Foundations of Modern Macroeconomics Third Edition

Chapter 9: Dynamic inconsistency in public and private decision making

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Outline

Aims of this lecture

- What do we mean by *dynamic inconsistency*?
- How can reputation effects help in solving the problem?
- Why do we appoint conservative central bankers?
- Why does taxing capital lead to dynamic inconsistency?

Dynamic inconsistency (1)

- Monetary policy: policy maker exploits the Lucas supply curve
- The Lucas supply curve is:

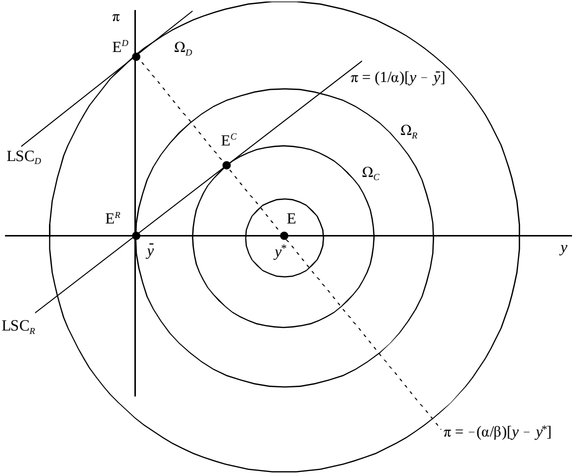
$$y = \bar{y} + \alpha \left[\pi - \pi^e \right] + \varepsilon, \quad \alpha > 0$$

  - y ($\bar{y}$) is the logarithm of (full employment) output
  - π is actual inflation
  - π^e is expected inflation
  - ε is a stochastic supply shock [observable to policy maker but not to public]
- LSC can be inverted:

$$\pi = \pi^e + (1/\alpha) \left[y - \bar{y} - \varepsilon \right]$$

In terms of **Figure 9.1** the LSC curves are upward sloping lines with a vertical intercept at the level of π^e

Figure 9.1: Consistent and optimal monetary policy



Dynamic inconsistency (2)

- The objective function of the policy maker [social welfare function]:

$$\Omega \equiv \frac{1}{2}\left[y - y^*\right]^2 + \frac{\beta}{2}\pi^2, \quad \beta > 0$$

  - y^* is desired output target of the policy maker
  - $y^* > \bar{y}$; policy maker deems $\bar{y}$ to be too low [overly ambitious? $\bar{y}$ distorted?]
  - β measures the relative inflation-aversion of the policy maker [high β is a right-winger]
- Policy maker chooses π (by monetary policy) and thus y to minimize Ω subject to the Lucas supply curve
- The Lagrangian is:

$$\min_{\{\pi, y\}} \mathcal{L} \equiv \frac{1}{2}\left[y - y^*\right]^2 + \frac{\beta}{2}\pi^2 + \lambda\left[y - \bar{y} - \alpha(\pi - \pi^e) - \varepsilon\right]$$

Dynamic inconsistency (3)

- First-order conditions:

$$\frac{\partial \mathcal{L}}{\partial y} = (y - y^*) + \lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial \pi} = \beta\pi - \alpha\lambda = 0$$

where λ is the Lagrange multiplier

- Combining the two FONCs yields the "social expansion path" [combinations of π and y for which Ω is minimized]:

$$y - y^* = -(\beta/\alpha)\pi \quad \Leftrightarrow$$
$$\pi = -(\alpha/\beta)\left[y - y^*\right] \tag{S1}$$

In terms of Figure 9.1, the FONC is a downward sloping [dashed] line through y^*

Dynamic inconsistency (4)

- The optimal solution under **discretionary policy** is computed by combining (S1) with the constraint and solving for the inflation rate, π_D:

$$\pi_D = \frac{\alpha^2 \pi^e + \alpha \left[y^* - \bar{y} - \varepsilon \right]}{\alpha^2 + \beta} \tag{S2}$$

  In terms of Figure 9.1, all points on the line between E^D and E are solutions for π_D for a particular expected price level (π^e)
- By invoking the rational expectations hypothesis [REH] we find a unique solution for the inflation rate under discretionary policy

Dynamic inconsistency (5)

- Derivation:
  - By REH we have $\pi^e = E(\pi_D)$
  - From (S2) we get:

$$E(\pi_D) = \frac{\alpha^2\pi^e + \alpha[y^* - \bar{y} - \overbrace{E(\varepsilon)}^{=0}]}{\alpha^2 + \beta} = \pi^e$$

  so that we can solve for π^e:

$$\pi^e = \frac{\alpha}{\beta}\left[y^* - \bar{y}\right] \qquad \text{(S3)}$$

  - Substituting (S3) into (S2) and the LSC we find the actual inflation rate:

$$\pi_D = (\alpha/\beta)\left[y^* - \bar{y}\right] - \frac{\alpha}{\alpha^2 + \beta}\varepsilon$$

$$y_D = \bar{y} + \frac{\beta}{\alpha^2 + \beta}\varepsilon$$

  In Figure 9.1 this is represented by point E^D

Dynamic inconsistency (6)

- But the discretionary solution (π_D, y_D) is sub-optimal! If the policy maker commits to a zero-inflation rule ($\pi_R = 0$) and households would expect it to stick to the rule [so that $\pi^e = 0$ also] then output would be:

$$y_R = \bar{y} + \varepsilon$$

In terms of Figure 9.1 the rule-based solution (π_R, y_R) is found in point E^R. [Later on we shall use "R" to denote **r**eputation.] Social welfare is higher in E^R than in E^D

- But unfortunately the rule-based solution is (π_R, y_R) inconsistent! If the policy maker is able to convince the public that it will follow the rule [so that $\pi^e = 0$] then the policy maker is tempted to produce "surprise inflation" to steer the economy towards y^*. In terms of Figure 9.1 the "cheating solution" [subscript C] lies at point E^C

Dynamic inconsistency (7)

- We find:

$$\pi_C = \frac{\alpha\left[y^* - \bar{y} - \varepsilon\right]}{\alpha^2 + \beta}$$

$$y_C = \frac{\beta}{\alpha^2 + \beta}\bar{y} + \frac{\alpha^2}{\alpha^2 + \beta}y^* + \frac{\beta}{\alpha^2 + \beta}\varepsilon$$

- It follows from the diagram that:

$$\Omega_D > \Omega_R > \Omega_C > 0$$

  - *Discretion*: satisfies REH but is sub-optimal [worst of all cases]
  - *Rule*: optimal and satisfies REH. But is open to temptation and thus not credible
  - *Cheating*: closest to bliss but inconsistent with REH

Reputation as an enforcement mechanism (1)

- Idea presented by Barro & Gordon (1983). *Key idea*:
  - Monetary policy is like a prisoners' dilemma [PD] game. If we only consider solution consistent with the REH then (π_R, y_R) is preferable over (π_D, y_D) but society nevertheless ends up with the worst case
  - Repeated interactions may help mitigate the PD problem. Barro and Gordon suggest that the reputation of the policy maker may act as an enforcement mechanism which makes the rule-based solution credible
- Model is inherently dynamic [reputation is an asset that can be accumulated or decumulated!]

Reputation as an enforcement mechanism (2)

- The social welfare function is now:

$$V \equiv \Omega_0 + \frac{\Omega_1}{1+r} + \frac{\Omega_2}{(1+r)^2} + \cdots = \sum_{t=0}^{\infty} \frac{\Omega_t}{(1+r)^t}$$

where r is the discount factor [interest rate] and Ω_t is:

$$\Omega_t \equiv \tfrac{1}{2}\left[y_t - y^*\right]^2 + \tfrac{\beta}{2}\pi_t^2$$

- The Lucas supply is deterministic:

$$y_t = \bar{y} + \alpha\left[\pi_t - \pi_t^e\right], \quad \alpha > 0$$

- Again we look at three types of solution, discretion [D], rule-based [R], and cheating [C]

Policy under discretion

- From our previous discussion we see that under discretion we would have:

$$\pi_{D,t} = (\alpha/\beta)\left[y^* - \bar{y}\right]$$

- So that:

$$V^D \equiv \frac{1+r}{r}\Omega_D$$

$$\Omega_D \equiv \tfrac{1}{2}\frac{\alpha^2+\beta}{\beta}\left[\bar{y} - y^*\right]^2$$

Policy under a constant-inflation rule

- The policy maker follows the rule $\pi_t = \pi_R$ [a constant]. The REH implies $E(\pi_t) = \pi_R$
- From our earlier discussion we find that:

$$\Omega_R = \tfrac{1}{2}\left[\bar{y} - y^*\right]^2$$

can be generalized [for a non-zero π_R] to:

$$\Omega_R(\pi_R) = \Omega_R + \tfrac{\beta}{2}\pi_R^2$$

- The social welfare function under the rule-based solution is:

$$V^R(\pi_R) \equiv \frac{1+r}{r}\left[\Omega_R + \tfrac{\beta}{2}\pi_R^2\right]$$

Cheating solution

- If the policy maker manages to make the agent expect that the rule will be followed $[\pi^e = \pi_R]$ then he has the incentive to cheat by exploiting the Lucas supply curve associated with $\pi^e = \pi_R$. The result is:

$$\pi_C = \frac{\alpha^2 \pi_R + \alpha \left[y^* - \bar{y}\right]}{\alpha^2 + \beta}$$

$$y_C = \frac{\beta}{\alpha^2 + \beta}\bar{y} + \frac{\alpha^2}{\alpha^2 + \beta}y^* - \frac{\alpha\beta}{\alpha^2 + \beta}\pi_R$$

so that the objective function under cheating is:

$$\Omega_C(\pi_R) = \tfrac{1}{2}\left[\frac{\beta}{\alpha^2 + \beta}\left[\bar{y} - y^*\right] - \frac{\alpha\beta}{\alpha^2 + \beta}\pi_R\right]^2 + \tfrac{\beta}{2}\left[\frac{\alpha^2}{\alpha^2 + \beta}\pi_R + \frac{\alpha}{\alpha^2 + \beta}\left[y^* - \bar{y}\right]\right]^2$$

Reputation (1)

- We now introduce the following **reputation mechanism** ["tit-for-tat"]:

$$\pi_t^e = \begin{cases} \pi_R & \text{if } \pi_{t-1} = \pi_{t-1}^e \\ \pi_{D,t} & \text{if } \pi_{t-1} \neq \pi_{t-1}^e \end{cases}$$

  - If the policy maker did in the last period what the public expected him to do ($\pi_{t-1} = \pi_{t-1}^e$) then this policy maker has credibility and the public expects that the rule inflation rate (π_R) will be produced in the present period
  - If the policy maker did not do in the last period what the public expected him to do ($\pi_{t-1} \neq \pi_{t-1}^e$) then this policy maker has no credibility and the public expects that the discretionary inflation rate ($\pi_{D,t}$) will be produced in the present period
  - The public adopt the "tit-for-tat" strategy in the repeated prisoner's dilemma game that it plays with the policy maker. If the policy maker "misbehaves" it gets punished by the public for one period

Reputation (2)

- Consider a policy maker in period 0 which kept its promise and produced the rule inflation in the period before [i.e. in period -1 it set $\pi_{-1} = \pi_R$]. This policy maker has credibility in period 0 and the public expects $\pi_0^e = \pi_R$. The policy maker can do two things in period 0:
  - Keep its promise and maintain its reputation [produce $\pi_0 = \pi_R$]. No punishment takes place!
  - Cheat in period 0 by producing π_C in that period [*temptation* is present because $\Omega_R(\pi_R) > \Omega_C(\pi_R)$]. But because he broke his promise, the public punishes the policy maker and expect the discretionary solution next period [$\pi_1^e = \pi_D$]. This involves *punishment* because $\Omega_D > \Omega_R(\pi_R)$ in period 1. In period 1 the public expects $\pi_1^e = \pi_D$ and, given this expectation, it is optimal for the policy maker to produce π_D. So policy maker has reputation again in period 2 [as it kept its promise in period 1] and the public expects $\pi_2^e = \pi_R$

Reputation (3)

- The benefits of cheating [*temptation*] are:

$$
\begin{aligned}
T(\pi_R) &\equiv \Omega_R(\pi_R) - \Omega_C(\pi_R) \\
&= \tfrac{1}{2}\left[\bar{y} - y^*\right]^2 + \tfrac{\beta}{2}\pi_R^2 - \tfrac{1}{2}\left[\frac{\beta}{\alpha^2+\beta}\left[\bar{y} - y^*\right] - \frac{\alpha\beta}{\alpha^2+\beta}\pi_R\right]^2 \\
&\quad - \tfrac{\beta}{2}\left[\frac{\alpha^2}{\alpha^2+\beta}\pi_R + \frac{\alpha}{\alpha^2+\beta}\left[y^* - \bar{y}\right]\right]^2
\end{aligned}
$$

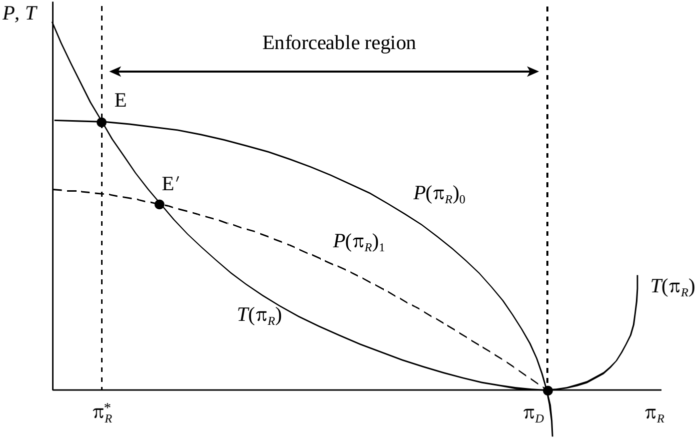
- The costs of cheating [*punishment*] are:

$$
\begin{aligned}
P(\pi_R) &\equiv \frac{\Omega_D - \Omega_R(\pi_R)}{1+r} \\
&= \left[\tfrac{1}{2}\frac{\alpha^2+\beta}{\beta}\left[\bar{y} - y^*\right]^2 - \tfrac{1}{2}\left[\bar{y} - y^*\right]^2 - \tfrac{\beta}{2}\pi_R^2\right]\frac{1}{1+r} \\
&= \left[\tfrac{1}{2}\frac{\alpha^2}{\beta}\left[\bar{y} - y^*\right]^2 - \tfrac{\beta}{2}\pi_R^2\right]\frac{1}{1+r}
\end{aligned}
$$

Reputation (4)

- In **Figure 9.2** we plot these two curves as a function of the rule inflation rate π_R
  - Rule inflation rates between 0 and π_R^* and the ones exceeding π_D are such that the policy maker will always deviate from the rule. The temptation is too big
  - Rule inflation rates between π_R^* and π_D are enforceable. The punishment exceeds the temptation and it is not worthwhile to deviate from the rule
  - Since social welfare depends negatively on inflation, the optimal enforceable inflation rate is the lowest enforceable one, i.e. π_R^*
  - If the interest rate rises, $P(\pi_R)$ rotates counter-clockwise and the optimal enforceable inflation rate rises. Punishment more heavily discounted

Figure 9.2: Temptation and enforcement



Voting and optimal inflation (1)

- Rogoff (1985) and Alesina & Grilli (1992) ask themselves why central bankers tend to be conservative economists
- The median voter model of A & G can be used to cast some light on this issue. Which agent is elected to head the central bank?
- Person i has the following cost function:

$$\Omega_i \equiv \frac{1}{2}\left[y - y^*\right]^2 + \frac{\beta_i}{2}\pi^2 \tag{S4}$$

- Note that β_i appears in (S4). The higher is β_i the more “right wing” we call this person
- The Lucas supply curve is still given by:

$$y = \bar{y} + \alpha\left[\pi - \pi^e\right] + \varepsilon, \quad \alpha > 0$$

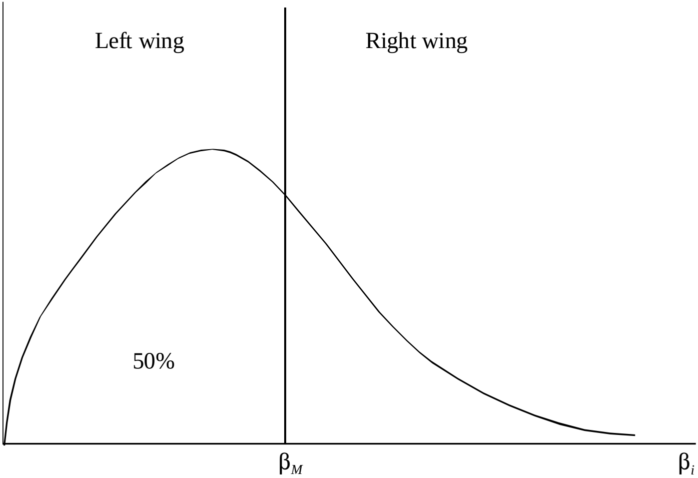
Voting and optimal inflation (2)

- If person i would be the central banker then he/she would set inflation according to:

$$\pi_D^i = \frac{\alpha}{\beta_i}\left[y^* - \bar{y}\right] - \frac{\alpha}{\alpha^2 + \beta_i}\varepsilon$$
$$y_D^i = \bar{y} + \frac{\beta_i}{\alpha^2 + \beta_i}\varepsilon$$

- Assume that the distribution of β_i across the population is as in **Figure 9.3**. The person with preference parameter β_M is the *median voter* and effectively decides the election. [There is a single issue and preferences are single-peaked, so the median voter theorem holds]

Figure 9.3: The frequency distribution of the inflation aversion parameter



Voting and optimal inflation (3)

- The median voter's cost function is:

$$
\begin{aligned}
\Omega_M &\equiv \tfrac{1}{2} E\left(\left(y_D^i - y^*\right)^2 + \beta_M \left(\pi_D^i\right)^2\right) \\
&= \tfrac{1}{2} E\left(\left(\underbrace{\bar{y} - y^* + \frac{\beta}{\alpha^2+\beta}\varepsilon}_{(a)}\right)^2 + \underbrace{\beta_M}_{(b)} \left(\underbrace{\frac{\alpha}{\beta}\left(y^* - \bar{y}\right) - \frac{\alpha}{\alpha^2+\beta}\varepsilon}_{(c)}\right)^2\right) \\
&= \tfrac{1}{2}\left[1 + \beta_M \left(\frac{\alpha}{\beta}\right)^2\right]\left(\bar{y} - y^*\right)^2 + \tfrac{1}{2}\frac{\beta^2 + \beta_M \alpha^2}{(\alpha^2+\beta)^2}\sigma^2
\end{aligned}
$$

→ Median voter cannot observe ε but he knows how banker of type β reacts to ε

(a) Output gap a central banker of type β would create

(b) Evaluated from the point of view of the median voter

(c) Inflation a central banker of type β would create

Voting and optimal inflation (4)

- The median voter elects central banker such that Ω_M is minimized by choice of β
- The first-order condition is:

$$\frac{d\Omega_M}{d\beta} = -\tfrac{1}{2} 2\beta_M \frac{\alpha^2}{\beta^3}\left(\bar{y}-y^*\right)^2 + \tfrac{1}{2}\frac{2(\alpha^2+\beta)^2\beta - 2(\beta^2+\beta_M\alpha^2)(\alpha^2+\beta)}{(\alpha^2+\beta)^4}\sigma^2 = 0 \quad \Rightarrow$$

$$\frac{d\Omega_M}{d\beta} = -\frac{\beta_M}{\beta}\left(\frac{\alpha}{\beta}\right)^2\left(\bar{y}-y^*\right)^2 + \frac{(\beta-\beta_M)\alpha^2}{(\alpha^2+\beta)^3}\sigma^2 = 0$$

- It follows that the optimal β exceeds β_M. The median voter delegates the conduct of monetary policy to someone more conservative than he is himself. This way the median voter commits to a lower inflation rate

Dynamic consistency and capital taxation (1)

- Dynamic inconsistency can also play a role in fiscal policy. We give the example of capital taxation
- Two-period model ($t = 1, 2$)
- Household utility:

$$U \equiv \frac{C_1^{1-1/\varepsilon_1}}{1-1/\varepsilon_1} + \frac{1}{1+\rho}\left[C_2 + \alpha\frac{(1-N_2)^{1-1/\varepsilon_2}}{1-1/\varepsilon_2} + \beta\frac{G_2^{1-1/\varepsilon_3}}{1-1/\varepsilon_3}\right]$$

Dynamic consistency and capital taxation (2)

- Technology:

$$F(N_t, K_t) = aN_t + bK_t$$

  - Production factors perfect substitutes
  - Inessential production factors
  - Constant marginal products

- Resource constraints:

$$C_1 + [K_2 - K_1] = bK_1$$

$$C_2 + G_2 = F(N_2, K_2) + K_2 = aN_2 + (1+b)K_2$$

Note that these are expressions like "$Y = C + I + G$"

First-best command optimum

- A benevolent social planner would choose C_1, C_2, N_2, and G_2 such that household utility is maximized subject to the consolidated resource constraint:

$$C_1 + \frac{C_2 + G_2 - aN_2}{1+b} = (1+b)K_1$$

- The solutions are:

$$C_1 = \left(\frac{1+b}{1+\rho}\right)^{-\varepsilon_1}$$
$$1 - N_2 = (a/\alpha)^{-\varepsilon_2}$$
$$G_2 = \beta^{-\varepsilon_3}$$

- The FBCO can be decentralized [i.e. reproduced in a free market setting] provided the policy maker has access to lump-sum taxes

Second-best optimum (1)

- What happens if lump-sum tax is not available and only distorting taxes can be used to obtain revenue [needed to pay for the public good]?
- The GBC becomes:

$$G_2 = t_K b K_2 + t_L a N_2$$

- The market solution becomes:

$$C_1 = \left(\frac{1 + b(1 - t_K)}{1 + \rho} \right)^{-\varepsilon_1}$$

$$\begin{aligned} C_2 = {} & a(1 - t_L) + (1 + b) \left[1 + b(1 - t_K) \right] K_1 \\ & - (1 + \rho)^{\varepsilon_1} \left[1 + b(1 - t_K) \right]^{1 - \varepsilon_1} - \alpha^{\varepsilon_2} \left[a(1 - t_L) \right]^{1 - \varepsilon_2} \end{aligned}$$

$$1 - N_2 = \left(\frac{a(1 - t_L)}{\alpha} \right)^{-\varepsilon_2}$$

Second-best optimum (2)

- Non-zero t_K and/or t_L drive the market solution away from the FBCO. We cannot set $t_L = t_K = 0$ because that would imply zero G [which is not optimal]. What do we do?
- We trade off the distortions in the tax system as well as we can by choosing G, t_L, and t_K such that welfare of the household is maximized *given the absence of lump-sum taxes*!
- The optimality conditions are the GBC plus:

$$\beta G_2^{-1/\varepsilon_3} = \eta \tag{S5}$$

$$\eta = \frac{1}{1 - \left(\frac{t_L}{1-t_L}\right)\varepsilon_L} \tag{S6}$$

$$\eta = \frac{1}{1 - \left(\frac{t_K}{1-t_K}\right)\varepsilon_K} \tag{S7}$$

Second-best optimum (3)

- Continued
  - η is the marginal cost of public funds [MCPF]
  - ε_L is the uncompensated wage elasticity of labour supply
  - ε_K is the uncompensated interest elasticity of gross saving
  - Equation (S5) is the "modified Samuelson rule"
- Equations (S6) and (S7) can be solved for the optimal tax rates:

$$\frac{t_L}{1-t_L} = \left(1 - \frac{1}{\eta}\right)\frac{1}{\varepsilon_L} \quad \text{(S8)}$$

$$\frac{t_K}{1-t_K} = \left(1 - \frac{1}{\eta}\right)\frac{1}{\varepsilon_K} \quad \text{(S9)}$$

  The intuition is as follows: the objective is to tax in the least distorting fashion by taxing most heavily the most inelastic tax base (e.g. if $\varepsilon_L = 0$ then $1/\varepsilon_L \to \infty$, $\eta = 1$, and $t_K = 0$. Labour income source of inelastic tax base in this special case)

Second-best optimum (4)

- BUT!!! In the general case, with both taxes non-zero, taxing labour in period 2 is not efficient. Once period 2 comes along, K_2 is inelastic and $t_L = 0$ and $t_K > 0$ is optimal. Hence, solutions in (S8) and (S9) are dynamically inconsistent
- To find the consistent solution we would have to work backwards. we know that $t_L = 0$ and $t_K > 0$ in period 2. Then we can figure out what t_L and t_K should be in the first period

Punchlines

- Dynamic inconsistency is all around us
- In the context of monetary policy a reputational mechanism can make a rule-based inflation rate enforceable
- The median voter can commit to a lower inflation rate by electing a central banker who is more conservative than himself
- The optimal taxes on labour and capital suffer from the dynamic inconsistency problem