0709	-0.0677	-1.6668	0.0000	-4.7892	-1.4530	-0.0685	0.0000	-2.6525	-0.2448
$ ilde{C}_0$	0.1449	0.2446	0.2583	0.1428	0.0000	0.4158	0.1306	0.2467	0.0000	0.1440	0.0349
\tilde{I}_0	10.6669			10.4082	0.0000	30.4470	9.6976	0.2107	0.0000	16.8642	2.9623
$\tilde{I}_1 - \tilde{I}_0$	-11.1295			-10.8386	0.0000	-31.6107	-9.6205	-0.0756	0.0000	-17.5352	-1.7041
\tilde{L}_0	2.0596	0.2369	0.1872	2.0317	0.0000	5.8862	1.8711	0.2325	0.0000	2.5217	0.5745
$\tilde{L}_1 - \tilde{L}_0$	-2.1149	-0.0849	-0.0608	-2.0869	0.0000	-6.0303	-1.8260	-0.0836	0.0000	-2.5928	-0.3234
$ ilde{P}_0$	0.2261	0.1125	0.1001	0.2279	0.3494	0.0000	0.2155	0.1116	0.0000	0.2295	0.3249
\tilde{P}_0^a	0.2253	0.1123	0.0999			0.0000	0.2148	0.1114	0.0000	0.2287	0.3240
\tilde{P}_0^n	0.9332	0.4571	0.4059			0.0197	0.8881	0.4532	0.0000	0.9479	0.5705
$ ilde w_0$	2.2075	0.4821	0.4460	2.1773	0.0000	6.3265	2.0042	0.4798	0.0000	2.6693	0.6097
\tilde{r}_0^K	0.1740	0.0286		0.1716	0.0000	0.5082	0.1579	0.0283	0.0000	0.1260	0.0479
\tilde{R}_0	-0.0191	-0.0210	-0.0211	-0.0191	-0.0177	-0.0219	0.0016	-0.0210	0.0000	-0.0190	-0.0178
\widetilde{mc}_0	2.7873	0.5482	0.4460	2.7492	0.0000	8.0238	2.5303	0.5446	0.0000	2.8246	0.7698
$\widetilde{mc}_1 - \widetilde{mc}_0$	-2.7921	-0.1975	-0.1457	-2.7542	0.0000	-8.0168	-2.4060	-0.1966	0.0000	-2.8313	-0.4138
\tilde{P}_0^K		0.2506									
\tilde{M}_1							0.2179		-0.9453		
\widetilde{q}_0								0.2693			
$ ilde{u}$										2.1826	

Notes: \tilde{x}_0 is the relative change in variable x at impact. Column (a) is the solution for the benchmark New Keynesian model featuring a constant money supply. Column (b): model with a fixed capital stock. Column (c) model without capital. Columns (d) model with Rotemberg pricing with χ set according to $\chi = \zeta(\theta - 1)(1 + \rho)/[(1 - \zeta)(1 + \rho - \zeta)]$. Column (e) model with flexible prices. Column (f) model with a predetermined price level. The entry for P_0^n refers to the new price to be effective one period after the shock. Column (g) model with a Keynesian (countercyclical) money supply rule featuring $\mu_y = 0.5$. Column (h) model with capital adjustment costs. Column (i) model with a Taylor rule featuring $\phi_{\pi} = 1.5$ and $\phi_y = 0.5$. Column (j) is the case with an endogenously chosen utilization rate of capital. Column (k) is the case with a fraction $1 - \psi$ of firms featuring perfectly flexible prices with the rest of the firms facing the Calvo friction (we set $\psi = 0.8$).



Figure A.15: Transitory monetary supply shock in the NK-FMS model





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