Foundations of Modern Macroeconomics Third Edition

Chapter 19: DSGE – New Keynesian models

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Outline

Dynamic Stochastic General Equilibrium

- Let us build a New Keynesian Dynamic Stochastic General Equilibrium (NK-DSGE) model
- Can be seen as an RBC model with monetary features: Monetary Business Cycle (MBC) model
- Very loosely based on: Yun (1996 JME), Bernanke *et al.* (1999), and Christiano *et al.* (2005)
- Main features:
    - Infinitely-lived representative household
    - Final good constructed with differentiated inputs
    - Differentiated input produced by monopolistically competitive firms (D-S approach)
    - Calvo pricing (green-light-red-light approach)
    - Perfectly flexible factor prices (perfect factor mobility)
    - Discrete time
    - Stochastic shocks

Representative firm in the homogeneous goods sector

- Continuum approach to product differentiation is adopted
- Technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{1-1/\theta} di \right]^{1/(1-1/\theta)}, \qquad \theta > 1$$

- Unit cost:

$$P_t \equiv \left[\int_0^1 P_t(i)^{1-\theta} di \right]^{1/(1-\theta)}$$

- Derived (inverse) demand:

$$Y_t(i) = Y_t \cdot \left(\frac{P_t(i)}{P_t} \right)^{-\theta} \qquad \Leftrightarrow \qquad P_t(i) = P_t \cdot \left(\frac{Y_t(i)}{Y_t} \right)^{-1/\theta}$$

Firm producing input variety i (1)

- Monopolistically competitive
- Bertrand competitor: in setting $P_t(i)$ it takes aggregate demand and the prices of other firms as given ($\frac{\partial P_t(j)}{\partial P_t(i)} = 0$ for $j \neq i$)
- Uses capital and labour to produce output
- Faces fixed cost in the form of "overhead labour"
- Production function:

$$\begin{aligned} Y_t(i) &= F(K_t(i), Z_t \cdot (L_t(i) - \bar{L})) \\ &\equiv K_t(i)^\alpha \left[Z_t \cdot (L_t(i) - \bar{L})\right]^{1-\alpha}, \qquad 0 < \alpha < 1 \end{aligned}$$

  - $K_t(i)$ is capital used by firm i in period t
  - $L_t(i)$ is labour used by firm i in period t
  - Z_t is a labour-augmenting technological shock term (stochastic; common to all firms)

Firm producing input variety i (2)

- Cost function (definition):

$$TC_t(i) \equiv \min_{\{K_t(i), L_t(i)\}} \quad R_t^K K_t(i) + W_t L_t(i)$$
$$\text{subject to} \quad Y_t(i) = F(K_t(i), Z_t \cdot (L_t(i) - \bar{L}))$$

  - perfectly competitive input markets
  - capital and labour perfectly mobile across firms
  - common nominal rental rates at time t: R_t^K for capital and W_t for labour

Firm producing input variety i (3)

- Cost function (solution):

$$TC_t(i) = MC_t \cdot Y_t(i) + W_t \bar{L}$$

$$MC_t \equiv \left(\frac{R_t^K}{\alpha} \right)^{\alpha} \left(\frac{W_t}{(1-\alpha) Z_t} \right)^{1-\alpha}$$

  - Total cost equals variable cost, $MC_t \cdot Y_t(i)$, plus fixed cost, $W_t \bar{L}$
  - Marginal cost is the same for all firms

- Duality relationships (Shephard's Lemma):

$$K_t(i) = \frac{\partial TC_t(i)}{\partial R_t^K} = \frac{\alpha}{R_t^K} \cdot MC_t \cdot Y_t(i)$$

$$L_t(i) = \frac{\partial TC_t(i)}{\partial W_t} = \bar{L} + \frac{1-\alpha}{W_t} \cdot MC_t \cdot Y_t(i)$$

Price-setting decision of firm i (no PAC)

- In the absence of price adjustment costs the choice facing the firm is static (benchmark)
- (Nominal) Profit definition:

$$\begin{aligned} NP_t(i) &\equiv P_t(i) \cdot Y_t(i) - TC_t(i) \\ &= [P_t(i) - MC_t] \cdot Y_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta} - W_t \bar{L} \end{aligned}$$

- Profit maximizing output choice:

$$\frac{dNP_t(i)}{dP_t(i)} = Y_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta} \cdot \left[1 - \theta \, \frac{P_t(i) - MC_t}{P_t(i)} \right] = 0$$

Price-setting decision of firm i (no PAC) (2)

- Firms sets its price such that:
  - ...it equals a markup times marginal cost:

$$P_t^f(i) = \frac{\theta}{\theta - 1} \cdot MC_t$$

[superscript "f" for flex-price]

  - ...and its profit level is:

$$\begin{aligned} NP_t^f(i) &\equiv P_t^f(i) \cdot Y_t^f(i) - TC_t(i) \\ &= \frac{\theta}{\theta - 1} \cdot MC_t \cdot Y_t^f(i) - MC_t \cdot Y_t^f(i) - W_t \bar{L} \\ &= \frac{1}{\theta - 1} \cdot MC_t \cdot Y_t^f(i) - W_t \bar{L} \end{aligned}$$

note: ceteris paribus factor rental prices, profit is increasing in output

Price-setting decision of firm i (with PAC) (1)

- In the presence of price adjustment costs the choice facing the firm is dynamic
- The metaphor of Calvo pricing is used
- Main features:
  - adapted to a zero-inflation world (no indexation)
  - each period a fraction $1 - \zeta$ of firms gets to charge a **new** price, $P_t(i) = P_t^n(i)$
  - ... whilst the remaining fraction ζ of firms must charge their old price

Price-setting decision of firm i (with PAC) (2)

- Nominal profit at some future time $t+\tau$ from the perspective of time t:

$$
\begin{aligned}
NP_{t+\tau}(i) &= \left[P_t(i) - MC_{t+\tau}\right] Y_{t+\tau} \left(\frac{P_t\,(i)}{P_{t+\tau}}\right)^{-\theta} - W_{t+\tau}\bar{L} \\
&\equiv \Phi\left(P_t(i), X_{t+\tau}\right) \qquad \text{(S1)}
\end{aligned}
$$

- $X_{t+\tau}$ is the vector of macroeconomic variables (expressed in nominal terms) that are taken as given by the firm:

$$
X_{t+\tau} \equiv \left(P_{t+\tau}, Y_{t+\tau}, W_{t+\tau}, MC_{t+\tau}\right)
$$

Price-setting decision of firm i (with PAC) (3)

- The nominal value of a firm that has just received a green light and decides on $P_t(i)$:

$$V_t^0(i) = \Phi\left(P_t(i), X_t\right) + E_t\left[\sum_{\tau=1}^{\infty} \zeta^\tau \mathcal{N}_{t,t+\tau} \Phi\left(P_t(i), X_{t+\tau}\right) + \ldots\right]$$

  [we have suppressed terms not involving $P_t^n(i)$]

- $\mathcal{N}_{t,s}$ is the *nominal* stochastic discount factor (used for discounting *nominal* profits):

$$\mathcal{N}_{t,s} \equiv \left(\frac{1}{1+\rho}\right)^{s-t} \frac{U_C(C_s, 1-L_s, M_{s+1}/P_s)}{U_C(C_t, 1-L_t, M_{t+1}/P_t)} \frac{P_t}{P_s}, \qquad s \geq t \tag{S2}$$

Price-setting decision of firm i (with PAC) (4)

- The firms sets $P_t^n(i)$ in order to maximize $V_t^0(i)$
- The FONC is:

$$\frac{dV_t^0(i)}{dP_t^n(i)} = 0$$

- The solution is:

$$P_t^n(i) = P_t^n = \frac{\theta}{\theta - 1} \frac{E_t \sum_{\tau=0}^{\infty} \zeta^\tau \mathcal{N}_{t,t+\tau} P_{t+\tau}^{\theta} Y_{t+\tau} MC_{t+\tau}}{E_t \sum_{\tau=0}^{\infty} \zeta^\tau \mathcal{N}_{t,t+\tau} P_{t+\tau}^{\theta} Y_{t+\tau}}$$

  - Derivation: non-trivial (see Intermezzo 19.2)
  - Note: every green-light firm sets the same price!
  - Note: if $\zeta = 0$ then we get $P_t^n(i) = P_t^n = \frac{\theta}{\theta-1} MC_t$ (flex-price solution)

Aggregate price level (1)

- Let us derive a recursive relationship between P_t, P_t^n, and P_{t-1}
- Recall new price set at time s is the same for all green-light firms:

$$P_s^n(i) = P_s^n$$

- The price index can be written as:

$$P_t^{1-\theta} \equiv \int_0^1 P_t(i)^{1-\theta} di$$

- Poisson and law of large numbers says that $(1-\zeta)\,\zeta^s$ is the fraction of firms which determined their new price s period ago

Aggregate price level (2)

- Including indexing we get:

$$P_t^{1-\theta} = (1-\zeta)\left(P_t^n\right)^{1-\theta} + (1-\zeta)\Big[\zeta\left(P_{t-1}^n\right)^{1-\theta} + \zeta^2\left(P_{t-2}^n\right)^{1-\theta} + \dots\Big]$$
$$= (1-\zeta)\Big[\left(P_t^n\right)^{1-\theta} + \zeta\left(P_{t-1}^n\right)^{1-\theta} + \zeta^2\left(P_{t-2}^n\right)^{1-\theta} + \dots\Big] \quad \text{(S3)}$$

- For the lagged price it follows that:

$$P_{t-1}^{1-\theta} = (1-\zeta)\Big[\left(P_{t-1}^n\right)^{1-\theta} + \zeta\left(P_{t-2}^n\right)^{1-\theta} + \zeta^2\left(P_{t-3}^n\right)^{1-\theta} + \dots\Big]$$

Aggregate price level (3)

- ... and thus:

$$\zeta \left(P_{t-1}\right)^{1-\theta} = (1-\zeta)\left[\zeta \left(P_{t-1}^{n}\right)^{1-\theta} + \zeta^{2}\left(P_{t-2}^{n}\right)^{1-\theta} + \zeta\left({}^{3}P_{t-3}^{n}\right)^{1-\theta} + \ldots\right] \tag{S4}$$

- Combining (S3) and (S4) gives:

$$P_t^{1-\theta} = (1-\zeta)\left(P_t^{n}\right)^{1-\theta} + \zeta P_{t-1}^{1-\theta}$$

- The current price is a CES aggregate of the new price and the indexed lagged price

Representative household (1)

- Household utility function in the planning period t:

$$E_t \Lambda_t \equiv E_t \sum_{\tau=t}^{\infty} \left(\frac{1}{1+\rho} \right)^{\tau - t} \cdot U\left(C_\tau, 1 - L_\tau, M_{\tau+1}/P_\tau\right) \quad \text{(S5)}$$

  - C_τ is goods consumption in period τ
  - L_τ is labour supply in period τ
  - $M_{\tau+1}$ is nominal money balances at the end of period τ
  - E_t is the conditional expectations operator

- Specific felicity function:

$$U\left(\cdot\right) \equiv \varepsilon_c \ln C_\tau + \varepsilon_l \ln\left(1 - L_\tau\right) + \varepsilon_m \ln \frac{M_{\tau+1}}{P_\tau} \quad \text{(S6)}$$

  - $0 < \varepsilon_c, \varepsilon_l, \varepsilon_m < 1$ and $\varepsilon_c + \varepsilon_l + \varepsilon_m = 1$
  - weakly separable in consumption, leisure, and real money balances

Representative household (2)

- We let the households make the capital accumulation decision
- Household periodic budget identity (in nominal terms):

$$P_\tau \left[C_\tau + I_\tau\right] + M_{\tau+1} + B_{\tau+1} + \sum_{s=0}^{\infty} Q_\tau^s S_{\tau+1}^s =$$

$$W_\tau L_\tau + R_\tau^K K_\tau + \left(1 + R_{\tau-1}\right) B_\tau$$

$$+ \sum_{s=0}^{\infty} X_\tau^s S_\tau^s + M_\tau - P_\tau T_\tau \qquad \text{(S7)}$$

  - B_τ single-period risk-free bond; $R_{\tau-1}$ is its nominal interest rate
  - R_τ^K is the nominal rental rate on capital K_τ
  - Q_τ^s is the nominal price of shares of type s; X_τ^s is their payoff
  - S_τ^s is the number of shares of type s
  - $P_t T_t$ is nominal transfers

Representative household (3)

- Law of motion for the capital stock:

$$K_{\tau+1} = I_\tau + (1-\delta) K_\tau \tag{S8}$$

  - $0 < \delta < 1$ is the depreciation rate of capital
  - no adjustment costs of capital
- The household:
  - chooses consumption C_τ, labour supply L_τ, investment I_τ, single-period bonds $B_{\tau+1}$, shares $S^s_{\tau+1}$, money balances $M_{\tau+1}$, and capital $K_{\tau+1}$ for $\tau \in \{t, \infty\}$
  - ...in order to maximize expected utility (S5) subject to the budget identity (S7) and the capital accumulation identity (S8)
  - ...taking as given its initial stocks, B_t, S^s_t, M_t, and K_t

Representative household (4)

- The main first-order conditions for the planning period $\tau = t$
- For the consumption-leisure choice:

$$\frac{U_{1-L}(C_t, 1-L_t, M_{t+1}/P_t)}{U_C(C_t, 1-L_t, M_{t+1}/P_t)} = \frac{W_t}{P_t} \tag{S9}$$

- For consumption over time:

$$1 = E_t \left[\frac{r^K_{t+1} + 1 - \delta}{1+\rho} \frac{U_C(C_{t+1}, 1-L_{t+1}, M_{t+2}/P_{t+1})}{U_C(C_t, 1-L_t, M_{t+1}/P_t)} \right] \tag{S10}$$

where $r^K_{t+1} \equiv R^K_{t+1}/P_{t+1}$ is the next period's real rental rate on capital

Representative household (5)

- For the risk-free bond:

$$1 = E_t\left[(1+R_t)\mathcal{N}_{t,t+1}\right] \tag{S11}$$

- For money balances:

$$\frac{U_{M/P}(C_t, 1-L_t, M_{t+1}/P_t)}{U_C(C_t, 1-L_t, M_{t+1}/P_t)} = \frac{R_t}{1+R_t} \tag{S12}$$

- For shares of type s:

$$Q_t^s = E_t\left[\mathcal{N}_{t,t+1}X_{t+1}^s\right] \tag{S13}$$

Loose ends (1)

- Equilibrium in the final goods sector:

$$Y_t = C_t + I_t + G_t$$

- Equilibrium in the labour market:

$$L_t = \int_0^1 L_t(i)di$$

- Equilibrium in the (rental) market for capital:

$$K_t = \int_0^1 K_t(i)di$$

- Alternative output measure:

$$Y_t^a \equiv \int_0^1 Y_t(i)di = K_t^\alpha \left[Z_t \cdot (L_t - \bar{L})\right]^{1-\alpha}$$

Loose ends (2)

- Government budget identity (nominal terms):

$$B_{t+1} + M_{t+1} = (1 + R_{t-1})\, B_t + M_t + P_t[T_t + G_t]$$

- Like households the government faces a solvency condition
- Model is closed by choosing a specification for monetary policy
  - Set the money supply (and let the nominal interest rate equilibrate the money market)
  - Set the nominal interest rate (and let the nominal money supply equilibrate the money market)

The basic Monetary Business Cycle model

- The full model is listed in **Table 19.1**
- Endogenous: Y_t, Y_t^a, C_t, I_t, K_t, L_t, w_t, r_t^K, mc_t, P_t^n, P_t, P_t^a, and one of R_t and M_{t+1}
- Exogenous: G_t, Z_t, and one of R_t and M_{t+1}
- In the background the government solvency condition is satisfied by means of debt and/or lump-sum taxes (Ricardian equivalence)

Table 19.1: The basic MBC model

$$K_{t+1} = I_t + (1-\delta)K_t \tag{T1.1}$$

$$Y_t = C_t + I_t + G_t \tag{T1.2}$$

$$\frac{\varepsilon_c}{C_t} = E_t\left[\frac{1+R_t}{1+\rho}\,\frac{\varepsilon_c}{C_{t+1}}\,\frac{P_t}{P_{t+1}}\right] \tag{T1.3}$$

$$\frac{\varepsilon_c}{C_t} = E_t\left[\frac{1+r^K_{t+1}-\delta}{1+\rho}\,\frac{\varepsilon_c}{C_{t+1}}\right] \tag{T1.4}$$

$$\frac{M_{t+1}}{P_t} = \frac{\varepsilon_m}{\varepsilon_c}\,C_t\,\frac{1+R_t}{R_t} \tag{T1.5}$$

$$L_t = 1 - \frac{\varepsilon_l}{\varepsilon_c}\,\frac{C_t}{w_t} \tag{T1.6}$$

$$w_t = (1-\alpha)\,mc_t\,\frac{Y^a_t}{L_t - \bar{L}} \tag{T1.7}$$

Table 19.1: The basic MBC model

$$r_t^K = \alpha\, mc_t \frac{Y_t^a}{K_t} \tag{T1.8}$$

$$P_t^n = \frac{\theta}{\theta-1} \frac{E_t\left[\sum_{\tau=0}^{\infty}\left(\frac{\zeta}{1+\rho}\right)^{\tau} C_{t+\tau}^{-1} P_{t+\tau}^{\theta} Y_{t+\tau}\, mc_{t+\tau}\right]}{E_t\left[\sum_{\tau=0}^{\infty}\left(\frac{\zeta}{1+\rho}\right)^{\tau} C_{t+\tau}^{-1} P_{t+\tau}^{\theta-1} Y_{t+\tau}\right]} \tag{T1.9}$$

$$P_t = \left[(1-\zeta)\left(P_t^n\right)^{1-\theta} + \zeta P_{t-1}^{1-\theta}\right]^{1/(1-\theta)} \tag{T1.10}$$

$$Y_t^a = K_t^{\alpha}\left[Z_t(L_t - \bar{L})\right]^{1-\alpha} \tag{T1.11}$$

$$P_t^a = \left[(1-\zeta)\left(P_t^n\right)^{-\theta} + \zeta\left(P_{t-1}^a\right)^{-\theta}\right]^{-1/\theta} \tag{T1.12}$$

$$Y_t = \left(\frac{P_t^a}{P_t}\right)^{\theta} Y_t^a \tag{T1.13}$$

Simplified model

- We first study a special case of the model
- Often studied: claimed by many to replace the IS-LM-AS model
  - "the new consensus"
  - "the new neoclassical synthesis" (Goodfriend & King, 1998)
- Main simplifying features:
  - No capital: $\alpha = 0$, $K_t = 0$, and $I_t = 0$ for all t
  - No government consumption: $G_t = 0$ so $Y_t = C_t$
  - See **Table 19.2**

Table 19.2: The canonical NK-DSGE model

$$\frac{1}{Y_t} = \frac{1+R_t}{1+\rho} E_t \left[\frac{1}{Y_{t+1}} \frac{P_t}{P_{t+1}} \right] \tag{T2.1}$$

$$\frac{M_{t+1}}{P_t} = \frac{\varepsilon_m}{\varepsilon_c} Y_t \frac{1+R_t}{R_t} \tag{T2.2}$$

$$L_t = 1 - \frac{\varepsilon_l}{\varepsilon_c} \frac{Y_t}{w_t} \tag{T2.3}$$

$$mc_t = \frac{w_t}{Z_t} \tag{T2.4}$$

$$P_t^n = \frac{\theta}{\theta - 1} \frac{E_t \left[\sum_{\tau=0}^{\infty} \left(\frac{\zeta}{1+\rho} \right)^{\tau} P_{t+\tau}^{\theta} mc_{t+\tau} \right]}{E_t \left[\sum_{\tau=0}^{\infty} \left(\frac{\zeta}{1+\rho} \right)^{\tau} P_{t+\tau}^{\theta-1} \right]} \tag{T2.5}$$

$$P_t = \left[(1-\zeta) \left(P_t^n \right)^{1-\theta} + \zeta P_{t-1}^{1-\theta} \right]^{1/(1-\theta)} \tag{T2.6}$$

Table 19.2: The canonical NK-DSGE model

$$Y_t^a = Z_t(L_t - \bar{L}) \tag{T2.7}$$

$$P_t^a = \left[(1-\zeta) \left(P_t^n \right)^{-\theta} + \zeta \left(P_{t-1}^a \right)^{-\theta} \right]^{-1/\theta} \tag{T2.8}$$

$$Y_t = \left(\frac{P_t^a}{P_t} \right)^{\theta} Y_t^a \tag{T2.9}$$

- Things to note:
  - Consumption equals output (no government consumption, no investment)
  - Consumption–output drops out of pricing rule (log felicity)
  - Marginal cost is just the scaled wage rate
  - Despite all these simplifications the model is still very complex!
  - Method of first resort: linearize the object to pieces and see if that helps Skip the linearization

Preview of Linearized Simplified Model (LSM)

- Monetary policy: the CB sets the money supply and lets the nominal interest rate equilibrate the money market
- The linearized model can now be written as:

$$\tilde{Y}_t = E_t\tilde{Y}_{t+1} - \left[\tilde{R}_t - E_t\tilde{\pi}_{t+1}\right] \qquad \text{(DIS)}$$

$$\tilde{\pi}_t = \gamma\left[\tilde{Y}_t - \tilde{Z}_t\right] + \frac{1}{1+\rho}E_t\tilde{\pi}_{t+1} \qquad \text{(NKPC)}$$

$$\tilde{M}_{t+1} - \tilde{P}_t = \tilde{Y}_t - \frac{\tilde{R}_t}{\rho} \qquad \text{(MME)}$$

  - Endogenous: $\tilde{Y}_t$, $\tilde{R}_t$, and $\tilde{P}_t$
  - Exogenous: $\tilde{M}_t$ (policy instrument) and $\tilde{Z}_t$ (mother nature)
  - Predetermined: $\tilde{P}_{t-1}$

- Derivation is non-trivial (a bit of work to do) Skip the linearization

Preparation for linearization (1)

- Notation:
  - Steady-state values are denoted with stars
  - For output, employment, prices, wages, etcetera we use proportional rates of change:

$$\tilde{x}_t \equiv \frac{x_t - x^*}{x^*}$$

  - For the interest rate we use:

$$\tilde{R}_t \equiv \frac{R_t - \rho}{1 + \rho}$$

- Features of the deterministic steady state:
  - Evaluate under the assumption that the unconditional mean of Z_t equals $EZ_t = Z^* = 1$
  - No real growth so $Y_t = Y^*$ and $C_t = C^*$
  - No money growth so $P_{t+1} = P_t = P^*$ and inflation is zero, $\pi^* = 0$

Preparation for linearization (2)

- Features of the deterministic steady state:
  - From the (deterministic) Euler equation we find that $R^* = \rho$

$$\frac{\varepsilon_c}{C_t} = \frac{1+R_t}{1+\rho}\frac{\varepsilon_c}{C_{t+1}}\frac{P_t}{P_{t+1}} \qquad \Leftrightarrow \qquad 1 = \frac{1+R^*}{1+\rho}$$

  - From (T2.5)–(T2.6) and (T2.8) we find that $P^* = (P^a)^* = (P^n)^*$ and from (T2.4) and (T2.5) we thus get that:

$$mc^* = w^* \equiv \frac{W^*}{P^*} = \frac{\theta - 1}{\theta}$$

  - From (T2.7) and (T2.9) we obtain:

$$Y^* = (Y^a)^* = L^* - \bar{L}$$

  - From (T2.2)–(T2.3) we get:

$$\frac{M^*}{P^*} = \frac{\varepsilon_m}{\varepsilon_c} Y^* \frac{1+R^*}{R^*} \quad \text{and} \quad \frac{\varepsilon_l}{1-L^*} = \frac{\varepsilon_c}{Y^*} w^*$$

Linearization of (T2.1) (1)

- Write (T2.1) as:

$$\frac{1}{Y_t} = \frac{1+R_t}{1+\rho}\frac{1}{Y_{t+1}}\frac{1}{1+\pi_{t+1}}$$

where future inflation, π_{t+1}, is given by:

$$\frac{P_{t+1}}{P_t} = \frac{P_t + \Delta P_{t+1}}{P_t} = 1 + \pi_{t+1}$$

- **Step 1** Work on the LHS:

$$\frac{1}{Y_t} \approx \frac{1}{Y^*} - \left(\frac{1}{Y^*}\right)^2 [Y_t - Y^*] = \frac{1-\tilde{Y}_t}{Y^*}$$

Linearization of (T2.1) (2)

- **Step 2** Work on the RHS:
  - First-order approximation:

$$\begin{aligned}\frac{1+R_t}{1+\rho}\frac{1}{Y_{t+1}}\frac{1}{1+\pi_{t+1}} \approx\; & \frac{1+R^*}{1+\rho}\frac{1}{Y^*}\frac{1}{1+\pi^*} \\ & + \frac{1}{1+\rho}\frac{1}{Y^*}\frac{1}{1+\pi^*}\left[R_t - R^*\right] \\ & - \frac{1+R^*}{1+\rho}\left(\frac{1}{Y^*}\right)^2\frac{1}{1+\pi^*}\left[Y_{t+1} - Y^*\right] \\ & - \frac{1+R^*}{1+\rho}\frac{1}{Y^*}\left(\frac{1}{1+\pi^*}\right)^2\left[\pi_{t+1} - \pi^*\right]\end{aligned}$$

Linearization of (T2.1) (3)

- **Step 3** Work on the RHS:
  - Recall that $R^* = \rho$ and $\pi^* = 0$:

$$\frac{1+R_t}{1+\rho}\frac{1}{Y_{t+1}}\frac{1}{1+\pi_{t+1}} \approx \frac{1}{Y^*} + \frac{1}{Y^*}\left[\frac{R_t-\rho}{1+\rho}\right] - \frac{1}{Y^*}\left[\frac{Y_{t+1}-Y^*}{Y^*}\right] - \frac{1}{Y^*}\pi_{t+1}$$

- **Step 4** Combine results and put E_t back in:

$$\frac{1-\tilde{Y}_t}{Y^*} = \frac{1}{Y^*} + \frac{1}{Y^*}\tilde{R}_t - \frac{1}{Y^*}\tilde{Y}_{t+1} - \frac{1}{Y^*}\tilde{\pi}_{t+1}$$

$$\boxed{\tilde{Y}_t = E_t\tilde{Y}_{t+1} - \left[\tilde{R}_t - E_t\tilde{\pi}_{t+1}\right]} \qquad \text{(DIS)}$$

Linearization of (T2.2) (1)

- Write:

$$\frac{M_{t+1}}{P_t} = \frac{\varepsilon_m}{\varepsilon_c} C_t \frac{1+R_t}{R_t}$$

- **Step 1** Work on the LHS:

$$\begin{aligned}\frac{M_{t+1}}{P_t} &\approx \frac{M^*}{P^*} + \frac{M^*}{P^*}\left[\frac{M_{t+1}-M^*}{M^*}\right] - \frac{M^*}{P^*}\left[\frac{P_t-P^*}{P^*}\right] \\ &= \frac{M^*}{P^*} + \frac{M^*}{P^*}\tilde{M}_{t+1} - \frac{M^*}{P^*}\tilde{P}_t\end{aligned}$$

- **Step 2** Work on the LHS:

$$\begin{aligned}C_t\frac{1+R_t}{R_t} \approx C^*\frac{1+R^*}{R^*} + C^*\frac{1+R^*}{R^*}\left[\frac{C_t-C^*}{C^*}\right] \\ - \frac{C^*}{(R^*)^2}\left[R_t - R^*\right]\end{aligned}$$

Linearization of (T2.2) (2)

- **Step 3** Combine results:

$$\frac{M^*}{P^*} + \frac{M^*}{P^*}\tilde{M}_{t+1} - \frac{M^*}{P^*}\tilde{P}_t = \frac{\varepsilon_m}{\varepsilon_c}\left[C^*\frac{1+R^*}{R^*} + C^*\frac{1+R^*}{R^*}\tilde{C}_t - \frac{C^*}{R^*}\frac{1+R^*}{R^*}\tilde{R}_t\right]$$

- **Step 4** Recall that $\frac{M^*}{P^*} = \frac{\varepsilon_m}{\varepsilon_c}C^*\frac{1+R^*}{R^*}$ and simplify:

$$\frac{M^*}{P^*} + \frac{M^*}{P^*}\tilde{M}_{t+1} - \frac{M^*}{P^*}\tilde{P}_t = \frac{M^*}{P^*} + \frac{M^*}{P^*}\tilde{C}_t - \frac{1}{R^*}\frac{M^*}{P^*}\tilde{R}_t$$

$$\boxed{\tilde{M}_{t+1} - \tilde{P}_t = \tilde{C}_t - \frac{\tilde{R}_t}{\rho}}$$

Linearization of (T2.3) (1)

- Write (T2.3) as:

$$\frac{\varepsilon_l}{1-L_t} = \frac{\varepsilon_c}{C_t} w_t$$

- **Step 1** Work on LHS:

$$\frac{\varepsilon_l}{1-L_t} \approx \frac{\varepsilon_l}{1-L^*} - \frac{\varepsilon_l L^*}{(1-L^*)^2}\left[\frac{L_t - L^*}{L^*}\right]$$

- **Step 2** Work on RHS:

$$\frac{\varepsilon_c}{C_t} w_t \approx \frac{\varepsilon_c}{C^*} w^* + \frac{\varepsilon_c}{C^*} w^* \left[\frac{w_t - w^*}{w^*}\right] - \frac{\varepsilon_c}{C^*} w^* \left[\frac{C_t - C^*}{C^*}\right]$$

Linearization of (T2.3) (2)

- **Step 3** Recall that $\frac{\varepsilon_l}{1-L^*} = \frac{\varepsilon_c}{C^*} w^*$, combine results and simplify:

$$\frac{\varepsilon_c}{C^*} w^* - \frac{\varepsilon_c}{C^*} w^* \frac{L^*}{1-L^*} \left[\frac{L_t - L^*}{L^*} \right] = \frac{\varepsilon_c}{C^*} w^* + \frac{\varepsilon_c}{C^*} w^* \left[\frac{w_t - w^*}{w^*} \right] - \frac{\varepsilon_c}{C^*} w^* \left[\frac{C_t - C^*}{C^*} \right]$$

$$\boxed{\frac{L^*}{1-L^*} \tilde{L}_t = \tilde{w}_t - \tilde{C}_t}$$

Linearization of (T2.4)

- Write:

$$mc_t = \frac{w_t}{Z_t}$$

- **Step 1** Work on RHS:

$$mc_t \approx \frac{w^*}{Z^*} + \frac{w^*}{Z^*}\left[\frac{w_t - w^*}{w^*}\right] - \frac{w^*}{Z^*}\left[\frac{Z_t - Z^*}{Z^*}\right]$$

- **Step 2** Recall that $mc^* = \frac{w^*}{Z^*}$ and combine results:

$$mc_t - mc^* = mc^*\left[\frac{w_t - w^*}{w^*}\right] - mc^*\left[\frac{Z_t - Z^*}{Z^*}\right]$$

$$\boxed{\widetilde{mc}_t = \tilde{w}_t - \tilde{Z}_t}$$

Linearization of (T2.6) and (T2.8) (1)

- Rewrite (T2.6) as:

$$P_t^{1-\theta} = (1-\zeta)\left(P_t^n\right)^{1-\theta} + \zeta P_{t-1}^{1-\theta}$$
$$1 = (1-\zeta) X_t^{1-\theta} + \zeta (1+\pi_t)^{\theta-1} \qquad \text{(S14)}$$

where X_t is short-hand notation for the relative new price at time t and π_t is the inflation rate:

$$X_t \equiv \frac{P_t^n}{P_t}, \qquad \frac{1}{1+\pi_t} = \frac{P_{t-1}}{P_t}$$

- **Step 1** Linearize RHS of (S14) around X^* and π^*:

$$\begin{aligned}(1-\zeta) X_t^{1-\theta} + \zeta (1+\pi_t)^{\theta-1} &\approx (1-\zeta)(X^*)^{1-\theta} + \zeta (1+\pi^*)^{\theta-1} \\ &+ (1-\zeta)(1-\theta)(X^*)^{1-\theta}\left[\frac{X_t - X^*}{X^*}\right] \\ &- \zeta (1-\theta)(1+\pi^*)^{\theta-2}\left[\pi_t - \pi^*\right]\end{aligned}$$

Linearization of (T2.6) and (T2.8) (2)

- **Step 2** Substitute into (S14) and recall that $X^* = 1$ and $\pi^* = 0$:

$$1 = 1 + (1-\zeta)(1-\theta)\tilde{X}_t - \zeta(1-\theta)\tilde{\pi}_t$$

$$\boxed{\tilde{X}_t \equiv \tilde{P}_t^n - \tilde{P}_t = \frac{\zeta}{1-\zeta}\tilde{\pi}_t}$$

- **Step 3** For P_t^a we define $X_t^a \equiv P_t^n / P_t^a$ and follow the same steps to obtain:

$$\boxed{\tilde{X}_t^a \equiv \tilde{P}_t^n - \tilde{P}_t^a = \frac{\zeta}{1-\zeta}\tilde{\pi}_t}$$

Linearization of (T2.7) (1)

- Write:

$$Y_t^a = Z_t \left(L_t - \bar{L} \right)$$

- **Step 1** Work on RHS:

$$Z_t \left(L_t - \bar{L} \right) \approx Z^* \left(L^* - \bar{L} \right) + Z^* \left[L_t - L^* \right] + \left(L^* - \bar{L} \right) \left[Z_t - Z^* \right]$$

- **Step 2** Combine results:

$$Y_t^a = \underbrace{Z^* \left(L^* - \bar{L} \right)}_{(Y^a)^*} + Z^* \left[L_t - L^* \right] + \left(L^* - \bar{L} \right) \left[Z_t - Z^* \right]$$

$$Y_t^a - (Y^a)^* = Z^* L^* \left[\frac{L_t - L^*}{L^*} \right] + \left(L^* - \bar{L} \right) \left[Z_t - Z^* \right]$$

Linearization of (T2.7) (2)

- **Step 3** Divide by $(Y^a)^*$ and use notation:

$$\frac{Y_t^a - (Y^a)^*}{(Y^a)^*} = \frac{Z^* L^*}{Z^* \left(L^* - \bar{L}\right)} \left[\frac{L_t - L^*}{L^*}\right] + \frac{L^* - L}{L^* - \bar{L}} \left[\frac{Z_t - Z^*}{Z^*}\right]$$

$$\boxed{\tilde{Y}_t^a = \frac{L^*}{L^* - \bar{L}} \tilde{L}_t + \tilde{Z}_t}$$

Linearization of (T2.9) (1)

- Write:

$$\frac{Y_t}{Y_t^a} = \left(\frac{P_t^a}{P_t}\right)^{\theta}$$

- **Step 1** Work on the LHS:

$$\frac{Y_t}{Y_t^a} \approx \frac{Y^*}{(Y^a)^*} + \frac{1}{(Y^a)^*}\left[Y_t - Y^*\right] - \frac{Y^*}{(Y^a)^*}\left[\frac{Y_t^a - (Y^a)^*}{(Y^a)^*}\right]$$
$$= 1 + \tilde{Y}_t - \tilde{Y}_t^a$$

where we recall that $Y^* = (Y^a)^*$

Linearization of (T2.9) (2)

- **Step 2** Work on the RHS:

$$\left(\frac{P_t^a}{P_t}\right)^{\theta} \approx \left(\frac{(P^a)^*}{P^*}\right)^{\theta} + \theta\left(\frac{(P^a)^*}{P^*}\right)^{\theta-1}\left[\frac{P_t^a-(P^a)^*}{P^*}\right]$$
$$-\theta\left(\frac{(P^a)^*}{P^*}\right)^{\theta-1}\frac{(P^a)^*}{P^*}\left[\frac{P_t-P^*}{P^*}\right]$$
$$=1+\theta\left(\tilde{P}_t^a-\tilde{P}\right)$$

where we recall that $P^* = (P^a)^*$

- **Step 3** Combining these results we get:

$$\boxed{\tilde{Y}_t-\tilde{Y}_t^a=\theta(\tilde{P}_t^a-\tilde{P}_t)}$$

Linearization of (T2.5) (1)

- Save the hardest one for last!
- Define:

$$X_t \equiv \frac{P_t^n}{P_t}, \qquad Q_{t+\tau} \equiv \frac{P_{t+\tau}}{P_t}$$

and note that $X^* = Q^* = 1$ and:

$$\tilde{Q}_{t+\tau} = \tilde{P}_{t+\tau} - \tilde{P}_t$$

- Drop the expectations operator E_t and write (T2.5) as:

$$\Xi_D \cdot X_t = \frac{\theta}{\theta - 1} \cdot \Xi_N \tag{S15}$$

with:

$$\Xi_D \equiv \sum_{\tau=0}^{\infty} \left(\frac{\zeta}{1+\rho} \right)^{\tau} Q_{t+\tau}^{\theta-1}; \quad \Xi_N \equiv \sum_{\tau=0}^{\infty} \left(\frac{\zeta}{1+\rho} \right)^{\tau} Q_{t+\tau}^{\theta} mc_{t+\tau}$$

Linearization of (T2.5) (2)

- Note that:

$$\Xi_D^* \equiv \sum_{\tau=0}^{\infty} \left(\frac{\zeta}{1+\rho}\right)^{\tau} (Q^*)^{\theta-1} = \frac{1+\rho}{1-\zeta+\rho} (Q^*)^{\theta-1}$$

$$\Xi_N^* \equiv \sum_{\tau=0}^{\infty} \left(\frac{\zeta}{1+\rho}\right)^{\tau} (Q^*)^{\theta} mc^* = \frac{1+\rho}{1-\zeta+\rho} (Q^*)^{\theta} mc^*$$

- **Step 1** Work on LHS of (S15):

$$\Xi_D X_t \approx \Xi_D^* X^* + \Xi_D^* X^* \left[\frac{X_t - X^*}{X^*}\right] + (\theta-1) X^* (Q^*)^{\theta-1} \sum_{\tau=0}^{\infty} \left(\frac{\zeta}{1+\rho}\right)^{\tau} \left[\frac{Q_{t+\tau} - Q^*}{Q^*}\right]$$

Linearization of (T2.5) (3)

- **Step 2** Work on RHS of (S15):

$$\Xi_N \approx \Xi_N^* + \theta mc^* \left(Q^*\right)^\theta \sum_{\tau=0}^{\infty} \left(\frac{\zeta}{1+\rho}\right)^\tau \left[\frac{Q_{t+\tau} - Q^*}{Q^*}\right]$$
$$+ mc^* \left(Q^*\right)^\theta \sum_{\tau=0}^{\infty} \left(\frac{\zeta}{1+\rho}\right)^\tau \left[\frac{mc_{t+\tau} - mc^*}{mc^*}\right]$$

- **Step 3** Use these results in (S15) and use notation:

$$\Xi_D^* X^* + \Xi_D^* X^* \tilde{X}_t + (\theta - 1) X^* \left(Q^*\right)^{\theta-1} \sum_{\tau=0}^{\infty} \left(\frac{\zeta}{1+\rho}\right)^\tau \tilde{Q}_{t+\tau}$$
$$= \frac{\theta}{\theta - 1} \left[\Xi_N^* + \theta mc^* \left(Q^*\right)^\theta \sum_{\tau=0}^{\infty} \left(\frac{\zeta}{1+\rho}\right)^\tau \tilde{Q}_{t+\tau}\right.$$
$$\left. + mc^* \left(Q^*\right)^\theta \sum_{\tau=0}^{\infty} \left(\frac{\zeta}{1+\rho}\right)^\tau \widetilde{mc}_t\right]$$

Linearization of (T2.5) (4)

- **Step 4** We can get rid of common terms by noting that:

$$\Xi_D^* X^* = \frac{\theta}{\theta - 1}\Xi_N^*, \qquad \frac{\Xi_N^*}{\Xi_D^*} = Q^* mc^*$$

$$\frac{(Q^*)^{\theta-1}}{\Xi_D^*} = \frac{mc^* (Q^*)^{\theta}}{\Xi_N^*} = \frac{1-\zeta+\rho}{1+\rho}$$

and thus solve for $\tilde{X}_t$:

$$\tilde{X}_t = (1-\theta)\frac{(Q^*)^{\theta-1}}{\Xi_D^*}\sum_{\tau=0}^{\infty}\left(\frac{\zeta}{1+\rho}\right)^{\tau}\tilde{Q}_{t+\tau} + \frac{mc^* (Q^*)^{\theta}}{\Xi_N^*}\left[\theta\sum_{\tau=0}^{\infty}\left(\frac{\zeta}{1+\rho}\right)^{\tau}\tilde{Q}_{t+\tau} + \sum_{\tau=0}^{\infty}\left(\frac{\zeta}{1+\rho}\right)^{\tau}\widetilde{mc}_{t+\tau}\right]$$

Linearization of (T2.5) (5)

- **Step 5** Simplify, note that $\tilde{Q}_{t+\tau} = \tilde{P}_{t+\tau} - \tilde{P}_t$, and put E_t back in:

$$\tilde{X}_t = \frac{1-\zeta+\rho}{1+\rho} E_t \sum_{\tau=0}^{\infty} \left(\frac{\zeta}{1+\rho}\right)^{\tau} \left[\tilde{P}_{t+\tau} - \tilde{P}_t + \widetilde{mc}_{t+\tau}\right]$$

$$\tilde{X}_t + \tilde{P}_t = \frac{1-\zeta+\rho}{1+\rho} E_t \sum_{\tau=0}^{\infty} \left(\frac{\zeta}{1+\rho}\right)^{\tau} \left[\tilde{P}_{t+\tau} + \widetilde{mc}_{t+\tau}\right]$$

- **Step 6** Note the recursive form:

$$\begin{aligned}\tilde{X}_t + \tilde{P}_t = {} & \frac{1-\zeta+\rho}{1+\rho} \left[\tilde{P}_t + \widetilde{mc}_t\right] \\ & + \frac{1-\zeta+\rho}{1+\rho} E_t \sum_{\tau=1}^{\infty} \left(\frac{\zeta}{1+\rho}\right)^{\tau} \left[\tilde{P}_{t+\tau} + \widetilde{mc}_{t+\tau}\right]\end{aligned}$$

Linearization of (T2.5) (6)

- ... **Step 6** Note the recursive form:

$$\begin{aligned}\tilde{X}_t + \tilde{P}_t &= \frac{1-\zeta+\rho}{1+\rho}\left[\tilde{P}_t + \widetilde{mc}_t\right] \\ &\quad + \frac{\zeta}{1+\rho}\frac{1-\zeta+\rho}{1+\rho}E_t\sum_{\tau=0}^{\infty}\left(\frac{\zeta}{1+\rho}\right)^{\tau}\left[\tilde{P}_{t+1+\tau} + \widetilde{mc}_{t+1+\tau}\right] \\ &= \frac{1-\zeta+\rho}{1+\rho}\left[\tilde{P}_t + \widetilde{mc}_t\right] + \frac{\zeta}{1+\rho}E_t\left[\tilde{X}_{t+1} + \tilde{P}_{t+1}\right]\end{aligned}$$

$$\begin{aligned}\tilde{X}_t &= \frac{1-\zeta+\rho}{1+\rho}\widetilde{mc}_t + \frac{\zeta}{1+\rho}E_t\left[\tilde{X}_{t+1} + \tilde{P}_{t+1} - \tilde{P}_t\right] \\ &= \frac{1-\zeta+\rho}{1+\rho}\widetilde{mc}_t + \frac{\zeta}{1+\rho}E_t\left[\tilde{X}_{t+1} + \tilde{\pi}_{t+1}\right]\end{aligned}$$

Linearization of (T2.5) (7)

- **Step 7** Recall from a couple of slides back that:

$$\tilde{X}_t = \frac{\zeta}{1-\zeta}\tilde{\pi}_t$$

It follows that:

$$\frac{\zeta}{1-\zeta}\tilde{\pi}_t = \frac{1-\zeta+\rho}{1+\rho}\widetilde{mc}_t + \frac{\zeta}{1+\rho}E_t\left[\frac{\zeta}{1-\zeta}\tilde{\pi}_{t+1} + \tilde{\pi}_{t+1}\right]$$

$$\boxed{\tilde{\pi}_t = \frac{1-\zeta}{\zeta}\frac{1-\zeta+\rho}{1+\rho}\widetilde{mc}_t + \frac{1}{1+\rho}E_t\tilde{\pi}_{t+1}} \qquad \text{(NKPC)}$$

Output Gap (1)

- In a traditional Phillips Curve one typically finds an Output Gap variable
- Output Gap: difference between **actual** and **potential** output (Arthur Okun)
- In our micro-founded model we can relate $\widetilde{mc}_t$ to an OG-like measure
- Hypothetical flex-price equilibrium $(\zeta = 0)$:
  - All firms set the same price, i.e. $P_t^n = P_t = P_t^a = \frac{\theta}{\theta-1} MC_t = \frac{\theta}{\theta-1} \frac{W_t}{Z_t}$
  - Hence, $w_t^f = \frac{\theta-1}{\theta} Z_t$ which is lower than Z_t
  - Solve for flex-price output and employment:

$$Y_t^f = Z_t \left(L_t^f - \bar{L} \right)$$

$$\frac{\varepsilon_l}{1 - L_t^f} = \frac{\varepsilon_c}{Y_t^f} w_t^f$$

Output Gap (2)

- . . . Hypothetical flex-price equilibrium ($\zeta = 0$):
  - This gives:

$$Y_t^f = \frac{\varepsilon_c(1-\bar{L})}{\varepsilon_l \frac{\theta}{\theta-1} + \varepsilon_c} Z_t$$

$$L_t^f = \bar{L} + \frac{\varepsilon_c(1-\bar{L})}{\varepsilon_l \frac{\theta}{\theta-1} + \varepsilon_c}$$

  - Hence: output fluctuates with the technology shocks (recall that the flex-price MBC model is just an RBC model because money is neutral)
  - In linearized form we get for output:

$$\tilde{Y}_t^f = \tilde{Z}_t$$

Output Gap (3)

- Sticky-price equilibrium $(0 < \zeta < 1)$
  - In proportional rates of change we have:

$$\tilde{Y}_t = \tilde{w}_t - \frac{L^*}{1-L^*}\tilde{L}_t \tag{S16}$$

$$\tilde{Y}_t = \tilde{Z}_t + \frac{L^*}{L^*-\bar{L}}\tilde{L}_t \tag{S17}$$

  - Solve for $\tilde{w}_t$ in terms of $\tilde{Z}_t$ and $\tilde{Y}_t$:

$$\tilde{w}_t = \tilde{Y}_t + \frac{L^*-\bar{L}}{1-L^*}\left[\tilde{Y}_t - \tilde{Z}_t\right]$$

  - Substitute in $\widetilde{mc}_t$ expression (and recall that $\tilde{Y}_t^f = \tilde{Z}_t$):

$$\begin{aligned} \widetilde{mc}_t = \tilde{w}_t - \tilde{Z}_t &= \tilde{Y}_t - \tilde{Y}_t^f + \frac{L^*-\bar{L}}{1-L^*}\left[\tilde{Y}_t - \tilde{Y}_t^f\right] \\ &= \frac{1-\bar{L}}{1-L^*}\left[\tilde{Y}_t - \tilde{Y}_t^f\right] \end{aligned} \tag{S18}$$

Output Gap (4)

- ...Sticky-price equilibrium $(0 < \zeta < 1)$
  - Substitute into (NKPC):

$$\tilde{\pi}_t = \gamma \left[\tilde{Y}_t - \tilde{Y}_t^f\right] + \frac{1}{1+\rho} E_t \tilde{\pi}_{t+1}$$

  where γ is a composite parameter:

$$\gamma \equiv \frac{1-\zeta}{\zeta} \frac{1-\zeta+\rho}{1+\rho} \frac{1-\bar{L}}{1-L^*} > 0$$

  - If output exceeds its potential level, $\tilde{Y}_t > \tilde{Y}_t^f$, then real marginal cost rises. Ceteris paribus expected inflation, actual inflation rises

Summary of the LSM (1)

- LSM = Linearized Simplified Model (in case you wonder)
- Assume that monetary policy is of the familiar type: the CB sets the money supply and lets the nominal interest rate equilibrate the money market
- The linearized model can now be written as:

$$\tilde{Y}_t = E_t \tilde{Y}_{t+1} - \left[\tilde{R}_t - E_t \tilde{\pi}_{t+1} \right] \qquad \text{(DIS)}$$

$$\tilde{\pi}_t = \gamma \left[\tilde{Y}_t - \tilde{Z}_t \right] + \frac{1}{1+\rho} E_t \tilde{\pi}_{t+1} \qquad \text{(NKPC)}$$

$$\tilde{M}_{t+1} - \tilde{P}_t = \tilde{Y}_t - \frac{\tilde{R}_t}{\rho} \qquad \text{(MME)}$$

  - Endogenous: $\tilde{Y}_t$, $\tilde{R}_t$, and $\tilde{P}_t$
  - Exogenous: $\tilde{M}_{t+1}$ (policy instrument) and $\tilde{Z}_t$ (mother nature)
  - Predetermined: $\tilde{P}_{t-1}$

Summary of the LSM (2)

- Eq. (DIS) is hailed as the "forward-looking" IS curve representing the demand side of the economy
  - expected real interest rate exerts negative influence on current output
  - ...but not because of its effect on investment (but rather via the consumer's Euler equation)
  - future expected income conditions current income (very un-Keynesian)
- Eq. (NKPC) is hailed as the "forward-looking" New Keynesian Phillips Curve representing the supply side of the economy
- Eq. (MME) represents a micro-based version of the LM curve
- Keynes, Phillips, Krugman and other "card-carrying" Keynesians would not recognize this model as Keynesian!

Solving the LSM (1)

- Recall that $\tilde{\pi}_t = \tilde{P}_t - \tilde{P}_{t-1}$ and $\tilde{\pi}_{t+1} = \tilde{P}_{t+1} - \tilde{P}_t$
- We must somehow impose that $\tilde{P}_{t-1}$ is pre-determined.
- Usual trick is to define an auxiliary variable:
  - Lagged price:
  $$\widetilde{LP}_t \equiv \tilde{P}_{t-1}$$
  - Current price:
  $$\widetilde{LP}_{t+1} = \tilde{P}_t$$
  - Relationship with inflation:
  $$\tilde{\pi}_t = \widetilde{LP}_{t+1} - \widetilde{LP}_t$$

Solving the LSM (2)

- After substituting (MME) into (IS) we can write the system as:

$$\Gamma \cdot \begin{bmatrix} E_t \widetilde{LP}_{t+1} \\ E_t \tilde{Y}_{t+1} \\ E_t \tilde{\pi}_{t+1} \end{bmatrix} = \Delta^* \cdot \begin{bmatrix} \widetilde{LP}_t \\ \tilde{Y}_t \\ \tilde{\pi}_t \end{bmatrix} + \begin{bmatrix} 0 \\ -\rho \tilde{M}_{t+1} \\ \gamma (1+\rho) \tilde{Z}_t \end{bmatrix}$$

where Γ and Δ^* are defined as:

$$\Gamma \equiv \begin{bmatrix} 1 & 0 & 0 \\ -\rho & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Delta^* \equiv \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1+\rho & 0 \\ 0 & -\gamma (1+\rho) & 1+\rho \end{bmatrix}$$

Solving the LSM (3)

- The inverse of Γ is:

$$\Gamma^{-1} \equiv \begin{bmatrix} 1 & 0 & 0 \\ \rho & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

- Hence $\Delta \equiv \Gamma^{-1}\Delta^{*}$ is:

$$\Delta \equiv \begin{bmatrix} 1 & 0 & 1 \\ \rho & (1+\gamma)(1+\rho) & -1 \\ 0 & -\gamma(1+\rho) & 1+\rho \end{bmatrix}$$

- The system features a unique solution for output, inflation, and the price level if and only if Δ features:
  - two roots outside the unit circle (because $\tilde{Y}_t$ and $\tilde{\pi}_t$ are jumping variables)
  - one root inside the unit circle (because $\widetilde{LP}_t$ is a pre-determined variable)

Solving the LSM (4)

- The characteristic equation of Δ is:

$$\Psi(s) \equiv |sI - \Delta| = [s - (1+\rho)] \cdot \Phi(s) \tag{S19}$$

where $\Phi(s)$ is given by:

$$\begin{aligned} \Phi(s) &\equiv (s-1)\left(s-(1+\gamma)(1+\rho)\right) - \gamma(1+\rho) \\ &= s^2 - [1+(1+\gamma)(1+\rho)]\, s + 1 + \rho \end{aligned} \tag{S20}$$

- So from (CE) one unstable root is immediately obvious:

$$\xi_3 = 1 + \rho > 1$$

- The function $\Phi(s)$ can be written as:

$$\begin{aligned} \Phi(s) &= (s-\xi_1)(s-\xi_2) \\ &= s^2 - (\xi_1+\xi_2)\, s + \xi_1\xi_2 \end{aligned} \tag{S21}$$

Solving the LSM (5)

- So it follows from (S20) and (S21) that:

$$\begin{aligned}\xi_1 + \xi_2 &= 1 + (1+\gamma)(1+\rho) > 2 \\ \xi_1 \xi_2 &= 1 + \rho > 1\end{aligned}$$

- Write:

$$\begin{aligned}\Phi(1) &= (1-\xi_1)(1-\xi_2) \\ &= 1 - (\xi_1 + \xi_2) + \xi_1 \xi_2 \\ &= 1 - [1 + (1+\gamma)(1+\rho)] + 1 + \rho \\ &= -\gamma(1+\rho) < 0\end{aligned}$$

- Hence: the roots are positive and lie on either side of unity, say:

$$0 < \xi_1 < 1, \qquad \xi_2 > 1$$

Example calibration of the LSM (1)

- The typical approach is to calibrate the model and simulate it numerically
- How can we calibrate the Simplified Model?
- Recall the simplified model: Jump to Model Listing
- Structural parameters: ε_c, ε_l, ε_m, ζ, θ, and ρ
- Choose these parameters such that a plausible steady state is obtained (to the extent that is possible given the absence of capital, investment, and government consumption)

Example calibration of the LSM (2)

- Assume that we want to do a quarterly calibration (remember: this is a short-run model)
- Steady-state nominal (and real) interest rate is four percent per annum, so $1 + R^a = 1.04$. On a quarterly basis we would thus get:

$$(1 + R^*)^4 = 1 + R^a \qquad \Leftrightarrow \qquad 1 + R^* = 1.00985$$

  Since $R^* = \rho$ in this model we have pinned down one of the structural parameters
- Remember that $\frac{\theta}{\theta - 1}$ is the steady-state markup of price over marginal cost. A reasonable range of values for that markup is between 1.2 and 1.3 (some monopoly power but not an outrageous amount).

Example calibration of the LSM (3)

- Sticking to the highest value for the markup we get a value for θ:

$$\frac{\theta}{\theta-1}=1.3 \qquad \Leftrightarrow \qquad \theta=\frac{1.3}{1.3-1}=\frac{13}{3}\approx 4.3333$$

- This also pins down $mc^{*}=w^{*}=(\theta-1)/\theta$
- Bernanke *et al.* assume that the probability that a firm does not change its price in a given quarter is 75 percent. The average period between adjustments is thus 4 quarters. Our structural parameter is thus $\zeta=0.75$
- People have 24 hours of time per day of which they typically work 8 hours (and consume the rest in the form of active or passive leisure). We thus want to get a steady-state such that $(1-L^{*})/L^{*}=2$ or $L^{*}=\frac{1}{3}$

Example calibration of the LSM (4)

- We know that in the steady state:

$$\frac{\varepsilon_l}{1-L^*} = \frac{\varepsilon_c w^*}{L^* - \bar{L}} \quad \left[= \frac{\varepsilon_c w^*}{Y^*}\right]$$

- Assume that ten percent of employment consists of overhead labour ("useless managers"), i.e. $\bar{L} = L^*/10$. This means that our target is met if:

$$\frac{9}{10}\frac{\varepsilon_l}{\varepsilon_c w^*} = \frac{1-L^*}{L^*} = 2 \qquad \Leftrightarrow \qquad \frac{\varepsilon_l}{\varepsilon_c} = \frac{20}{9} w^* \equiv \zeta_l$$

  Though w^* is fixed already we can always choose $\varepsilon_l/\varepsilon_c$ such that this expression holds. Note also that $Y^* = Z^*(L^* - \bar{L}) = 0.3$

Example calibration of the LSM (5)

- If we normalize $P^* = 1$ then we know that $W^* = w^* P^* = \frac{10}{13} \approx 0.769$
- If we normalize $M^* = 1$ then we find from money demand:

$$\frac{\varepsilon_m}{\varepsilon_c} = \frac{R^*}{1+R^*} \frac{M^*}{P^* Y^*} \equiv \zeta_m$$

- Since $1 = \varepsilon_c + \varepsilon_l + \varepsilon_m$ we find that the calibration is consistent provided:

$$\begin{aligned} \varepsilon_c &= \frac{1}{1+\zeta_l+\zeta_m} && = 0.36471 \\ \varepsilon_l &= \zeta_l \varepsilon_c && = 0.62343 \\ \varepsilon_m &= \zeta_m \varepsilon_c && = 0.01186 \end{aligned}$$

Example calibration of the LSM (6)

- For our linearized example the only structural parameters we need are ζ and ρ, and we need to know L^* and $\bar{L}$. On the basis of the calibration we find:

$$\zeta = 0.75 \qquad 1+\rho = 1.00985$$
$$L^* = \frac{1}{3} \qquad \bar{L} = \frac{1}{30}$$
$$\gamma = \frac{1-\zeta}{\zeta}\frac{1-\zeta+\rho}{1+\rho}\frac{1-\bar{L}}{1-L^*} = 0.12437$$

- The spectral decomposition of $\Delta = S\Lambda S^{-1}$ gives:

$$S \equiv \begin{bmatrix} -0.79414 & 0.55690 & 0.99995 \\ 0.56139 & -0.79555 & 0 \\ 0.23275 & 0.23865 & 0.00985 \end{bmatrix}$$

Example calibration of the LSM (7)

- . . . and

$$\Lambda \equiv \mathrm{diag}\{\xi_i\} = \begin{bmatrix} 0.70692 & 0 & 0 \\ 0 & 1.42853 & 0 \\ 0 & 0 & 1.00985 \end{bmatrix}$$

- This calibration can be used to simulate the dynamic effects of the stochastic shocks or of monetary policy
- How can we solve this model under rational expectations?

Rational expectations solution (1)

- Useful technique proposed by Blanchard & Kahn (1980)
- System in general:

$$\begin{bmatrix} \boldsymbol{B}_{t+1} \\ E_t \boldsymbol{F}_{t+1} \end{bmatrix} = \Delta \cdot \begin{bmatrix} \boldsymbol{B}_t \\ \boldsymbol{F}_t \end{bmatrix} + \Psi \cdot \boldsymbol{X}_t$$

  - $\boldsymbol{B}_t$ is an $(n_b \times 1)$ vector of predetermined variables ("backward looking")
  - $\boldsymbol{F}_t$ is an $(n_f \times 1)$ vector of non-predetermined variables ("forward looking")
  - $\boldsymbol{X}_t$ is a $(k \times 1)$ vector of exogenous variables
  - Δ is an $(n_b + n_f) \times (n_b + n_f)$ matrix of coefficients
  - Ψ is an $(n_b + n_f) \times k$ matrix of coefficients

Rational expectations solution (2)

- **Proposition 1** of B-K: If the number of eigenvalues of Δ outside the unit circle (say n_f') is equal to the number of non-predetermined variables (n_f) then there exists a unique solution.
- **Proposition 2** of B-K: If n_f' exceeds n_f there no solution.
- **Proposition 3** of B-K: If n_f' falls short of n_f there is an infinity of solutions ("indeterminacy").
- In our model the condition mentioned in Proposition 1 is met
  - We have $n_b = 1$ and $n_f = n_f' = 2$ so there is a unique solution
  - Ready-made solution expressions are available
  - See, for example, Harald Uhlig's Matlab toolkit
  - Or use Dynare (`www.dynare.org`)

Interest rate policy and (in)stability (1)

- Assume now that monetary policy takes the form of a **policy rule** for the nominal interest rate, i.e. $\tilde{R}_t$ is set by the CB (and the real money supply follows residually from (MME))
- The linearized model can now be written as:

$$\tilde{Y}_t = E_t\tilde{Y}_{t+1} - \left[\tilde{R}_t - E_t\tilde{\pi}_{t+1}\right] \quad \text{(DIS)}$$

$$\tilde{\pi}_t = \gamma\left[\tilde{Y}_t - \tilde{Z}_t\right] + \frac{1}{1+\rho}E_t\tilde{\pi}_{t+1} \quad \text{(NKPC)}$$

  - Endogenous: $\tilde{Y}_t$ and $\tilde{\pi}_t$
  - Exogenous: $\tilde{R}_t$ and $\tilde{Z}_t$
- Assume first that the policy rule is very simple:

$$\tilde{R}_t = \tilde{U}_t \quad (\text{PR}_1)$$

where $\tilde{U}_t$ is a stationary stochastic process

Interest rate policy and (in)stability (2)

- Combining (PR$_1$) with (IS)–(NKPC) we get:

$$\Gamma \cdot \begin{bmatrix} E_t\tilde{Y}_{t+1} \\ E_t\tilde{\pi}_{t+1} \end{bmatrix} = \Delta^* \cdot \begin{bmatrix} \tilde{Y}_t \\ \tilde{\pi}_t \end{bmatrix} + \begin{bmatrix} \tilde{U}_t / (1+\rho) \\ \gamma (1+\rho) \tilde{Z}_t \end{bmatrix}$$

with:

$$\Gamma \equiv \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \qquad \Delta^* \equiv \begin{bmatrix} 1 & 0 \\ -\gamma (1+\rho) & 1+\rho \end{bmatrix}$$

- We find that Δ is:

$$\Delta \equiv \Gamma^{-1}\Delta^* = \begin{bmatrix} 1+\gamma(1+\rho) & -(1+\rho) \\ -\gamma(1+\rho) & 1+\rho \end{bmatrix}$$

Interest rate policy and (in)stability (3)

- Our usual trick can be used to establish the signs and magnitude of the characteristic roots:
  - Compute the characteristic equation:

$$\Phi(s) = s^2 - [1 + (1+\gamma)(1+\rho)]\, s + 1 + \rho$$

  - Factor the characteristic equation:

$$\begin{aligned}\Phi(s) &= (s - \xi_1)(s - \xi_2) \\ &= s^2 - (\xi_1 + \xi_2)\, s + \xi_1 \xi_2\end{aligned}$$

    and observe that:

$$\begin{aligned}\xi_1 + \xi_2 &= 1 + (1+\gamma)(1+\rho) > 2 \\ \xi_1 \xi_2 &= 1 + \rho > 1\end{aligned}$$

Interest rate policy and (in)stability (4)

- ...up to our usual tricks:
  - Compute $\Phi(1)$:

$$\Phi(1) = 1 - [2 + \rho + \gamma(1 + \rho)] + 1 + \rho = -\gamma(1 + \rho) < 0$$

  - It follows that both roots are positive and lie on either side of unity, say:

$$0 < \xi_1 < 1, \qquad \xi_2 > 1$$

- Oops! **Proposition 3** of Blanchard-Kahn applies: $n_f = 2$ but $n'_f = 1$ so there is an infinity of solutions ("indeterminacy")

Interest rate policy and (in)stability (5)

- **Lesson #1** an exogenous policy rule (such as (PR_1)) induces indeterminacy in an economy that is stable if the CB would just control the money supply.
- Intuition:
  - Assume that expected future inflation rises ($E_t\tilde{\pi}_{t+1}\uparrow$)
  - Under (PR_1) $\tilde{R}_t$ does not react so the real interest rate falls
  - This prompts an increase in output via (IS) ($\tilde{Y}_t\uparrow$)
  - Which in turn boosts current inflation via (NKPC) ($\tilde{\pi}_t\uparrow$)
  - ... self-fulfilling increase in inflation emerges

Interest rate policy and (in)stability (6)

- Next let us try a feed-back rule that chokes off this inflation spiral:

$$\tilde{R}_t = \delta_\pi (1+\rho)\tilde{\pi}_t + \tilde{U}_t, \qquad \delta_\pi > 0 \qquad \text{(PR}_2\text{)}$$

- Γ is unchanged but element $(1,2)$ of Δ^* changes from 0 to δ_π and Δ becomes:

$$\Delta \equiv \begin{bmatrix} 1+\gamma(1+\rho) & \delta_\pi - (1+\rho) \\ -\gamma(1+\rho) & 1+\rho \end{bmatrix}$$

- We find that:

$$\begin{aligned} \xi_1 + \xi_2 &= 2+\rho+\gamma(1+\rho) > 2 \\ \xi_1\xi_2 &= (1+\rho)\left[1+\delta_\pi\gamma\right] > 1 \\ \Phi(1) &= 1 - \left[2+\rho+\gamma(1+\rho)\right] + (1+\rho)\left[1+\delta_\pi\gamma\right] \\ &= (\delta_\pi - 1)\gamma(1+\rho) \end{aligned}$$

Interest rate policy and (in)stability (7)

- . . . We find that:

$$\Phi(1) = (\delta_\pi - 1)\,\gamma\,(1+\rho)$$

- Provided $\delta_\pi > 1$ we find that both roots are larger than unity, i.e.:

$$\xi_1 > 1, \qquad \xi_2 > 1$$

  and **Proposition 1** of Blanchard-Kahn applies: $n_f = n_f' = 2$

- **Lesson #2** a feed-back policy rule (such as (PR$_2$)) eliminates indeterminacy provided $\delta_\pi > 1$. This is called the **Taylor Principle** after John Taylor who stressed that an interest rate rule should react more than one-for-one to inflation

Interest rate policy and (in)stability (8)

- Finally, let use try a more complicated feedback rule that responds both to inflation and to output:

$$\tilde{R}_t = \delta_\pi(1+\rho)\tilde{\pi}_t + \delta_y(1+\rho)\tilde{Y}_t + \tilde{U}_t, \qquad \delta_\pi, \delta_y > 0 \qquad (\text{PR}_3)$$

- This is an example of a **Taylor Rule** and empirically such a rule seems to have been followed by many CBs in the world
- Γ is unchanged but element $(1,1)$ of Δ^* changes from 1 to $1+\delta_y$ and Δ becomes:

$$\Delta \equiv \begin{bmatrix} 1 + \delta_y + \gamma(1+\rho) & \delta_\pi - (1+\rho) \\ -\gamma(1+\rho) & 1+\rho \end{bmatrix}$$

Interest rate policy and (in)stability (9)

- We find that:

$$\begin{aligned} \xi_1 + \xi_2 &= 2 + \delta_y + \rho + \gamma\left(1+\rho\right) > 2 \\ \xi_1\xi_2 &= \left(1+\rho\right)\left[1 + \delta_y + \delta_\pi\gamma\right] > 1 + \rho \end{aligned}$$

$$\begin{aligned} \Phi\left(1\right) &= 1 - \left[2 + \delta_y + \rho + \gamma\left(1+\rho\right)\right] + \left(1+\rho\right)\left[1 + \delta_y + \delta_\pi\gamma\right] \\ &= \rho\delta_y + \left(\delta_\pi - 1\right)\gamma\left(1+\rho\right) \end{aligned}$$

- The stability condition is that $\Phi\left(1\right) > 0$, because then both roots exceed unity and **Proposition 1** of Blanchard-Kahn applies: $n_f = n_f' = 2$
  - Since $\delta_y > 0$ we now have that $\delta_\pi > 1$ is no longer a NC for stability
  - $\ldots\delta_\pi < 1$ can still be consistent with stability provided δ_y is large enough

Figure 19.1: Transitory productivity shock

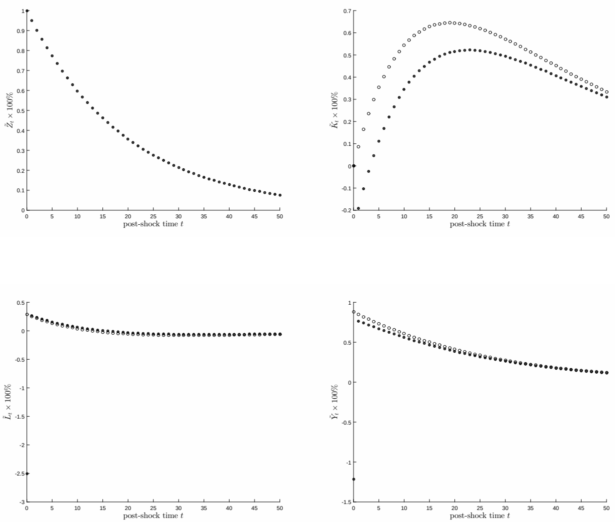


Figure 19.1: Transitory productivity shock

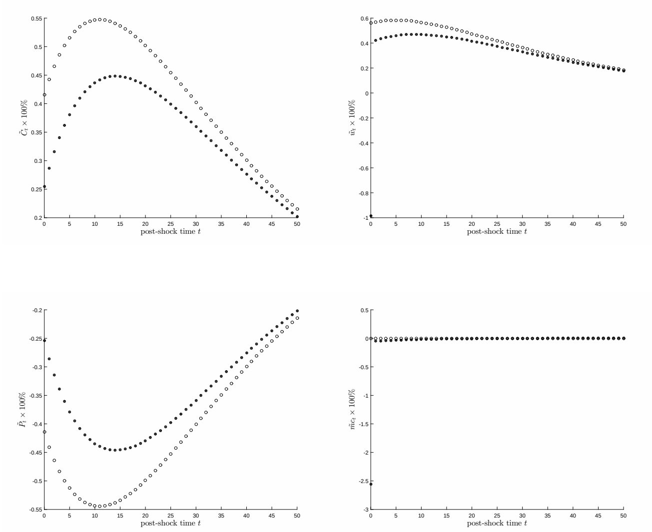


Figure 19.2: Transitory money supply shock

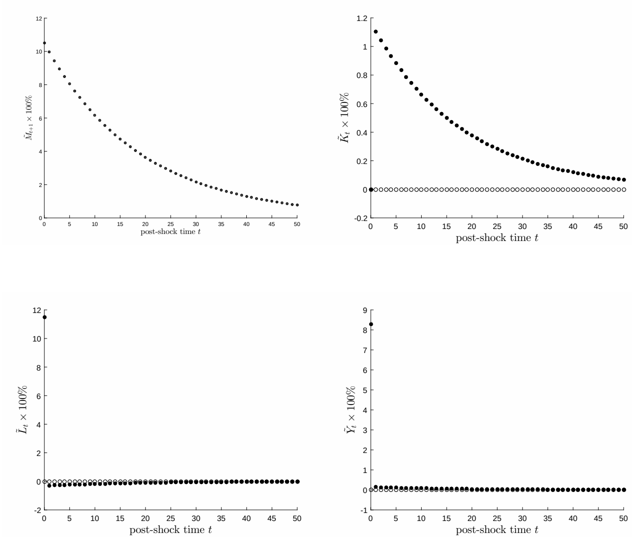


Figure 19.2: Transitory money supply shock

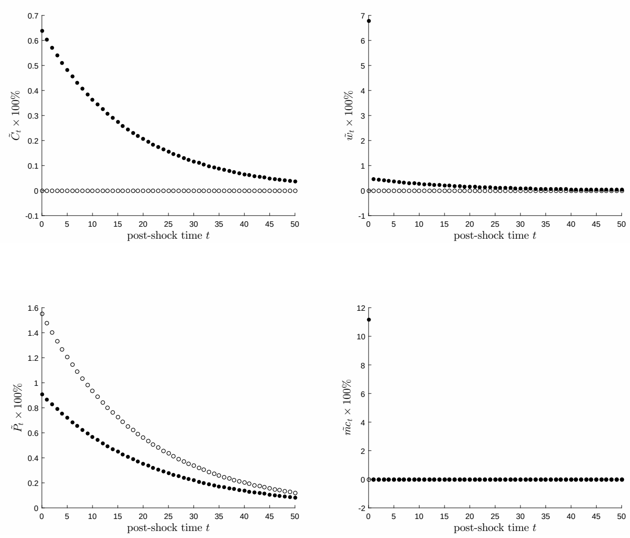


Figure 19.3: Transitory government consumption shock

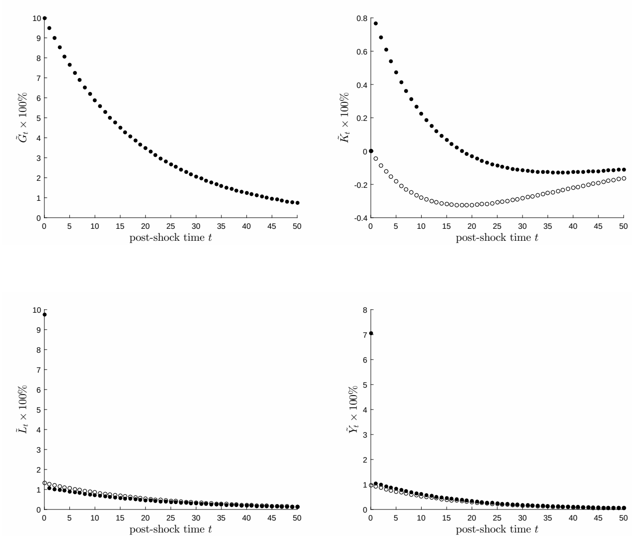
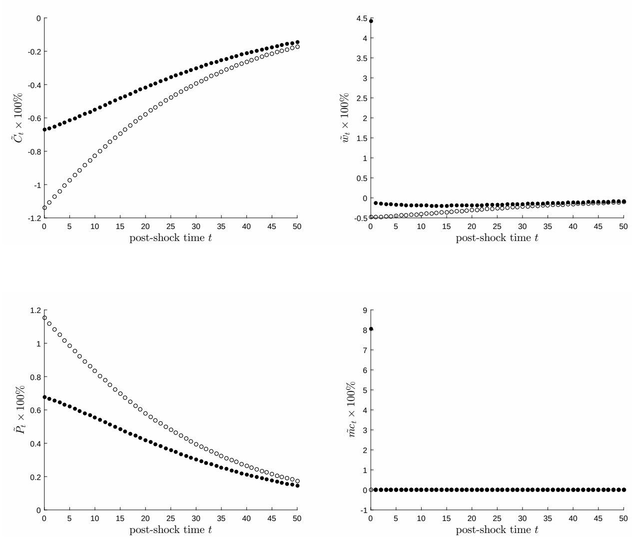


Figure 19.3: Transitory government consumption shock



The CEE Model (1)

- Second wave of MBC models do not just calibrate. Some parameters are estimated directly
- Famous examples of this approach: Smets and Wouters (2003 JEEA) and Christiano *et al.* (2005 JPE)
- Punchlines from Christiano *et al.* (2005) – CEE hereafter
- Question: "Can models with moderate degrees of nominal rigidities generate inertial inflation and persistent output movements in response to monetary policy shocks?"
- CEE's emphatic answer: "yes, they can!"

The CEE Model (2)

- Our general model is expanded–augmented in the following directions:
  - Also Calvo-style pricing of labour
  - Habit formation in the agent's preferences for consumption
  - Adjustment costs in investment
  - Variable utilization rate of the capital stock
  - Firms borrow working capital to pay workers up front
  - The lagged inflation rate is used for indexation purposes by red-light firms and workers
- Let us briefly look at some of these things

The CEE Model (3)

- Calvo pricing of labour:
  - Each household supplies a slightly unique variety of labour, $L_t(j)$
  - A representative, competitive firm buys all types of labour and transforms it into homogeneous labour L_t according to:

$$L_t = \left[\int_0^1 L_t(j)^{1-1/\theta_l} dj \right]^{1/(1-1/\theta_l)}, \qquad \theta_l > 1$$

  - Aggregate wage rate (unit cost of standardized labour):

$$W_t \equiv \left[\int_0^1 W_t(j)^{1-\theta_l} dj \right]^{1/(1-\theta_l)}$$

  - Derived (inverse) demand:

$$L_t(j) = L_t \cdot \left(\frac{W_t(j)}{W_t} \right)^{-\theta_l}$$

The CEE Model (4)

- Habit formation in preferences:
  - Felicity is separable in three arguments:

$$U\left(C_\tau, 1-L_\tau, m_\tau\right) \equiv \varepsilon_c \ln\left[C_\tau - \beta C_{\tau-1}\right] - \varepsilon_l L_\tau^2 + \varepsilon_m \frac{m_\tau^{1-1/\sigma} - 1}{1 - 1/\sigma}$$

  - For $\beta > 0$ there is habit formation in consumption preferences (which has been used in the asset pricing literature)
- Adjustment costs in investment
  - The law of motion for the capital stock is changed to:

$$K_{t+1} = \left[1 - \Phi\left(\frac{I_t}{I_{t-1}}\right)\right] \cdot I_t + (1-\delta) K_{t-1}$$

  - $\Phi(\cdot)$ captures the notion of installation costs
  - Properties: $\Phi(1) = \Phi'(1) = 0$ and $\Phi''(1) > 0$

The CEE Model (5)

- Variable utilization rate of the capital stock
  - Capital services, K_t^s, are related to the stock of capital by the specification:
  $$K_t^s \equiv u_t \cdot K_t$$
  where u_t is the utilitization rate
  - The household's nominal budget identity is affected on both sides:
  $$W_t L_t + R_t^K u_t K_t + \ldots = P_t \left[C_t + I_t + \Gamma(u_t) K_t \right] + \ldots$$
  - $\Gamma\left(u_t\right)$ represents cost of setting the utilization rate u_t
  - Properties: increasing and convex

The CEE Model (6)

- The lagged inflation rate is used for indexation purposes by red-light firms and workers
  - Red-light firm sets its price at:

$$P_t(i) = (1 + \pi_{t-1}) \cdot P_{t-1}(i)$$

    where $P_{t-1}(i)$ is the price it charged to the homogeneous goods producing firm last quarter
  - Red-light worker sets his/her wage rate at:

$$W_t(j) = (1 + \pi_{t-1}) \cdot W_{t-1}(j)$$

    where $W_{t-1}(j)$ is the wage it charged to the labour transforming firm last quarter

The CEE approach to calibration–estimation

- Estimate the impulse response of 8 key macroeconomic variables to a monetary policy shock using an identified VAR specification
- Calibrate the first group of structural parameters in the standard way (shown above)
- Postulate the monetary policy rule:

$$\mu_t = \mu + \theta_0\varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \dots$$

  where μ_t is the growth rate in the nominal money supply. Use the estimated VAR coefficients to obtain estimates for the θ_i parameters
- Estimate the third group of parameters such that the distance between the model-generated and empirical impulse-response functions is minimized

CEE's main findings

- Average period between price adjustments is about two quarters (for wage adjustments it is three quarters)
- Of the nominal frictions, the wage friction is the important one
- The effect of nominal frictions depends a lot on how the real side of the economy is modeled
- Variable capital utilization is important to get a good fit (it dampens fluctuations in the rental rate on capital and marginal cost and prices)
- Investment adjustment costs and habit formation are mainly important to account for the variables other than inflation and output
- In the absence of the working capital assumption the average duration of price contracts becomes unrealistically large
- Model contains a strong internal propagation mechanism

Keynesian critics (1)

- Kiyotaki (2011)
- Main points:
  - In the RBC–MBC framework heterogeneous firms and households all play a part in the social division of labour under ideal market mechanisms
  - "...markets are complete, that is, there exists a complete set of Arrow Securities so that state-contingent claims to goods and factors of production for every possible future state can be traded at the initial period"
  - With complete markets heterogeneity does not matter because we can always study the aggregate economy with the construct of the Representative Agent
  - In such an "Arrow-Debreu" Complete Markets Economy (CME), credit is just a frictionless exchange between future and present goods. It is always enforced by an auctioneer who has the authority and ability to enforce all contracts costlessly

Keynesian critics (2)

- . . . Main points:
  - In reality such an auctioneer is absent and we enter the realm of Incomplete Markets Economics (IME): borrowers might default, creditors require collateral, and agent heterogeneity becomes crucial again
  - Inherent problem with IME: whilst there is typically only one way to write down the CME there are many ways in which to formulate an IME, depending on which frictions you wish to stress (cf. the economist who loses his keys in the dark and goes to the nearest lamp post)
  - In a series of papers, Kiyotaki and Moore stress the importance of credit constraints

Classical critics (1)

- Chari *et al.* (2009)
- Marvellous piece of rhetoric
- Main points:
  - NK models not yet useful for policy analysis
  - NK economists add shocks that are "dubiously structural"
  - In their desire to fit the data closely, NK economists use too many "free parameters" and don't subject their models to the "discipline of microeconomic evidence"
  - Critique explicitly aimed at Smets & Wouters (SW hereafter)
  - Dubiously structural shocks in the SW framework:
    - shocks to wage markups
    - shocks to price markups
    - shocks to exogenous government spending
    - shocks to risk premia

Classical critics (2)

- Further weakness of the SW and CEE frameworks:
  - backward indexation of prices
  - the specification of the Taylor Rule
- With non-structural – reduced-form – shocks multiple structural explanations can be formulated that give rise to drastically different policy implications
- Example: a labour wedge ($\equiv$ difference between the real wage and the MRS between leisure and consumption) constitutes a reduced-form shock. Structural explanations:
  - Union wage setting. Policy advice: bust-the-unions policy
  - Fluctuating utility of leisure. Policy advice: pursue laissez faire

Keynesians *versus* Classicals

- Debate between Keynesians and Classicals reminds this commentator of the age-old debate between these groups of economists!