Foundations of Modern Macroeconomics Third Edition

Chapter 18: DSGE – New Classical models

Ben J. Heijdra

Department of Economics, Econometrics & Finance
University of Groningen

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Outline

1. Extended RCK model
   - Households
   - Firms
   - Equilibrium
2. Technology shocks
   - Model
   - Simulations
   - Puzzles
3. Numerical approach: Dynare

Getting started (1)

- This set of slides is based on: B.J. Heijdra (2017), *Foundations of Modern Macroeconomics* (Third Ed.), Chapter 18 and Section 13.5
- We augment the Ramsey–Cass–Koopmans model by:
    - Reformulating it in discrete time
    - Explaining the concept of a stochastic discount factor
    - Introducing (potentially) stochastic exogenous variables
- Introduce the Dynare package and show:
    - The effects of fiscal policy in a deterministic setting (permanent *versus* temporary and anticipated *versus* unanticipated)
    - The important role of a key structural parameter: the intertemporal labour supply elasticity

Getting started (2)

- *Key idea*: To build a micro-based theory of short-term fluctuations in macroeconomic variables (output, employment, consumption, investment, wages, etcetera) one can use the Ramsey-Cass-Koopmans model with an **endogenous labour supply decision** by households
- In Section 13.5 of the book this Extended RCK model is formulated in continuous time
- In Section 18.1 the Extended RCK model is developed in discrete time and incorporates uncertainty
- In what follows we abstract from population growth (population normalized at unity)

Expected utility function

- The household expected lifetime utility function is:

$$E_t\Lambda_t \equiv E_t \sum_{\tau=t}^{\infty} U(C_\tau, 1-L_\tau)\left(\frac{1}{1+\rho}\right)^{\tau-t} \qquad \text{(S1)}$$

  where E_t is the expectations operator (i.e. information dated up to and including period t is used)
    - The planning period is t so the time index τ runs from t to ∞
    - $\rho > 0$ is the pure rate of time preference
- We use the logaritmic felicity function for illustrative purposes:

$$U(C_\tau, 1-L_\tau) \equiv \varepsilon \ln C_\tau + (1-\varepsilon)\ln[1-L_\tau] \qquad \text{(S2)}$$

Household budget identity

- The agent's budget identity is given (for $\tau = t, t+1, t+2, \ldots$) by:

$$C_\tau + p_\tau S_{\tau+1} + B_{\tau+1} = w_\tau L_\tau + (1 + r^c_{\tau-1}) B_\tau$$
$$+ S_\tau (p_\tau + D_\tau) - T_\tau \qquad \text{(S3)}$$

  - w_τ is the wage rate
  - T_τ is the lump-sum tax
  - S_τ is the number of shares owned at the start of period τ (dividends are given by D_τ, and the stock market price of shares at time τ is p_τ)
  - B_τ is the number single-period corporate bonds owned at the start of period τ (such bonds pay a risk-free interest rate $r^c_{\tau-1}$, i.e. this rate is determined in period $\tau - 1$)

Household choices (1)

- The household chooses paths for consumption, labour supply, share holdings, and corporate bonds such that lifetime utility (S1) is maximized subject to the household budget identity
- S_t and B_t are taken as given (predetermined)
- Expected paths for w_τ, p_τ, r^c_τ, and D_τ (for $\tau > t$) are treated as given
- Optimal labour supply in the planning period t:

$$\frac{U_{1-L}(C_t, 1-L_t)}{U_C(C_t, 1-L_t)} = w_t \tag{S4a}$$

  where $U_C(\cdot)$ and $U_{1-L}(\cdot)$ denote the marginal felicity of, respectively, consumption and leisure

  - **Static choice** The MRS between consumption and leisure is equated to the real wage

Household choices (2)

- Optimal portfolio investment in the planning period t:

$$U_C(C_t, 1-L_t) = E_t\left[\frac{1+r_t^c}{1+\rho} U_C(C_{t+1}, 1-L_{t+1})\right] \qquad \text{(S4b)}$$

$$U_C(C_t, 1-L_t) = E_t\left[\frac{1+r_{t+1}^e}{1+\rho} U_C(C_{t+1}, 1-L_{t+1})\right] \qquad \text{(S4c)}$$

where r_{t+1}^e is the net yield on equity shares:

$$r_{t+1}^e \equiv \frac{p_{t+1} - p_t + D_{t+1}}{p_t} \qquad \text{(S4d)}$$

- **Dynamic choice** For both assets the marginal felicity in this period is equated to the resulting *expected* marginal felicity in the next period discounted for impatience

Asset pricing and the Stochastic Discount Factor

- The corporate bond rate is risk-free (as r_t^c is known in period t) so we can rewrite (S4b) to find:

$$1 = (1 + r_t^c) E_t \left[\mathcal{R}_{t,t+1} \right] \qquad \text{(S4b}')$$

where $\mathcal{R}_{t,s}$ is the real *stochastic discount factor*:

$$\mathcal{R}_{t,s} \equiv \left(\frac{1}{1+\rho} \right)^{s-t} \frac{U_C(C_s, 1-L_s)}{U_C(C_t, 1-L_t)}, \qquad \text{for } s \geq t \qquad \text{(S5)}$$

- Using (S4d) and (S5) we can rewrite (S4c) as:

$$p_t = E_t \left[(p_{t+1} + D_{t+1}) \, \mathcal{R}_{t,t+1} \right] \qquad \text{(S4c}')$$

- With identical households:

$$p_t = E_t \left[\sum_{i=1}^{\infty} D_{t+i} \, \mathcal{R}_{t,t+i} \right] \qquad \text{(S6)}$$

The firm's objective function

- Technology:

$$Y_\tau = F(Z_\tau, K_\tau, L_\tau) \equiv \Omega_0 Z_\tau K_\tau^\alpha L_\tau^{1-\alpha} \tag{S7}$$

with $0 < \alpha < 1$ and $\Omega_0 > 0$

- Objective function of the representative firm:

$$V_t = CF_t + E_t \left[\sum_{\tau=t+1}^{\infty} CF_\tau \mathcal{R}_{t,\tau} \right] \tag{S8}$$

where $\mathcal{R}_{t,\tau}$ is the real stochastic discount factor (see (S5)) and CF_τ is the net cash flow:

$$CF_\tau \equiv F(Z_\tau, K_\tau, L_\tau) - w_\tau L_\tau - I_\tau \tag{S9}$$

The firm's choices

- Capital accumulation:

$$K_{\tau+1} = I_\tau + (1-\delta)K_\tau \tag{S10}$$

  with $0 < \delta < 1$
- The firm chooses paths for investment and labour demand such that the value of the firm (S8) is maximized subject to the capital accumulation identity
- K_t is taken as given (predetermined)
- Expected paths for w_τ and $\mathcal{R}_{t,\tau}$ (for $\tau > t$) are treated as given
- Optimal labour demand and investment in period t:

$$w_t = F_L(Z_t, K_t, L_t) \tag{S11a}$$

$$1 = E_t\left[\left(F_K(Z_{t+1}, K_{t+1}, L_{t+1}) + 1 - \delta\right)\mathcal{R}_{t,t+1}\right] \tag{S11b}$$

Table 18.1: The basic RBC model

$$K_{t+1} = I_t + (1-\delta)K_t \tag{T1.1}$$

$$\frac{\varepsilon}{C_t} = E_t\left[\frac{1+r_{t+1}}{1+\rho}\frac{\varepsilon}{C_{t+1}}\right] \tag{T1.2}$$

$$w_t = (1-\alpha)\frac{Y_t}{L_t} \tag{T1.3}$$

$$r_t + \delta = \alpha\frac{Y_t}{K_t} \tag{T1.4}$$

$$Y_t = C_t + I_t + G_t \tag{T1.5}$$

$$L_t = 1 - \frac{1-\varepsilon}{\varepsilon}\frac{C_t}{w_t} \tag{T1.6}$$

$$Y_t = \Omega_0 Z_t K_t^{\alpha} L_t^{1-\alpha} \tag{T1.7}$$

$$T_t = G_t \tag{T1.8}$$

Table S1: Deterministic steady state

$$I^* = \delta K^* \tag{ST1.1}$$

$$r^* = \rho \tag{ST1.2}$$

$$w^* = (1-\alpha)\frac{Y^*}{L^*} \tag{ST1.3}$$

$$r^* + \delta = \alpha\frac{Y^*}{K^*} \tag{ST1.4}$$

$$Y^* = C^* + I^* + G_0 \tag{ST1.5}$$

$$L^* = 1 - \frac{1-\varepsilon}{\varepsilon}\frac{C^*}{w^*} \tag{ST1.6}$$

$$Y^* = \Omega_0 (K^*)^{\alpha} (L^*)^{1-\alpha} \tag{ST1.7}$$

$$T^* = G_0 \tag{ST1.8}$$

attained if $Z_t = 1$ and $G_t = G_0$ for all t for sure

Intermediate steps

- The deterministic steady state features a number of "Great Ratios" and constants
  - real interest rate
  - capital-output ratio
  - capital-labour ratio
  - investment-capital ratio
  - output per worker and the wage rate
  - consumption-leisure ratio
- The non-linear stochastic ERCK model can be approximately analyzed (both analytically and numerically) by
  - defining the perturbation variable:

$$\tilde{x}_t \equiv \ln\left(\frac{x_t}{x^*}\right)$$

  - and log-linearizing the stochastic model around the deterministic steady-state
  - ▶ This gives a linear system of expectational difference equations

Table 18.2: The log-linearized stochastic model

$$\tilde{K}_{t+1} - \tilde{K}_t = \delta \left[\tilde{I}_t - \tilde{K}_t \right] \tag{T2.1}$$

$$E_t \tilde{C}_{t+1} - \tilde{C}_t = \frac{\rho}{1+\rho} E_t \tilde{r}_{t+1} \tag{T2.2}$$

$$\tilde{w}_t = \tilde{Y}_t - \tilde{L}_t \tag{T2.3}$$

$$\rho \, \tilde{r}_t = (\rho + \delta) \left[\tilde{Y}_t - \tilde{K}_t \right] \tag{T2.4}$$

$$\tilde{Y}_t = \omega_C^* \tilde{C}_t + \omega_I^* \tilde{I}_t + \omega_G^* \tilde{G}_t \tag{T2.5}$$

$$\tilde{L}_t = \omega_{LL}^* \left[\tilde{w}_t - \tilde{C}_t \right] \tag{T2.6}$$

$$\tilde{Y}_t = \tilde{Z}_t + (1-\alpha) \tilde{L}_t + \alpha \tilde{K}_t \tag{T2.7}$$

$$\tilde{G}_t = \tilde{T}_t \tag{T2.8}$$

with $\omega_{LL}^* \equiv \frac{1-L^*}{L^*}$, $\omega_C^* \equiv \frac{C^*}{Y^*}$, $\omega_I^* \equiv \frac{I^*}{Y^*}$, and $\omega_G^* \equiv \frac{G_0}{Y^*}$

Quantification (1)

- The log-linearized ERCK model can be used to obtain quantitative results (for impact, transitional, and long-run effects)
- Advantages of using the loglinearized model:
  - It is expressed in terms of parameters which can be measured by empirically, e.g. income shares of various macro variables (ω_C^*, ω_I^*, ω_G^*, α) etcetera
  - In a deterministic setting we can solve the model for all kinds of policy shocks (anticipated/unanticipated, permanent/temporary)
  - In a stochastic setting we can solve the stochastic equilibrium paths for the different endogenous variables by postulating stochastic processes for the exogenous variables
  - We can simulate 'realistically calibrated' models on the computer and see how well they fit the real world data (the *Lucas research program*)

Quantification (2)

- In Section 13.5.3 of the book we show what a reasonable quarterly calibration looks like:
  - Pure rate of time preference: $\rho = 0.0159$
  - Depreciation rate of capital: $\delta = 0.0241$
  - Efficiency parameter of capital: $\alpha = \frac{1}{3}$
  - Taste parameter for consumption: $\varepsilon = 0.183$
  - Scale parameter in the production function $\Omega_0 = 1.442$
  - Initial level of government consumption $G_0 = 0.2$
- We find the following deterministic steady state:

$$\begin{array}{lll} Y^* = 1 & C^* = 0.599 & I^* = 0.201 \\ r^* = 0.0159 & L^* = 0.2 & K^* = 8.337 \\ T = G_0 = 0.2 & w^* = 3.333 & \end{array} \tag{S15}$$

Remaining tasks

- We use the Extended Ramsey–Cass–Koopmans (ERCK) model and:
  - Derive closed-form solutions for the loglinearized version of this model
  - Introduce stochastic technology shocks
- We thus obtain a prototypical RBC model which we use to:
  - Obtain a theory of fluctuations at business cycle frequencies
  - Derive impulse response functions (IRFs)
  - Match real world data (calibration)
- We end with an evaluation of the RBC approach

The Lucas research program

- *Key idea*: Macroeconomists should build so-called *structural models*, i.e. models that are based on microeconomic foundations (maximizing households and firms, flexible prices/wages, market clearing, etcetera)
- The Lucas Research Program (LRP) is the logical outcome of the Rational Expectations Revolution of the 1970s
- Kydland & Prescott accepted the challenge posed by Lucas: they built the first Real Business Cycle (RBC) model

Outline of the RBC methodology

- Postulate stochastic fluctuations in the level of general productivity; Future technology is unknown so agents form rational expectations about it
- Calibrate the model in a realistic fashion (done)
- Find the stochastic equilibrium process for the macroeconomic variables (output, employment, consumption, investment, the capital stock, and factor prices)
- Compute basic statistics (correlations, and standard deviations) for the different variables both for the artificial economy and for the actual economy
- Compare how well the model economy matches the actual economy's characteristics

Table 18.1: The basic RBC model

$$K_{t+1} = I_t + (1-\delta)K_t \tag{T1.1}$$

$$\frac{\varepsilon}{C_t} = E_t\left[\frac{1+r_{t+1}}{1+\rho}\frac{\varepsilon}{C_{t+1}}\right] \tag{T1.2}$$

$$w_t = (1-\alpha)\frac{Y_t}{L_t} \tag{T1.3}$$

$$r_t + \delta = \alpha\frac{Y_t}{K_t} \tag{T1.4}$$

$$Y_t = C_t + I_t + G_t \tag{T1.5}$$

$$L_t = 1 - \frac{1-\varepsilon}{\varepsilon}\frac{C_t}{w_t} \tag{T1.6}$$

$$Y_t = \Omega_0 Z_t K_t^{\alpha} L_t^{1-\alpha} \tag{T1.7}$$

$$T_t = G_t \tag{T1.8}$$

Table 18.2: The loglinearized stochastic model

$$\tilde{K}_{t+1} - \tilde{K}_t = \delta \left[\tilde{I}_t - \tilde{K}_t \right] \tag{T2.1}$$

$$E_t \tilde{C}_{t+1} - \tilde{C}_t = \frac{\rho}{1+\rho} E_t \tilde{r}_{t+1} \tag{T2.2}$$

$$\tilde{w}_t = \tilde{Y}_t - \tilde{L}_t \tag{T2.3}$$

$$\rho \, \tilde{r}_t = (\rho + \delta) \left[\tilde{Y}_t - \tilde{K}_t \right] \tag{T2.4}$$

$$\tilde{Y}_t = \omega_C^* \tilde{C}_t + \omega_I^* \tilde{I}_t + \omega_G^* \tilde{G}_t \tag{T2.5}$$

$$\tilde{L}_t = \omega_{LL}^* \left[\tilde{w}_t - \tilde{C}_t \right] \tag{T2.6}$$

$$\tilde{Y}_t = \tilde{Z}_t + \alpha \tilde{K}_t + (1-\alpha) \tilde{L}_t \tag{T2.7}$$

$$\tilde{T}_t = \tilde{G}_t \tag{T2.8}$$

Market equilibrium with stochastic technology

- Because general technology is stochastic, so is the future interest rate. For that reason, $E_t\tilde{r}_{t+1}$ appears in the log-linearized Euler equation. Recall:

$$r_{t+1} = F_K(\underbrace{Z_{t+1}}_{(a)}, \underbrace{K_{t+1}}_{(b)}, \underbrace{L_{t+1}}_{(c)}) - \delta$$

(a) Future general technology; unknown in period t (but may be partially forecastable if the shock is persistent)
(b) Future capital stock; known in period t as it depends only on present accumulation decisions
(c) Future labour supply; unknown in period t as it depends on w_{t+1} and C_{t+1} and thus on Z_{t+1}

The shock process

- The specification of the model is completed once the stochastic process for general productivity is specified. A commonly used specification is first-order autoregressive:

$$\ln Z_t = \alpha_Z + \xi_Z \ln Z_{t-1} + \eta_t, \qquad 0 \leq \xi_Z \leq 1 \qquad \Longrightarrow$$
$$\tilde{Z}_t = \xi_Z \tilde{Z}_{t-1} + \eta_t$$

  where $\tilde{Z}_t \equiv \ln(Z_t/Z^*)$ and:
  - ξ_Z is the *degree of persistence* of the shock [special cases: $\xi_Z = 0$ purely transitory shock; $\xi_Z = 1$ permanent shock]
  - η_t is the stochastic *innovation term* (identically and independently distributed with mean zero and variance σ_η^2).
  - If ξ_Z is nonzero, general productivity in the next period is partially forecastable. Under REH the agents best forecast is:

$$E_t \tilde{Z}_{t+1} = \xi_Z \tilde{Z}_t$$

Model solution

- The loglinearized model in Table 18.2 can be solved under the REH. In the text we show two methods. The easiest of these looks directly at so-called *impulse-response functions* for the different variables. *Key idea:*
  - Assume that the system is initially in steady state and trace the effect of a single innovation at time $t = 0$: $\eta_0 > 0$ and $\eta_t = 0$ for $t = 1, 2, \ldots$
  - We call η_0 the *impulse* hitting the economic system
  - Compute the implied *response* of the different variables to the impulse
  - In the text we derive the general case for which $0 \leq \xi_Z \leq 1$
  - To understand the general result it pays to look at the special cases

A purely temporary productivity improvement

- **A purely temporary shock**: $\xi_Z = 0$. The impulse-response functions for this type of shock are given in **Figure 18.2**
  - No long-run effect on general productivity and thus no long-run effect on any variable
  - Productivity only higher than normal in period $t = 0$
  - Agents are a little richer and thus $C_0 \uparrow$, and $I_0 \uparrow$ (agents spread gain over present and future consumption)
  - Strong incentive to work when productivity is high: $w_0 \uparrow$, $(1 - L_0) \downarrow$, $L_0 \uparrow$, $Y_0 \uparrow$ (see **Figure 18.1**)
  - For $t = 1, 2, 3, \ldots$ general productivity back to normal – agents gradually run down extra savings by consuming more than normal: $C_t \searrow$, $K_t \searrow$, Y_t, L_t, and I_t almost back to normal
  - Note: Output response looks virtually identical to impulse (lack of internal propagation)

Figure 18.2: Purely transitory productivity shock (1)

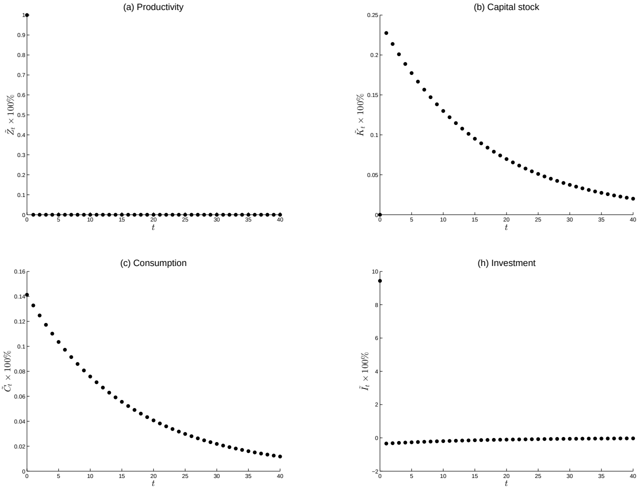


Figure 18.2: Purely transitory productivity shock (2)

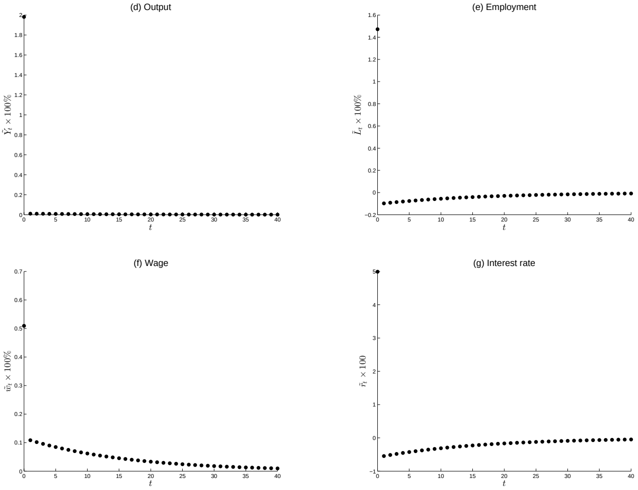
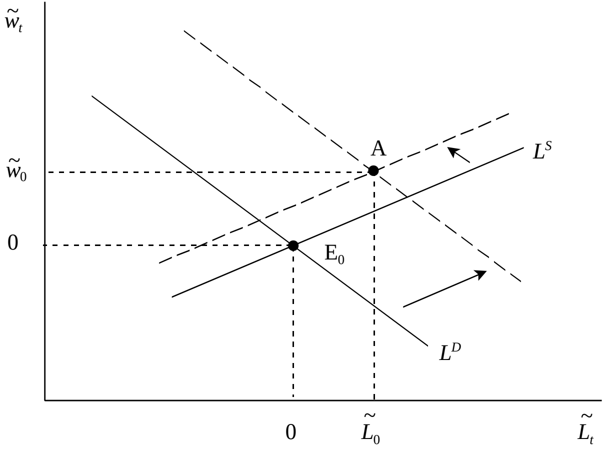


Figure 18.1: A shock to technology and the labour market



A purely permanent productivity improvement

- **A purely permanent shock**: $\xi_Z = 1$. The impulse-response functions for this type of shock are given in **Figure 18.3**.
  - There is a long-run effect on productivity and thus on most macro variables: the great ratios explain that $Y_\infty \uparrow$, $C_\infty \uparrow$, $K_\infty \uparrow$, $I_\infty \uparrow$, and $L_\infty \downarrow$ (if $\omega_G^* > 0$ so that IE effect dominates SE in labour supply)
  - Agents are a lot richer and thus $C_0 \uparrow$, and $I_0 \uparrow$ (agents spread gain over present and future consumption)
  - Even though $w_0 \uparrow$ and SE says $L_0 \uparrow$, there is a smaller upward jump in employment (than for temporary shock) because IE says $L_0 \downarrow$
  - For $t = 1, 2, 3, \ldots$ general productivity stays high. Agents gradually keep accumulating capital and consumption continues to rise: $C_t \nearrow$, $K_t \nearrow$, $L_t \searrow$, and $I_t \searrow$
  - Note: Output response again looks virtually identical to impulse (lack of internal propagation)

Figure 18.3: Permanent productivity shock (1)

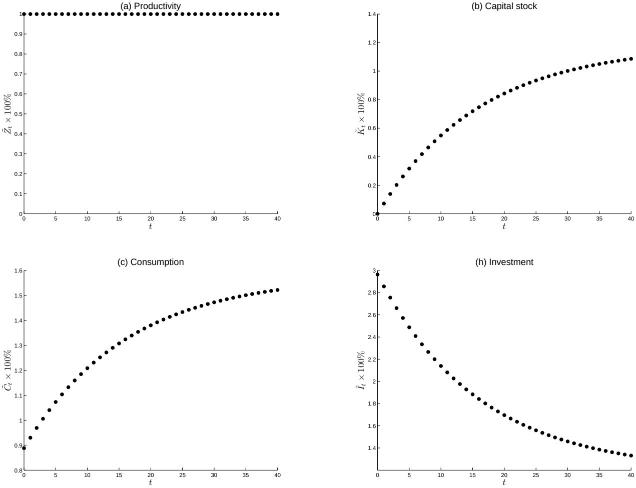
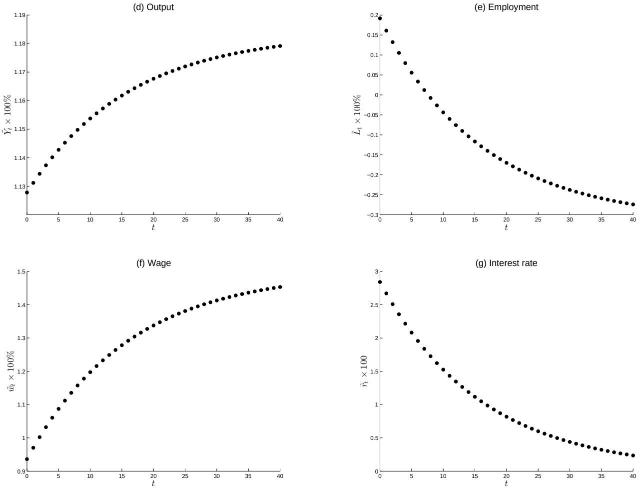


Figure 18.3: Permanent productivity shock (2)



What is a realistic shock process? (1)

- What would a **realistic shock** look like?
- The seminal work by Solow (1957) has been used to estimate the nature of technological change.
- *Solow residual*: compute how much of output growth can be explained by growth in inputs. The remainder is now called the Solow residual
- In our model the Solow residual is equal to the general productivity index Z_t:

$$\ln SR_t \equiv \ln Y_t - (1-\alpha)\ln L_t - \alpha \ln K_t = \ln Z_t$$

What is a realistic shock process? (2)

- We can obtain estimates for α_Z, ξ_Z, and σ_η^2 by regressing:

$$\ln SR_t = \alpha_Z + \xi_Z \ln SR_{t-1} + \eta_t$$

- For the US one finds:

$$\hat{\xi}_Z = 0.979$$

  which means that the technology shocks are not permanent but nevertheless display a very high degree of persistence
- **Figure 18.4**: impulse-response functions for a range of ξ_Z values (including the realistic one)
- Highly nonlinear behaviour of the IR functions for values of ξ_Z near unity

Figure 18.4: Temporary productivity shock (1)

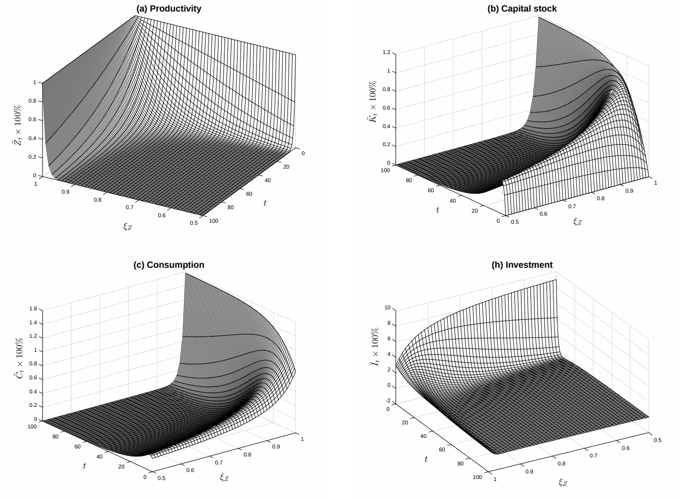
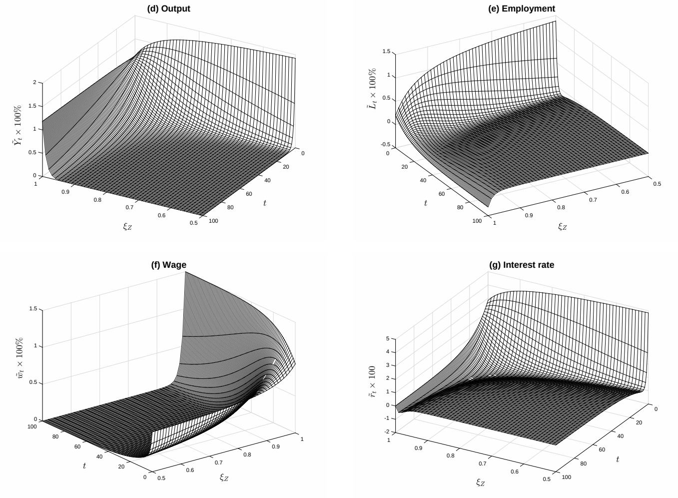


Figure 18.4: Temporary productivity shock (2)



Correlations and (co)variances

- Although the impulse-response functions display a lot of information about the model, most RBC modellers judge the performance of their model by looking at the match between model-generated and actual statistics
- **Table 18.3** shows an example of this approach. Model results in panel (b) and actual statistics for the US economy in panel (a)
  - Model captures that $\sigma(I_t) \gg \sigma(Y_t)$, $\sigma(C_t) < \sigma(Y_t)$
  - Model more or less matches $\rho(C_t, Y_t)$, $\rho(I_t, Y_t)$, $\rho(K_t, Y_t)$, and $\rho(L_t, Y_t)$, but overstates $\rho(Y_t/L_t, Y_t)$
  - Given the simple structure of the model, the fit is quite impressive
  - ... But recall the lack of propagation (explanation is almost entirely exogenous)

Table 18.3: The unit-elastic RBC model

	(a) *US economy*		(b) *Model economy I*		(c) *Model economy II*	
x_t:	$\sigma(x_t)$	$\rho(x_t, Y_t)$	$\sigma(x_t)$	$\rho(x_t, Y_t)$	$\sigma(x_t)$	$\rho(x_t, Y_t)$
Y_t	1.76		1.35		1.76	
C_t	1.29	0.85	0.42	0.89	0.51	0.87
I_t	8.60	0.92	4.24	0.99	5.71	0.99
K_t	0.63	0.04	0.36	0.06	0.47	0.05
L_t	1.66	0.76	0.70	0.98	1.35	0.98
Y_t/L_t	1.18	0.42	0.68	0.98	0.50	0.87

Problematic features of the RBC model

- A number of 'puzzles' remain
- Solving these puzzles constitutes current research in the area
- Puzzle (A): Employment variability
- Puzzle (B): Pro-cyclical real wage
- Puzzle (C): Productivity
- Puzzle (D): What about unemployment?
- Puzzle (E): Where is the money?

(A) Employment variability puzzle (1)

- *Key idea*: In reality $\sigma(Y_t)$ close to $\sigma(L_t)$, employment strongly pro-cyclical ($\rho(L_t, Y_t)$ near unity), and wages a-cyclical or mildly pro-cyclical ($\rho(w_t, Y_t)$ near zero). In the model:
  - With productivity shocks: η_t shifts labour demand, given upward sloping labour supply both w_t and L_t should be pro-cyclical
  - With low labour supply elasticity (micro-evidence) $\sigma(L_t)$ should be low and $\sigma(w_t)$ should be high
  - Hence, model under-predicts $\sigma(L_t)$ by quite a margin!
  - See **Figures S1 and 18.5** to visualize correlations

Figure S1: The good, the bad, and the average

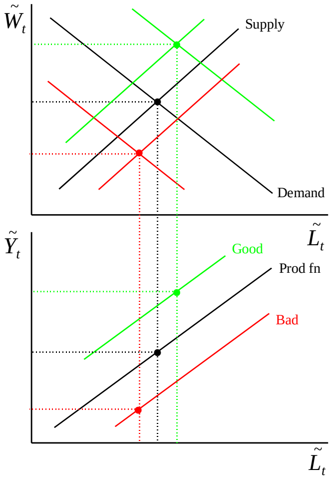
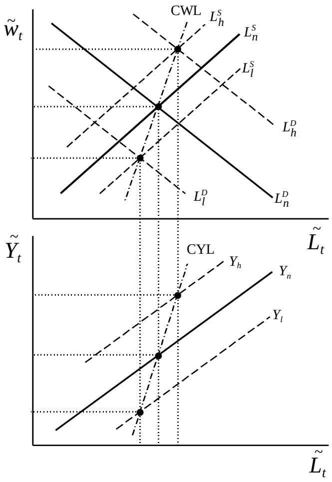


Figure 18.5: Visualizing contemporaneous correlations



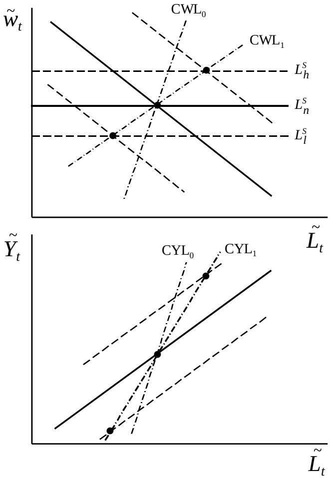
(A) Employment variability puzzle (2)

- *Solution to the puzzle*: We need a high substitution elasticity of labour supply [near horizontal labour supply curve] despite micro-evidence to the contrary. Indivisible labour model:
  - Either work 8 hours per day or 0 hours per day
  - Lottery determines which is which each period
  - Firm provides full insurance to the worker, and aggregate outcome is *as if* the representative agent has an infinite intertemporal substitution elasticity of labour supply
  - See **Figure 18.6**
  - In Table 18.3 panel (c), we observe that the lottery (or indivisible labour) model does better than the unit elastic model at matching $\sigma(L_t)$

Table 18.3: The unit-elastic RBC model

	(a) *US economy*		(b) *Model economy I*		(c) *Model economy II*	
x_t:	$\sigma(x_t)$	$\rho(x_t, Y_t)$	$\sigma(x_t)$	$\rho(x_t, Y_t)$	$\sigma(x_t)$	$\rho(x_t, Y_t)$
Y_t	1.76		1.35		1.76	
C_t	1.29	0.85	0.42	0.89	0.51	0.87
I_t	8.60	0.92	4.24	0.99	5.71	0.99
K_t	0.63	0.04	0.36	0.06	0.47	0.05
L_t	1.66	0.76	0.70	0.98	1.35	0.98
Y_t/L_t	1.18	0.42	0.68	0.98	0.50	0.87

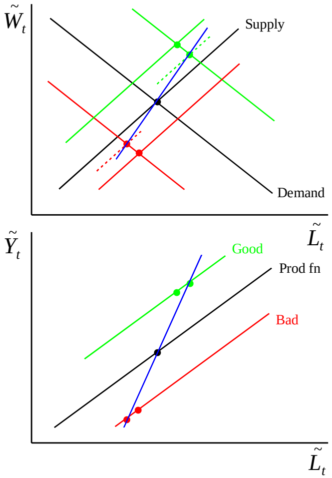
Figure 18.6: Lottery model and contemporaneous correlations



(B) Pro-cyclical wage

- *Key idea*: the unit-elastic model predicts too high a correlation between labour productivity [the wage] and output, $\rho(Y_t/L_t, Y_t) = 0.98$. In Hansen model we have $\rho(Y_t/L_t, Y_t) = 0.78$. In reality this correlation is much lower $(0.42$ for US).
- *Solution(s) of the puzzle*:
  - introduce shift factors in the labour supply function [both L^D and L^S shift out]
  - See **Figure S2**
  - Use any of the theories explaining real wage rigidity (efficiency wages, union model, etcetera)

Figure S2: Contemporaneous correlations and shift factors



(C) Productivity puzzle

- *Key idea*: If productivity shocks are predominant then L^D shifts explain high $\rho(Y_t/L_t, L_t)$ and $\rho(Y_t/L_t, Y_t)$. In reality $\rho(Y_t/L_t, L_t) \approx 0$ and $\rho(Y_t/L_t, Y_t)$ is weaker than predicted.
- *Solution(s) of the puzzle*:
  - Introduce shift factors in the labour supply function (both L^D and L^S shift out)
  - Nominal wage contracts and money supply shocks (nominal innovation shifts effective labour supply)
  - Labour hoarding by firms
  - Non-market sector also subject to technology shocks
  - Preference shocks affecting labour supply
  - Shocks to government spending

(D) Unemployment & No money (E)

- *Key idea (D)*: The standard RBC models assume market clearing in the labour market so all variation in employment is due to adjustment in hours worked. In reality 2/3 is explained by the extensive margin (in/out of employment) and 1/3 by the intensive margin (overtime etcetera)
- *Solution(s) to the puzzle*: Introduce unemployment model in the RBC framework, such as:
  - Search-theoretic approach
  - Efficiency wage theory, union models
- *Key idea (E)*: The standard RBC models does not feature money
- *Solution(s) to the puzzle*: Introduce money into the model:
  - Uninteresting with flexible prices and wages
  - With stickiness: New Keynesian DSGE models

Dynare documentation

- Dynare website: `www.dynare.org`
- For in-depth information on the Dynare program, see:
  - Adjemian, S. H. Bastani, F. Karamé, M. Juillard, J. Maih, F. Mihoubi, G. Perendia, J. Pfeifer, M. Ratto and S. Villemot (2013), *Dynare: Reference Manual*, Version 4.3.3
- For in-depth information on using Dynare for DSGE models, see:
  - Griffoli, T. M. (2013), *Dynare User Guide: An Introduction to the Solution and Estimation of DSGE Models*, Version January 2013"

What does it do for you?

- In the book we use the analytical – theoretical approach
- Steps to be taken:
  - Log-linearize the model around a steady state
  - Investigate (local) stability of the log-linearized model
  - Solve the linear system of difference equations under rational expectations
  - Study the effects of shocks in the exogenous variables
  - Etcetera, etcetera ...
- This approach is time-consuming and a bit tedious. Most people just do the numerical simulations
  - Advantage: Dynare is doing all the hard and tedious work for you
  - Disadvantage: easy to lose track of the key mechanisms accounting for your numerical results (model becomes black box)

Running Dynare

- So how did we get these results?
- Dynare is run from within Matlab
- Instructions to Dynare gathered in a socalled ***.mod file**
- See **Program18_01.mod** for the file we use
- Structure of the **Program18_01.mod** file
- Five steps:
  - Step 1: Define the variables
  - Step 2: Calibrate the model
  - Step 3: Formulate the model
  - Step 4: Simulate the model
  - Step 5: Show simulation results

Step 1: Define the variables

```
% Basic RBC model
%
% Dynare model file: Program18_01.mod
%

%----------------------------------------------------------------
% 1. Defining variables
%----------------------------------------------------------------

var Y C K I L W R Ztilde;
varexo G eta;

parameters Omega_0 alpha epsilon delta rho xi_Z sigma_eta ;
```

Step 2: Calibrate the model

```
%----------------------------------------------------------------
% 2. Calibration
%----------------------------------------------------------------

alpha     = 0.333333333333333;
delta     = 0.024113689084445;
Omega_0   = 1.442032886235652;
epsilon   = 0.183413993147403;
rho       = 0.015868284782784;
sigma_eta = 0.0015;
xi_Z      = 0.95;

% Guess for the initial steady state
Y0            = 1;
K0            = 8.337;
C0            = 0.599;
I0            = 0.201;
L0            = 0.200;
W0            = 3.333;
R0            = rho;
```

Step 3: Formulate the model

```
%-----------------------------------------------------------------
% 3. Model
%-----------------------------------------------------------------

model;
  K = I + (1 - delta)*K(-1);
  (1/C) = ((1 + R(+1))/(1+rho)) * (1/C(+1)) ;
  W = (1 - alpha) * Y / L;
  R = alpha * Y / K(-1) - delta;
  Y = C + I + G_0 ;
  W * (1 - L) = ((1 - epsilon)/epsilon) * C;
  Y = Omega_0 * exp(Ztilde) * K(-1)^(alpha) * L^(1-alpha);
  Ztilde = xi_Z * Ztilde(-1) + eta;
end;
```

Step 4: Simulate the model

```
%-----------------------------------------------------------------
% 4. Computation
%-----------------------------------------------------------------
% Compute initial steady state and verify the calibration
initval;
  K   = K0;
  C   = C0;
  L   = L0;
  I   = I0;
  Y   = Y0;
  W   = W0;
  R   = R0;
  eta = 0;
  G   = 0.2;
end;
steady;

% Postulate the variance of eta and simulate
shocks;
var eta = sigma_eta^2;
end;
```

Step 5: Show simulation results

```
% Postulate the variance of eta and simulate

shocks;
var eta = sigma_eta^2;
end;

stoch_simul(order = 1);

%-----------------------------------------------------------------
% 5. Some Results
%-----------------------------------------------------------------

statistic1 = 100*sqrt(diag(oo_.var(1:7,1:7)))./oo_.mean(1:7);
dyntable('Relative st devs in %',strvcat('VARIABLE','REL. S.D.'),
M_.endo_names(1:7,:),statistic1,10,8,4);
```

Punchlines (1)

- We have reformulated the Extended Ramsey–Cass–Koopmans (ERCK) in discrete time and incorporated uncertainty
- Model components:
  - Individuals have a fixed amount of time each day (time endowment)
  - Labour supply choice is implied by leisure choice ("buy leisure from yourself")
  - Economic actors have rational expectations and live in an inherently risky world hit by stochastic shocks
- Main findings:
  - Neoclassical structure gives rise to many "great ratios"
  - Fiscal spending multipliers are positive and may even exceed unity
  - Dynare is a very useful package to obtain quantitative results

Punchlines (2)

- The standard RBC model has a hard time matching data for real economies
- It is difficult to believe that the productivity shocks explain all fluctuations in the economy: "If they are so important then why don't we read about them in the Wall Street Journal"
- Link between micro-data and calibration values not strong
- Most important contribution of the approach is a methodological one:
  - Approach is flexible
  - Micro-foundations for macro are important and can be improved (alternative market structures, heterogeneous households, etcetera)
  - Other shocks can be introduced (government spending shocks, tax shocks, changes in the real exchange rate, etcetera)