Environmental abatement and the macroeconomy in the presence of ecological thresholds*

Ben J. Heijdra‡
University of Groningen; IHS (Vienna); CESifo; Netspar

Pim Heijnen♯
University of Groningen

September 2010 (Rev. July 2012)

Abstract: We study the environmental and economic effects of public abatement in the presence of multiple stable steady-state ecological equilibria featuring reversible hysteresis. The isocline for the stock of pollution possesses two stable branches. Assuming that the ecology is initially located on the upper (high pollution) branch of the isocline, a simple time-invariant temporary abatement policy can be used to steer the environment from the high- to the low-pollution equilibrium. In all models considered in this paper, a "cold turkey" abatement policy is optimal within the class of stepwise policies, i.e. the largest feasible shock should be administered for the shortest possible amount of time. The cold-turkey result is robust to alternative models for the economic system, although there is a capital feedback effect that either helps or hinders the speed of transition to the low-pollution equilibrium.

Keywords: Ecological thresholds, nonlinear dynamics, environmental policy, abatement, overlapping generations.

JEL Codes: D60, E62, H23, H63, Q20, Q28, Q50.

---

*We are grateful to the editor, Hassan Benchekroun, and two anonymous referees for valuable comments and suggestions. We also thank various participants at the macro breakfast seminar (IHS Wien), the TU Wien, the EEA 2010 meetings (University of Glasgow), the 2010 GAM Workshop (University of Warwick), the April 2011 Spring Meeting of Young Economists (University of Groningen), the July 2011 Econometric Society Australasian Meeting (University of Adelaide), and the June 2012 Annual Conference of the European Association of Environmental and Resource Economists (University of Economics, Prague).

‡Corresponding author. Faculty of Economics and Business, University of Groningen, P.O. Box 800, 9700 AV Groningen, The Netherlands. Phone: +31-50-363-7303, Fax: +31-50-363-7337, E-mail: b.j.heijdra@rug.nl.

♯Faculty of Economics and Business, University of Groningen, P. O. Box 800, 9700 AV Groningen, The Netherlands. Phone: +31-50-363-3789, Fax: +31-50-363-7337, E-mail: p.heijnen@rug.nl.

1 Introduction

In this paper we revisit an important theme in environmental macroeconomics, namely the environmental and economic effects of public abatement activities. The existing literature typically studies environmental policy with the aid of "monotone models" in which gradual changes in dirt emissions have gradual effects on the ecological system – Bovenberg and Heijdra (1998, 2002) are examples of this approach.

In recent years, however, prominent ecologists have argued that in many cases nature does not respond smoothly to gradual changes at all (e.g. Arrow *et al.* 1995). Scheffer *et al.* (2001) postulate the key elements of this new view. First, ecosystems do not respond smoothly to gradual changes in dirt flows, abrupt "catastrophic shifts" may be possible in the vicinity of threshold points, and there are typically no early warning signals for such shifts.[1] Second, there may be multiple stable equilibria. Third, irreversibility and hysteresis are all possible.

The nonlinear ecological dynamics described by Scheffer *et al.* (2001) now carries the name Shallow-Lake Dynamics (SLD hereafter).[2] But the same phenomenon holds for a range of other ecological systems such as coral reefs, boreal forests, savannas and grasslands, and seas.[3] Ecological systems featuring thresholds and flip points appear to be the rule rather than the exception. Indeed, there now exist several websites with databases documenting a broad variety of cases.[4]

The objective of this paper is to study the effects of public abatement on the environment and the economic system when the ecological system features SLD.[5] Regarding the ecological

[1] There is an emerging literature on early warning systems. See Biggs et al. (2009).

[2] For overviews of the SLD approach, see Muradian (2001), Mäler *et al.* (2003), Brock and Starrett (2003), and Wagener (2009). For economic applications of SLD, see Heijnen and Wagener (2009), Ranjan and Shortle (2007), and Wirl (2004). A related nonlinear approach is used by Prieur (2009).

[3] See, for example, Mäler and Li (2010), Crépin (2003, 2007), and Janssen, Anderies, and Walker (2004).

[4] See the threshold databases of the Resilience Alliance (`www.resalliance.org`) and the Stockholm Resilience Centre (`www.stockholmresilience.org`). Arrow *et al.* (1995, p. 521) define resilience as "a measure of the magnitude of disturbances that can be absorbed before a system centered on one locally stable equilibrium flips to another."

[5] We have chosen to focus on SLD not because we think that the ecology *literally* behaves just like a shallow lake but rather because SLD provides a simple and tractable framework in which there are potentially multiple stable steady states. For a survey on ecosystem dynamics, see Levin and Pacala (2003). They briefly discuss SLD on pages 79-80. In future work we will study alternative specifications for the ecological system. We believe our main results to be quite robust.

system we adopt the following assumptions. First, we assume that the flow of dirt is such that there exist two stable ecological steady-state equilibria. Second, we postulate that the ecological system has settled down at the "bad" equilibrium (from a steady-state welfare perspective) featuring a high stock of pollution.[6] Third, we assume that the flow of dirt depends negatively on public abatement activities but positively on the aggregate capital stock. Whereas the former is under the direct control of the government, the latter results from the aggregate savings behaviour of households.

To study the economic effects of abatement, we develop a number of dynamic general equilibrium models of a closed economy. In all these models households practice intertemporal consumption smoothing and accumulate capital that is rented out to perfectly competitive firms. Following Bovenberg and Heijdra (2002), we assume that the capital stock is the polluting production factor. Households enjoy living in a clean environment but act as free riders and thus fail to internalize the external effects caused by their capital accumulation decisions. As a result, there is a meaningful role for the policy maker.

Regarding public policy we adopt the following assumptions. First, we assume that public abatement is the only environmental policy instrument available. We thus abstract from Pigouvian corrective taxation that would take the form of a tax on capital since that is the polluting factor of production in our model. Second, we assume that the policy consists of a step-wise temporary increase in abatement spending. This policy is extremely simple as it involves only two parameters, namely the size and the duration of the policy shock. Third, we assume that the government has access to non-distorting lump-sum taxes to finance its spending.

Our motivation for endowing the policy maker with a restrictive set of instruments to conduct environmental policy is threefold. First, it enables us to make the comparison with the existing macroeconomic literature as exemplified by Bovenberg and Heijdra (2002). Second, since the main contribution of our paper lies in the study of the dynamic interactions between the economy and the environment in the presence of SLD, we wish to consider the simplest policy shock first. Indeed, in the perfect foresight models employed in this paper the time profile of the policy shock itself exerts an important influence on the impulse-response

[6] In the context of our model this sorry state of affairs could have arisen if the capital stock was much more polluting in the past than it is at present.

functions. However, as we demonstrate in the paper, a step-wise shock function is sufficiently flexible whilst still relatively easy to analyze both analytically and quantitatively. Third, by assuming the availability of lump-sum taxation, the mode of financing the abatement spending is itself non-distortionary (at least in two of the three macro models employed). In contrast, if only distortionary taxes would be available, any change in abatement would automatically cause additional behavioral changes originating from the financing side rather than from the abatement shock itself.

The paper is structured as follows. In Section 2 we provide a compact presentation of the core model, consisting of an ecological system featuring SLD and an economic system. The macroeconomic model is of the standard Ramsey type and features infinitely-lived representative agents that are endowed with perfect foresight and supply labour inelastically. Since preferences are assumed to be separable in consumption and environmental quality, the resulting dynamic system of differential equations is skew in the sense that the economy influences the ecology but not vice versa. It features two predetermined stock variables (the stock of pollution and the capital stock) and one non-predetermined variable (consumption).

Section 3 uses the core model to study the environmental and macroeconomic effects of a stepwise temporary abatement shock. We analytically characterize the qualitative effects at impact, during transition, and in the long run. To visualize and quantify these effects we also develop a plausibly calibrated version of the model. We demonstrate that a properly parameterized abatement policy is successful in steering the ecological equilibrium from the bad (high-pollution) to the good (low-pollution) steady state, thus increasing welfare of the representative agent. We identify an environmentally advantageous *capital feedback effect* that works as follows. The lump-sum tax financed increase in abatement prompts households to reduce their saving (and decumulate capital) during the early phase of the abatement policy which further reduces the inflow of dirt and "facilitates" environmental policy (compared to the case with a fixed capital stock). In addition, the model features a trade-off between shock size and shock duration. Given the form of the dirt flow equation, the best abatement policy in the class of stepwise shocks is a "cold turkey" policy, i.e. the maximum feasible shock for the shortest possible duration.

In Section 4 we study the robustness of this cold turkey result by augmenting the macroeconomic model in two distinct directions. Like the Ramsey model used in the previous section, both of these extensions represent canonical "workhorse" models in the field of dynamic macroeconomics. The first extension endogenizes the representative household's labour supply decision. We establish two main results. First, like in the core model, the cold turkey policy is welfare maximizing within the class of abatement policies. Second, unlike in the core model, the capital feedback effect hinders environmental policy. Intuitively, the tax increase that is needed to finance the abatement spending causes a wealth effect which prompts agents to cut back on consumption and leisure, increase labour supply, and boost saving (and thus the capital stock) during the early phase of transition. This in turn increases the inflow of dirt and "complicates" environmental policy (compared to the case with a fixed labour supply). The endogeneity of the labour supply decision thus reverses the sign of the capital feedback effect.

The second extension to the core model that is studied in Section 4 retains labour supply exogeneity but assumes that the economy is populated by overlapping generations of finite-lived agents. This introduces a heterogeneity between agents of different birth vintages. Three main results are established. First, like in the core model, the cold turkey policy is welfare improving. Second, with period-by-period budget balance the aggregate effects are qualitatively and quantitatively very similar to those predicted by the core model and the capital feedback effect simplifies environmental policy. Though positive, the welfare gains are very unevenly distributed across pre-shock and post-shock generations. Third, in the presence of deficit financing and public debt creation the distribution of benefits can be equalized by shifting some of the costs of the abatement policy to future generations. In addition to the capital feedback effect there is also a *public debt feedback effect* further facilitating environmental policy. Since Ricardian Equivalence does not hold, the long-run increase in public debt leads to capital crowding out and a reduction in the steady-state dirt flow.

Finally, in Section 5 we offer a brief summary of the main results and offer some thoughts on future work. A brief Appendix presents some technical details.

2 Core model

2.1 Ecological system

The environment is modelled as a renewable resource stock. Its quality depends negatively on the *flow* of dirt, $D(t)$, that is generated in the production process:

$$D(t) \equiv \kappa K(t) - \gamma G(t), \qquad \kappa > 0,\ \gamma > 0, \tag{1}$$

where $K(t)$ is the private capital stock (see below), and $G(t)$ represents abatement activities by the government. Capital is assumed to be the polluting factor of production, just as in Bovenberg and Heijdra (1998, 2002).[7] By definition the flow of dirt must be non-negative ($D(t) \geq 0$).[8] Denoting the *stock* of pollution at time t by $P(t)$, we write the general form of the emission equation as:

$$\dot{P}(t) = -\pi P(t) + \frac{P(t)^2}{P(t)^2 + 1} + D(t), \qquad \frac{1}{2} < \pi < \frac{3\sqrt{3}}{8}, \tag{2}$$

where $\dot{P}(t) \equiv dP(t)/dt$. The first term on the right-hand side shows that nature features a regenerative capacity (since $\pi > 0$). In combination, the first two terms give rise to shallow-lake dynamics (SLD) – see Mäler *et al.* (2003, p. 606).

We depict the isocline for the stock of pollution in Figure 1. The vertical arrows depict the dynamic forces operating on the stock of pollution off the $\dot{P}(t) = 0$ line. Our assumption that $\frac{1}{2} < \pi < \frac{3\sqrt{3}}{8}$ ensures that nature does not feature irreversible equilibria and that the P-isocline is S-shaped (see Appendix A). For time-invariant dirt flows satisfying $0 \leq D < D_L$ and $D > D_U$, there is a unique and stable ecological steady state to which nature converges. In contrast, for $D_L \leq D \leq D_U$ there exist two stable ecological steady-state equilibria, i.e. the lower branch (through points C′, A, and B) and the upper branch (from point C to point F and beyond) are both stable.[9] Which particular steady state is attained depends on initial conditions, i.e. the ecological model features *reversible hysteresis* (Mäler *et al.*, 2003, p. 607).

[7] Xepapadeas (2005, p. 1239) proposes a more general functional form of the type $D(t) = \Phi(K(t), G(t))$ with $\partial\Phi/\partial K(t) > 0$ and $\partial\Phi/\partial G(t) < 0$. We have chosen a linear form for convenience and because it allows us to zero in on the nonlinear features due to SLD.

[8] We interpret $D(t)$ as the *net* dirt flow which must be non-negative by definition. Initiatives such as Carbon Capture and Storage (CCS) can be seen as a way to increase the value of π in the ecological function (2). Since CCS is rather ineffective at present, we ignore this mechanism in this paper.

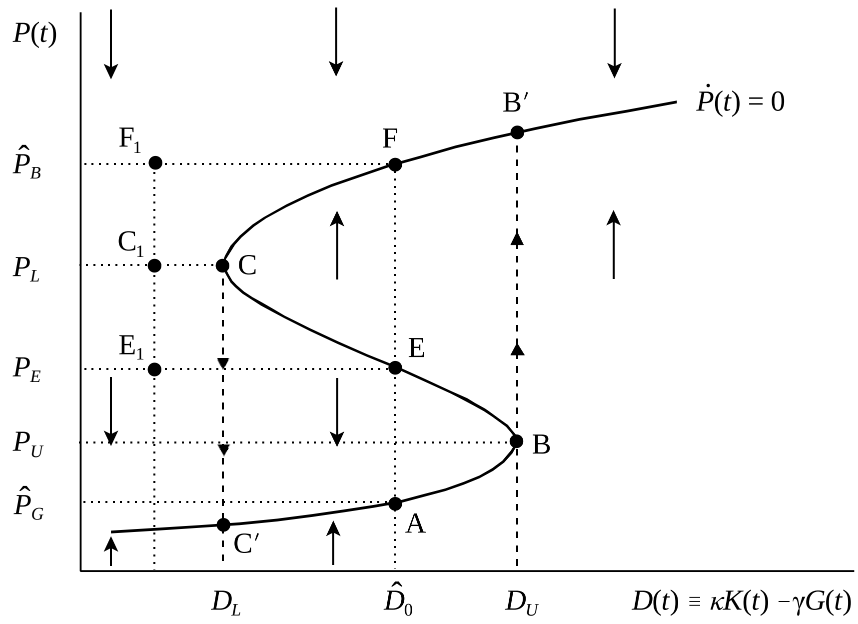
[9] The branch connecting points B and C is unstable, i.e. all vertical arrows point away from it.

An economy which starts out with a relatively low dirt flow will find itself on the lower branch. Even a sizeable increase in the dirt flow will only result in a small increase in the steady-state stock of pollution – see for example the move from point C′ to A. An economy which lets things get too dirty, however, and produces a dirt flow exceeding the upper threshold $(D(t) > D_U)$ will experience a catastrophic increase in the pollution stock and end up on the upper branch of the $\dot{P}(t) = 0$ line, say at point B′. A subsequent reduction in the dirt flow will move the ecological steady-state along the upper branch, say from point B′ to point F. Even though the dirt flow is the same in the dirty equilibrium F and in the clean equilibrium A, $D(t) = \hat{D}_0$, the stock of pollution is much higher in the dirty equilibrium, i.e. $\hat{P}_B > \hat{P}_G$. To make things worse, there is no way to get from F to A without reducing the dirt flow below its lower threshold value *for a long enough period of time.*

In a qualitative sense, to get from point F to point A, the following road must be traveled. First, the dirt flow should be set such that $D(t) < D_L$. In Figure 1 this produces, say, the shift from point F to F_1. This point lies in the basin of attraction of the lower branch of the $\dot{P}(t) = 0$ line as the vertical arrows indicate. Abstracting from economic feedback effects (see below), the ecology moves in the direction of points C_1 and E_1. Second, the stock of pollution must be allowed to fall below a critical level, P_E, representing the stock associated with point E in the figure. Third, once the ecology has passed point E_1, the dirt flow must be restored to its initial level $\hat{D}_0$. Since $P(t) < P_E$, the ecology will ultimately converge to point A.

But this is the mechanical story associated with SLD. As is shown in equation (1), the flow of dirt depends not only on the abatement activities of the government but also on the macroeconomic capital stock, i.e. on the aggregate savings behaviour of the economic agents in the economy. And to make things worse, the resources needed by the government to conduct its abatement activities will in general affect the behaviour of these very agents, i.e. $K(t)$ is not independent from $G(t)$. In the next subsection a simple general equilibrium model is postulated to capture this dependency.

Figure 1: Ecological dynamics



2.2 Economic system

In this subsection we formulate a simple general equilibrium model of the macroeconomy.[10] This core model describes a closed economy and is populated by representative households and firms who are blessed with perfect foresight.

2.2.1 Households

The representative household lives forever, and features the following utility functional:

$$\Lambda(t) \equiv \int_t^{\infty} \left[\ln C(\tau) + \varepsilon_E \ln \left[\bar{E} - P(\tau) \right] \right] \cdot e^{\rho(t-\tau)} d\tau, \tag{3}$$

where $C(\tau)$ denotes consumption of private commodities at time τ, $E(\tau) \equiv \bar{E} - P(\tau) > 0$ measures the quality of the environment, $\bar{E}$ is some pristine value attained in a non-polluting society, ε_E denotes the weight in overall utility attached to environmental amenities, and $\rho \geq 0$ stands for the pure rate of time preference. Since utility is separable in its two arguments, the quality of the environment does not directly affect household consumption. Since the felicity

[10]Our discussion of the standard economic models used in this paper is quite compact and intuitive. All technical details can be found in Heijdra and Heijnen (2012).

function for private consumption is logarithmic, the model features a unitary intertemporal elasticity of substitution.[11] Without leisure entering utility, labour supply is exogenously fixed.

Households face the following budget identity:

$$\dot{A}(\tau) = r(\tau)A(\tau) + w(\tau) - T(\tau) - C(\tau), \tag{4}$$

where $r(\tau)$ denotes the real rate of interest on financial assets, $w(\tau)$ represents the wage rate, $T(\tau)$ are net lump-sum taxes, and $A(\tau)$ stands for real financial assets owned in period τ. As usual we define $\dot{A}(\tau) \equiv dA(\tau)/d\tau$.

The representative agent chooses paths for $C(\tau)$ and $A(\tau)$ which maximize (3) subject to (4) and a solvency requirement. The solution for consumption at time t amounts to:

$$C(t) = \rho \cdot [A(t) + H(t)], \tag{5}$$

where human wealth, $H(t)$, is given by:

$$H(t) \equiv \int_t^\infty [w(\tau) - T(\tau)] \cdot e^{-\int_t^\tau r(s)ds} d\tau. \tag{6}$$

The optimal time profile for consumption is given by the Euler equation:

$$\frac{\dot{C}(\tau)}{C(\tau)} = r(\tau) - \rho, \qquad \tau \geq t. \tag{7}$$

Equation (5) shows that the agent consumes a constant proportion of total wealth in the planning period, whilst equation (7) shows that consumption growth over time is chosen to be equal to the rationally anticipated gap between the interest rate and the rate of time preference. Finally, the expression in (6) shows that human wealth is given by the discounted value of after-tax wage payments. Intuitively it thus represents the after-tax value of the agent's unitary time endowment.

2.2.2 Firms

The production sector of the economy is perfectly competitive. The production function is Cobb-Douglas, with constant returns to scale to the factors capital, $K(t)$, and labour, $L(t)$:

$$Y(t) \equiv F(K(t), L(t)) = \Omega_0 K(t)^{\varepsilon_L} L(t)^{1-\varepsilon_L}, \qquad \Omega_0 > 0, \ 0 < \varepsilon_L < 1, \tag{8}$$

[11] We adopt simple functional forms for felicity and production functions in order to keep our economic analysis as simple as possible. By adopting this approach we are better able to zoom in on the complications arising from the nonlinear environmental dynamics.

Table 1: The core model

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho, \qquad \rho > 0 \tag{T1.1}$$

$$\dot{K}(t) = Y(t) - C(t) - G(t) - \delta K(t) \tag{T1.2}$$

$$[r(t) + \delta] K(t) = (1 - \varepsilon_L) Y(t) \tag{T1.3}$$

$$w(t)L(t) = \varepsilon_L Y(t) \tag{T1.4}$$

$$Y(t) = \Omega_0 L(t)^{\varepsilon_L} K(t)^{1-\varepsilon_L}, \qquad \Omega_0 > 0,\ 0 < \varepsilon_L < 1 \tag{T1.5}$$

$$L(t) = 1 \tag{T1.6}$$

$$\dot{P}(t) = -\pi P(t) + \frac{P(t)^2}{P(t)^2 + 1} + D(t), \qquad \frac{1}{2} < \pi < \frac{3\sqrt{3}}{8} \tag{T1.7}$$

$$D(t) = \kappa K(t) - \gamma G(t), \qquad \kappa > 0,\ \gamma > 0 \tag{T1.8}$$

Endogenous variables: consumption, $C(t)$, capital stock, $K(t)$, output, $Y(t)$, interest rate, $r(t)$, wage rate, $w(t)$, employment, $L(t)$, pollution stock, $P(t)$, dirt flow, $D(t)$.

Exogenous variable: government abatement, $G(t)$.

Parameters: (economic) rate of time preference, ρ, depreciation rate of capital, δ, labour coefficient in the technology, ε_L, and scale factor in the technology, Ω_0. (ecological): lake resilience, π, capital dirt coefficient, κ, and abatement clean-up coefficient, γ.

---

where $Y(t)$ denotes gross output. The representative firm maximizes the value of the firm, $V(t)$, which is defined as follows:

$$V(t) = \int_t^{\infty} [Y(\tau) - w(\tau)L(\tau) - I(\tau)] \cdot e^{-\int_t^{\tau} r(s)ds} d\tau, \tag{9}$$

subject to the production function, and the capital accumulation identity:

$$\dot{K}(\tau) = I(\tau) - \delta K(\tau), \tag{10}$$

where $\dot{K}(\tau) \equiv dK(\tau)/d\tau$ denotes the rate of change in the capital stock and δ is the depreciation rate ($\delta > 0$). The first-order conditions for value maximization imply the usual marginal productivity conditions:

$$\frac{\partial Y(\tau)}{\partial K(\tau)} = r(\tau) + \delta, \qquad \frac{\partial Y(\tau)}{\partial L(\tau)} = w(\tau). \tag{11}$$

Since we abstract from adjustment costs in investment, the value of equity corresponds to the replacement value of the capital stock, i.e. $V(t) = K(t)$.

2.2.3 Equilibrium

For convenience, the key equations of the core model have been gathered in Table 1. Equation (T1.1) is the Euler equation (7), whilst equations (T1.5) and (T1.7)–(T1.8) just restate, respectively (8), (2), and (1). Labour supply is exogenous so $L(t) = 1$ – see (T1.6). The factor demand expressions in (11) have been rewritten by using the production function – see (T1.3) and (T1.4). Equation (T1.2) is obtained by combining (10) with the goods market clearing condition for a closed economy, i.e. $Y(\tau) = C(\tau) + I(\tau) + G(\tau)$. Finally, in the absence of government debt, claims on the capital stock are the only assets available, i.e. $A(t) = K(t)$.

By using (T1.3)–(T1.6) in (T1.1)–(T1.2) we obtain the nonlinear system of differential equations characterizing the economic system:

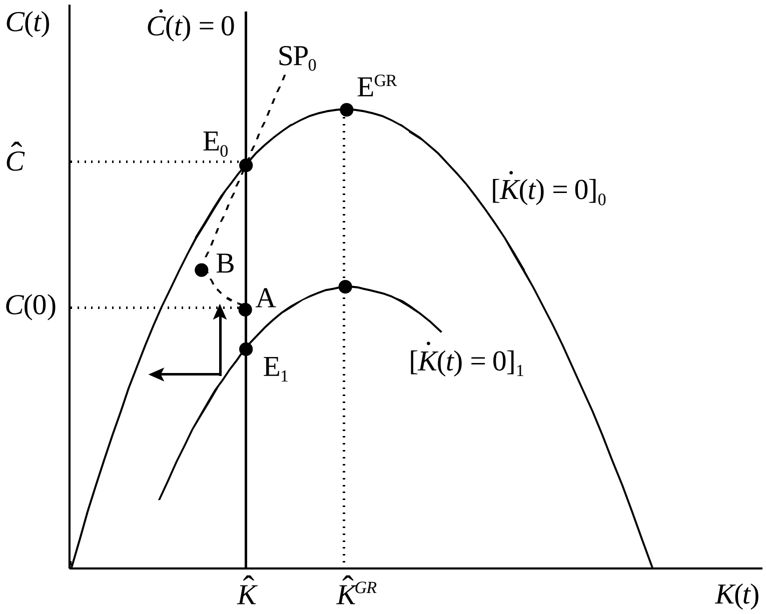
$$\begin{aligned}\dot{C}(t) &= \left[(1-\varepsilon_L)\Omega_0 K(t)^{-\varepsilon_L} - \rho - \delta\right] C(t),\\ \dot{K}(t) &= \Omega_0 K(t)^{1-\varepsilon_L} - C(t) - G(t) - \delta K(t).\end{aligned}$$

Because the economic system does not directly depend on the stock of pollution, its dynamic properties can be analyzed in the two-dimensional plane. We assume that in the initial (pre-shock) equilibrium there is no public abatement, i.e. $G(t) = 0$. The phase diagram is depicted in Figure 2 where $\dot{C}(t) = 0$ and $[\dot{K}(t) = 0]_0$ are the initial isoclines for, respectively, consumption and the capital stock. The initial equilibrium is at point E_0 and steady-state consumption and the capital stock are given by, respectively, $\hat{C}$ and $\hat{K}$. The equilibrium is saddle-point stable, with SP_0 representing the saddle path, and is dynamically efficient, i.e. $\hat{K}$ is strictly less than the golden-rule capital stock, $\hat{K}^{GR}$.

3 Environmental and macroeconomic effects of public abatement

In this section we study the effects of temporary public abatement on the environment, the macroeconomy, and on individual welfare. Much of the existing literature on environmental

Figure 2: Consumption-capital dynamics in the core model



macroeconomics only looks at "monotone" ecological systems – see for example Bovenberg and Heijdra (1998, 2002) and the references therein. In essence this literature assumes that the $\dot{P}(t) = 0$ curve is monotonically increasing, rather than S-shaped. As a result, for a given flow of dirt there exists a unique ecological equilibrium. Because the ecological system is non-hysteretic, a temporary abatement policy cannot be used to move the ecology to another steady-state equilibrium; only a permanent policy could possibly do that.

In stark contrast, as was argued above in our intuitive discussion of Figure 1, in the presence of SLD (featuring $\frac{1}{2} < \pi < \frac{3\sqrt{3}}{8}$) the ecological system is reversibly hysteretic, and there may be two welfare-rankable steady-state equilibria at the same inflow of dirt, namely a "clean" and a "dirty" one. Furthermore, a suitably designed *temporary* abatement policy can be used to shift the environment from the high- to the low-pollution equilibrium. Assume that the economy is initially at the steady-state equilibrium (point E_0 in Figure 2) and that the ecological equilibrium is at point F in Figure 1. Since $G(t) = 0$ initially, it follows from (1) that the dirt flow associated with point F is equal to $\hat{D}_0 = \kappa \hat{K}$. At this dirt flow there is another stable equilibrium at point A in Figure 1 which, from a steady-state perspective, features a higher level of welfare.

In order to move the ecological equilibrium from point F to point A, we assume that the policy maker engages in a simple abatement policy of the following type:

$$G(t) = \begin{cases} G & \text{for } 0 \leq t \leq t_E \\ 0 & \text{for } t > t_E \end{cases} \tag{12}$$

where the shock is administered at time $t = 0$, t_E represents the *duration* of the shock, and G is its *size*. Figure 2 shows the qualitative nature of the adjustment paths of consumption and the capital stock. The abatement shock shifts the K-isocline down (from $[\dot{K}(t) = 0]_0$ to $[\dot{K}(t) = 0]_1$) because less resources are available for private consumption and investment. If the shock were permanent ($t_E \to \infty$), the equilibrium would instantaneously shift from E_0 to E_1, i.e. the economy would feature a once off reduction in private consumption. Provided G is sufficiently large, such that $0 \leq \kappa\hat{K} - \gamma G < D_L$, the ecology would gradually move to the lower branch of the $\dot{P}(t) = 0$ line in Figure 1. It would reach a steady state to the left of point C′.

To reach point A from F a temporary policy ($0 < t_E < \infty$) must be used. In terms of Figure 2, such a shock produces the adjustment path from A through B to E_0. At impact ($t = 0$) the capital stock is predetermined and the economy shifts from point E_0 to A. The increased tax bill leads to an immediate reduction in human wealth and thus causes agents to cut back consumption – see (5) and (6) above.[12]

At point A, the dynamic forces are those indicated by the north-west arrows and for $0 < t < t_E$ the economy gradually moves from A to B. During transition the capital stock falls short of its steady-state level ($K(t) < \hat{K}$), the interest rate exceeds the rate of time preference ($r(t) > \rho$), and the optimal consumption profile is upward sloping – see (7) above. Also, since the economy is located above the then relevant $[\dot{K}(t) = 0]_1$ line, there is too little investment and the capital stock falls over time. Point B is reached at time t_E, at which moment the K-isocline shifts back to its original position. The resources previously used

[12] Taxes and government spending are related to each other via the intertemporal government budget constraint. In the absence of government debt at time t, this constraint is given by:

$$0 \equiv \int_t^\infty [T(\tau) - G(\tau)] \cdot e^{-\int_t^\tau r(s)ds} d\tau.$$

Table 2: Structural parameters and steady-state features

Economic system:					
$\rho = 0.04$	$\delta = 0.07$	$\varepsilon_L = 0.70$	$\Omega_0 = 0.7401$		
$\hat{r} = 0.04$	$\hat{K} = 2.7273$	$\hat{Y} = 1.000$	$\hat{C} = 0.8091$	$\hat{I} = 0.1909$	$G = 0$
Ecological system:					
$\pi = 0.52$	$\kappa = 0.0147$	$\gamma = 0.302$	$\varepsilon_E = 0.9$	$\bar{E} = 2$	
$D_L = 0.0196$	$D_U = 0.0735$	$\hat{D}_0 = 0.04$	$\hat{P}_B = 1.2482$	$\hat{P}_G = 0.0936$	$P_E = 0.6581$

for abatement can once again be used to restore the capital stock and consumption to their original levels.

The flow of dirt during the abatement policy features two jumps (namely at times $t = 0$ and $t = t_E$), is downward sloping for $0 < t < t_E$, and is upward sloping for $t \geq t_E$. Interestingly, the policy shock prompts a reaction from the private sector in the form of a temporarily lower capital stock which boosts the environmental cleanup.

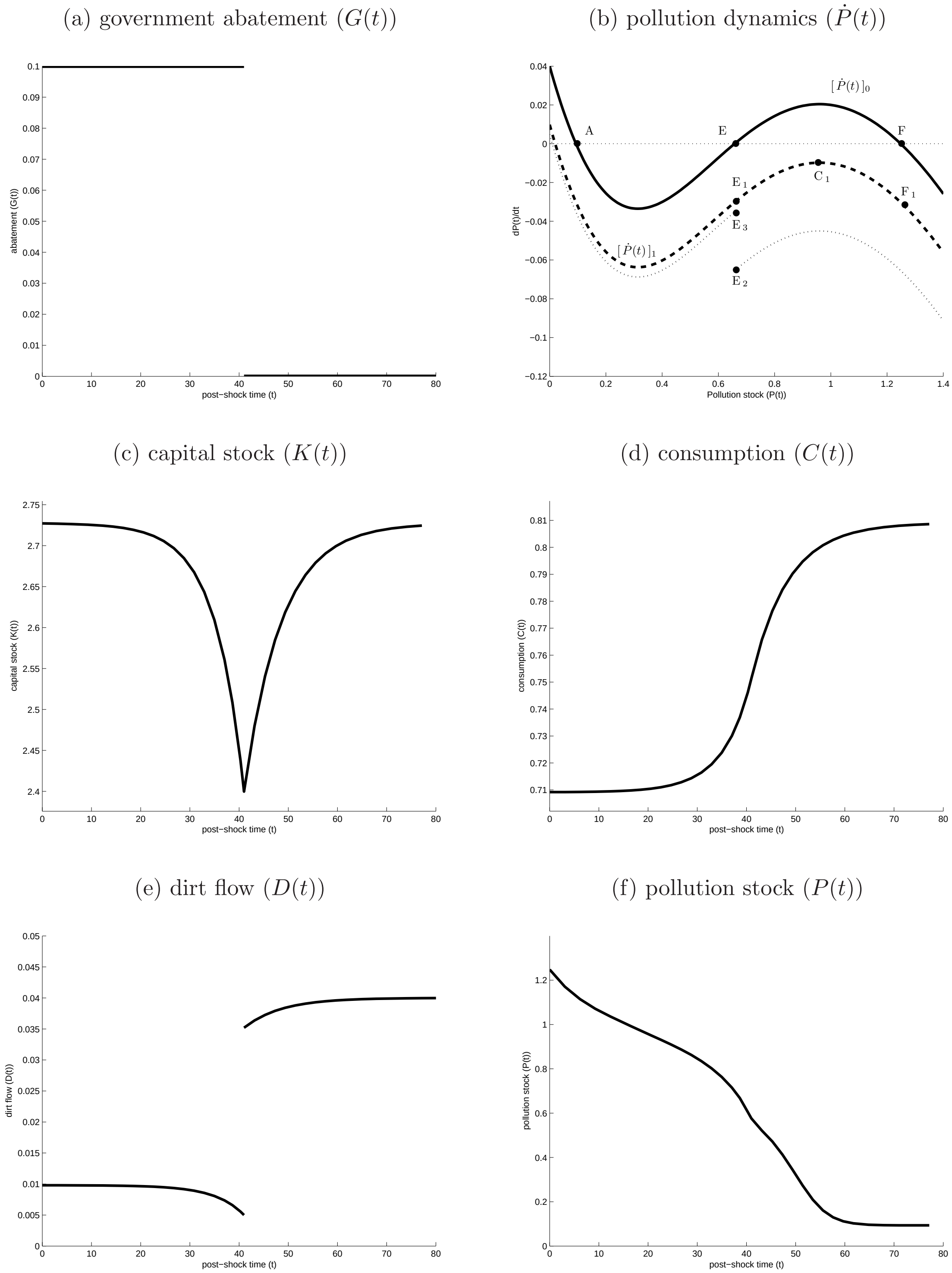
Of course, not just any temporary policy will result in a successful transition from point F to A in Figure 1. Indeed, there is a trade-off between the size of the abatement shock (G) and its minimum required duration (t_E). For a given value of G, t_E must be sufficiently large for the ecological system to "sail" past points C and E in Figure 1. Vice versa, for a given value of t_E, G must be large enough. We provide quantitative evidence for this trade-off below.

In Figure 3 we visualize the adjustment paths for the key variables using a plausibly calibrated version of the model.[13] The structural parameters and key features of the economic and ecological steady states are reported in Table 2. Since $\hat{D}_0$ is in between the lower and upper threshold values, there exist two stable ecological equilibria – see points A (with $\hat{P}_G = 0.0936$) and F (with $\hat{P}_B = 1.2482$) in Figure 3(b). The critical pollution stock associated with $\hat{P}_0$ is at point E where $P_E = 0.6581$.

The shock consists of a temporary increase in government abatement equal to ten percent of initial output, i.e. $G = 0.1$ for $0 \leq t < t_E$. The minimum duration for a shock of this

[13]The parameters of the economic system take the values typically assumed in the macroeconomic literature. The resulting steady-state interest rate, capital-output ratio, and output shares of consumption and investment are quite realistic. Much less is known about the parameters of ecological system. We have chosen these parameters such that the initial steady-state is in the region with multiple equilibria.

Figure 3: Dynamic effects of government abatement: Core model



Parameters: see Table 2. The initial ecological equilibrium is at point D in panel (b).

size to succeed in steering the ecology to the clean ecological equilibrium is $t_E = 41$ years – see Figure 3(a). The other panels in Figure 3 confirm the qualitative results relating to the economy. Panel (c) shows that the capital stock is reduced quite substantially during transition, reaching a minimum of $K(t_E) = 2.3998$. Similarly, as panel (d) shows, private consumption is reduced at impact to $C(0) = 0.7092$.

The ecological effects are as follows. At impact the abatement shock shifts the $\dot{P}(t)$ curve down (from $[\dot{P}(t)]_0$ to $[\dot{P}(t)]_1$ in panel (b)). For $P(t) = \hat{P}_B$, the abatement shock ensures that the stock of pollution starts to fall at impact, i.e. at point F_1 in panel (b) $\dot{P}(0) < 0$. Two things happen over time. First, the pollution stock gradually declines as $\dot{P}(t) < 0$, for $0 \leq t < \infty$. Second, the dashed $\dot{P}(t)$ line itself gradually shifts down in the direction of the thin dotted line as a result of capital crowding out. At time $t_E = 41$, the ecology arrives at point E_2 in panel (b), shortly thereafter $P(t) < P_E$, the abatement policy is terminated, and the $\dot{P}(t)$ line immediately shifts up to the thin dotted directly line below $[\dot{P}(t)]_1$. The ecology jumps from E_2 to E_3 in panel (b). Thereafter, the gradual increase in the capital stock shifts the $\dot{P}(t)$ line back to $[\dot{P}(t)]_0$ and the ecology converges to point A. Panels (e) and (f) of Figure 3 depict the time paths of, respectively, the flow of dirt and the pollution stock. The new ecological equilibrium is attained after more than sixty years.

Points F and A feature the same steady-state value for consumption but environmental quality is much higher in the latter point, so it follows that steady-state welfare is higher after the abatement policy. But is welfare also increased when we take the transitional dynamic effects on consumption and the pollution stock into account? Since the shock occurs at time $t = 0$, welfare is given by:

$$\Lambda_A(0) \equiv \int_0^{\infty} \Big[\ln C(t) + \varepsilon_E \ln \left[\bar{E} - P(t) \right] \Big] \cdot e^{-\rho t} dt. \tag{13}$$

Using the values for ε_E and $\bar{E}$ from Table 2, as well as the solution paths for $C(t)$ and $P(t)$ during transition, we find that $\Lambda_A(0) = -5.890$ with the abatement policy. At the initial high-pollution equilibrium, welfare is:

$$\Lambda_F(0) = \frac{1}{\rho} \cdot \Big[\ln \hat{C} + \varepsilon_E \ln[\bar{E} - \hat{P}_B] \Big], \tag{14}$$

which is equal to $\Lambda_F(0) = -11.71$. The welfare gain in utility terms is thus equal to $\Delta\Lambda(0) \equiv \Lambda_A(0) - \Lambda_F(0) = 5.82$. To facilitate the interpretation of this gain we compute an "equivalent-variation" type welfare measure by computing what $\hat{C}$ would have to be in

(14) to get $\Delta\Lambda(0)$ to be zero. Denoting this hypothetical consumption level by $\hat{C}'$ we find $\hat{C}' = e^{\rho\Lambda_A(0)-\varepsilon_E \ln\left[\bar{E}-\hat{P}_B\right]}$. An interpretable welfare measure is then:

$$EV(0) \equiv 100 \cdot \frac{\hat{C}' - \hat{C}}{\hat{C}}. \tag{15}$$

Intuitively, $EV(0)$ represents consumption that is missed out on if the abatement policy is not pursued. For the policy combination $(G, t_E) = (0.1, 41)$ we find that $EV(0)$ is 26.3%, i.e. lost consumption is more than one quarter of actual consumption at point F and the welfare gains of abatement are substantial.[14] Despite the fact that consumption is lower than its pre-shock level during transition, these costs are more than compensated for by the increased quality of the environment.

As was mentioned above, there is a trade-off between the size of the abatement shock (G) and its minimum required duration (t_E). We provide quantitative evidence for this trade-off in Figure 4(a). This figure plots the minimum shock size for shock durations ranging from 25 to 50 years.[15] For $t_E = 41$ a value of $G = 0.1$ is sufficient (as we saw above), but for $t_E = 30$ the shock must be increased to $G = 0.1166$, whilst for $t_E = 25$ it must be set at $G = 0.1293$. In short, Figure 4(a) shows that the size-duration locus is downward sloping. Not all points along the size-duration locus are feasible. Indeed, points to the left of the vertical dashed line are infeasible because the dirt flow constraint, $D(t) \geq 0$, is violated for some t during transition. It follows that only the size-duration locus to the right of the dashed line is feasible.

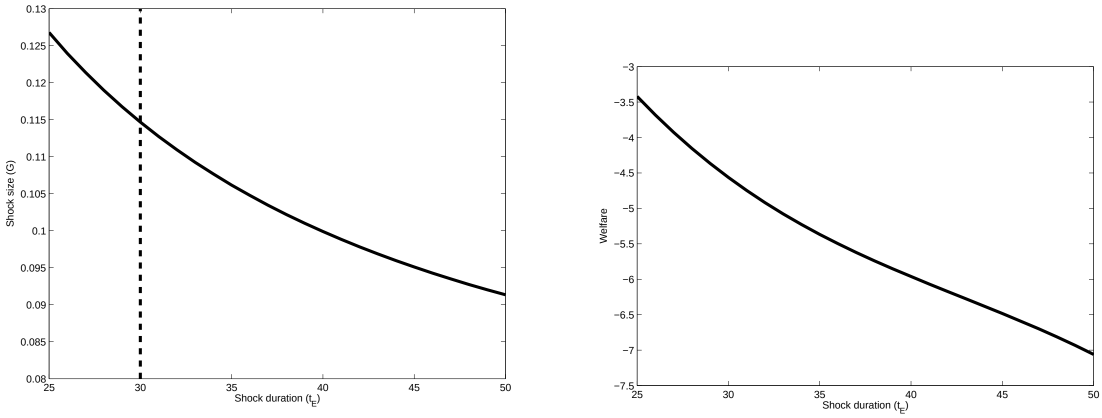
The clear trade-off between shock size and duration takes us to the social welfare issue. What is better from a welfare-theoretic point of view, a long-lasting small shock, or a short-lasting big shock? Figure 4(b) plots the optimized values of $\Lambda(0)$ for different values of t_E (and the associated values of G implied by the trade-off). It is clear from the figure that a "cold turkey" abatement policy is optimal, i.e. to get from F to A in Figure 1, the duration should be as small as possible and the shock size as large as needed. Indeed, for the cold turkey combination $(G, t_E) = (0.1166, 30)$ we find that $EV(0)$ is as much as 33.7% of initial consumption.

[14] Of course, the magnitude of the welfare gain from abatement depends critically on $\hat{P}_B - \hat{P}_G$ and ε_E about which little or no direct information is available.

[15] For $t_E < 25$ there is no feasible solution for the shock size.

Figure 4: Size, duration, and welfare

(a) Size G versus duration t_E (b) welfare effect ($\Lambda(0)$)



4 Extensions

In this section we study two modifications to the economic system embedded in the core model. Both model extensions are canonical “workhorse” models in the field of macroeconomics. By considering these particular two extensions it is possible to shed some light on the robustness to alternative macroeconomic closures of our finding regarding the constrained optimality of the cold turkey abatement policy.

The first extension endogenizes the labour supply decision of the infinitely lived representative agent. In such a setting, a lump-sum tax-financed abatement policy expands labour supply (as people get poorer), the capital stock, and output. This dirties the environment and makes it harder to steer the ecology from the dirty to the clean steady-state equilibrium.

In the second extension we take labour supply to be exogenous (as in the core model) but assume that the economy is populated by overlapping generations of finitely-lived agents. As in the core model, a lump-sum tax-financed abatement policy crowds out the capital stock during transition. Unlike the core model, however, the overlapping generations model features intergenerational redistribution (both during transition and in the long run). This opens up a useful role for public debt policy, namely to redistribute uneven welfare effects. The debt policy itself introduces hysteresis into the *economic* system in that a deficit-financed temporary abatement policy causes a permanent effect on the capital stock, consumption, output, wages, and the interest rate.

4.1 Endogenous labour supply

In the first extension we change the utility functional of the representative agent from (3) to:

$$\Lambda(t) \equiv \int_t^\infty \left[\varepsilon_C \ln C(\tau) + (1-\varepsilon_C)\ln[1-L(\tau)] + \varepsilon_E \ln\left[\bar{E} - P(\tau)\right]\right] \cdot e^{\rho(t-\tau)} d\tau, \tag{16}$$

with $0 < \varepsilon_C < 1$. Here, $L(\tau)$ is labour supply and, since the time endowment is equal to unity, $1 - L(\tau)$ represents the amount of leisure consumed by the household. The household's budget identity (4) is changed to reflect the endogeneity of labour supply:

$$\dot{A}(\tau) = r(\tau)A(\tau) + w(\tau)L(\tau) - T(\tau) - C(\tau), \tag{17}$$

where $w(\tau)L(\tau)$ represents labour income in period τ.

The representative agent chooses time profiles for $C(\tau)$, $L(\tau)$, and $A(\tau)$ which maximize (16) subject to (17) and a solvency requirement. The solutions for consumption and labour supply in the planning period t amounts are:

$$C(t) = \rho\varepsilon_C \cdot [A(t) + H(t)], \qquad w(t) \cdot [1 - L(t)] = \rho(1-\varepsilon_C) \cdot [A(t) + H(t)], \tag{18}$$

where $H(t)$ is defined in (6) above. Optimal consumption growth is still as given in (7) above. By combining the two expressions in (18), we find that the optimal labour supply decision leads to an equalization of the marginal rate of substitution between leisure and consumption to the wage rate, or:

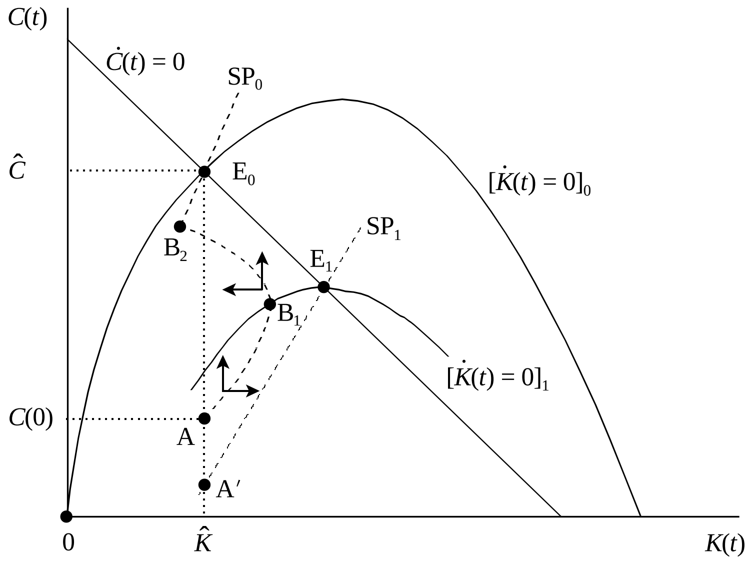
$$L(t) = 1 - \frac{1-\varepsilon_C}{\varepsilon_C} \cdot \frac{C(t)}{w(t)}. \tag{19}$$

Equation (19) replaces (T1.6) in Table 1.

By equating demand and supply on the labour market we find an implicit function, $L(t) = \Psi(C(t), K(t))$, featuring $\Psi_C \equiv \frac{\partial\Psi(C(t),K(t))}{\partial C(t)} < 0$ and $\Psi_K \equiv \frac{\partial\Psi(C(t),K(t))}{\partial K(t)} > 0$. Hence, equilibrium employment depends negatively on consumption (via labour supply) and negatively on the capital stock (via labour demand). The nonlinear system of differential equations characterizing the economic system can now be written as:

$$\begin{aligned}
\dot{C}(t) &= \left[(1-\varepsilon_L)\Omega_0 \left(\frac{\Psi(C(t), K(t))}{K(t)}\right)^{\varepsilon_L} - \rho - \delta\right] C(t), \\
\dot{K}(t) &= \Omega_0 \Psi(C(t), K(t))^{\varepsilon_L} K(t)^{1-\varepsilon_L} - C(t) - G(t) - \delta K(t).
\end{aligned}$$

Figure 5: Consumption-capital dynamics with endogenous labour supply



The phase diagram for the endogenous labour supply (ELS) model is given in Figure 5. In contrast to the core model, the ELS model features a downward sloping $\dot{C}(t) = 0$ line. For points above (below) the $\dot{C}(t) = 0$ line, consumption is too high (low), labour supply is too low (high), the capital-labour ratio is too high (low), the interest rate falls short of (exceeds) the rate of time preference, and consumption falls (rises) over time. The dynamics for the capital stock is qualitatively the same as in the core model, and the ELS model features a unique saddle-point stable steady state at point E_0.

Provided it is of sufficient duration, a temporary abatement policy of the form stated in (12) gives rise to the adjustment path in Figure 5, consisting of an immediate jump from E_0 to point A, a gradual move from A to B_1 and B_2, and a gradual move from B_2 back to point E_0. The intuition is as follows. At impact the tax increase reduces human wealth, $H(0)$, which prompts the agent to cut the consumption of goods and leisure, i.e. labour supply increases. Provided the policy is of sufficiently long duration,[16] point A lies below the $[\dot{K}(t) = 0]_1$ line and the agent saves part of the additional wage income. Since the capital-labour ratio is

[16] That such a value for t_E exists can be demonstrated graphically. In Figure 5 the thinly dotted line labeled SP_1 is the saddle path associated with a permanent increase in government abatement (a policy for which $t_E \to \infty$). For temporary abatement policies, the higher is t_E, the closer will be the adjustment path to SP_1during the early transition phase. Hence, a point like A can always be found.

low, the interest rate is high and both consumption and the capital stock increase over time immediately after the shock. This explains the gradual move from A to B_1.

At some time t such that $0 < t < t_E$ the economy arrives at point B_1, after which capital decumulation takes place though consumption continues to grow. At time $t = t_E$, the economy arrives at point B_2, the abatement policy is terminated, and the capital equilibrium locus shifts back to $[\dot{K}(t) = 0]_0$. From then on the dynamic forces are such as to increase both consumption and capital as the economy moves from B_2 to E_0.

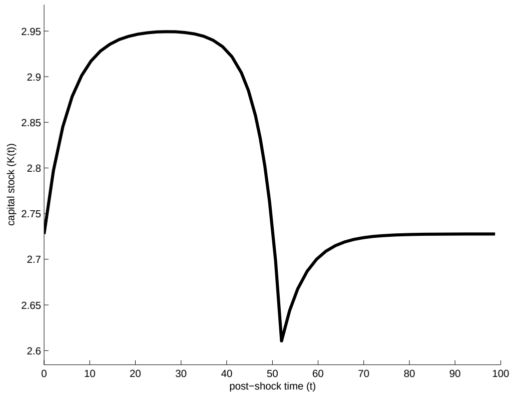
An interesting feature of the transition path for the capital stock is its non-monotonicity. More importantly, at least during the early transition phase capital is *crowded in* as a result of the tax-financed abatement shock, a phenomenon which complicates environmental policy because it leads to an increase in the flow of dirt. So whereas capital decumulation helps environmental policy in the core model during the early phases, it hinders policy in the ELS model.

To investigate the quantitative significance of the negative feedback effect via the capital stock we calibrate and simulate the ELS model. For ρ, δ, ε_L, κ, and γ we use the same parameters as before – see Table 2. We choose ε_C such that the steady-state intertemporal labour supply elasticity, $(1 - \hat{L})/\hat{L}$, is equal to two (i.e. 8 hours of work in a 24 hour day). This gives $\varepsilon_C = 0.3663$ and $\hat{L} = 1/3$. Finally, we choose Ω_0 such that steady-state output is equal to unity, $\hat{Y} = 1$. This gives $\Omega_0 = 1.5969$.

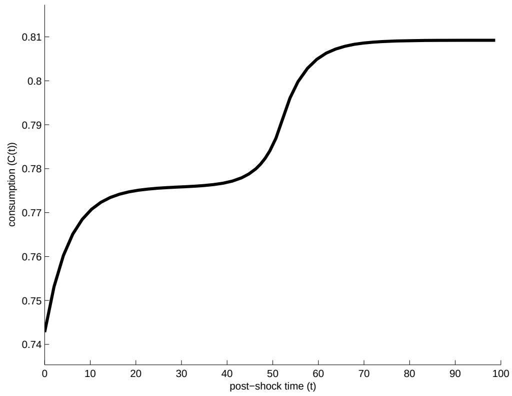
Figure 6 illustrates the transition paths for a shock featuring $G = 0.1$. The minimum feasible duration for this shock is $t_E = 52$ years! Despite the fact that the core model and the ELS model are initially in an identical steady state (as far as output, consumption, the capital stock, and pollution are concerned), the abatement policy must be maintained for a much longer period than in the core model when labour supply effects are taken into account. In a quantitative sense, therefore, the capital-feedback effect is significant. As is shown in panel (a), the capital stock features large fluctuations, reaching a maximum of about 2.95 for much of the early transition phase, and a minimum of $K(t_E) = 2.6106$. Similarly, output reaches a maximum of about 1.08 reflecting the large spending multiplier that exists in the ELS model (see also Heijdra, 2009, p. 511). Using the welfare measure developed above in (15) and adapting it to the endogenous choice of leisure, we find that for the policy combination $(G, t_E) = (0.1, 52)$, $EV(0)$ is 7.8%. Recall that in the core model there was a welfare gain of

Figure 6: Dynamic effects of government abatement: Endogenous labour supply

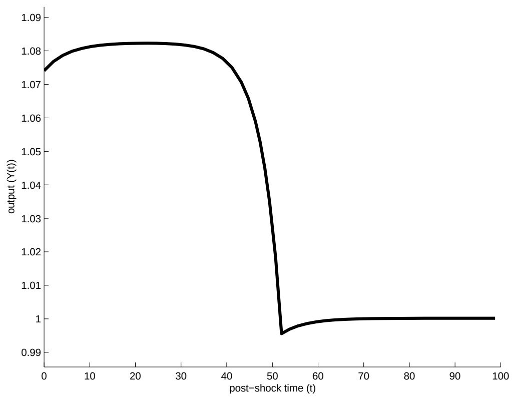
(a) capital stock ($K(t)$)



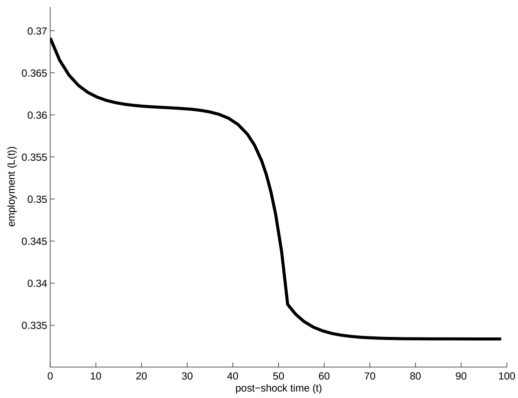
(b) consumption ($C(t)$)



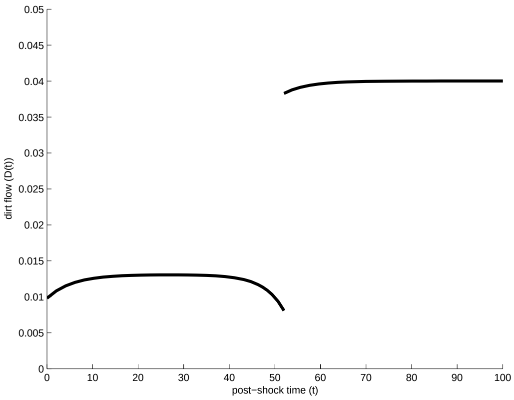
(c) output ($Y(t)$)



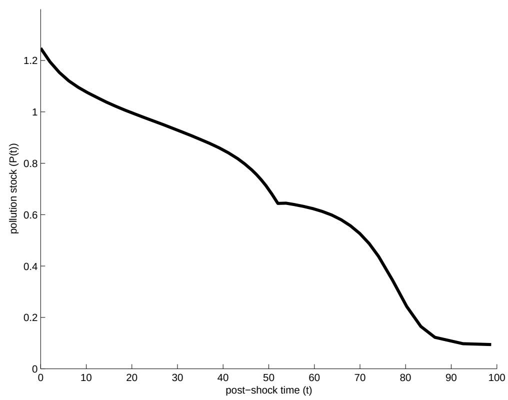
(d) employment ($L(t)$)



(e) dirt flow ($D(t)$)



(f) pollution stock ($P(t)$)



26.3% for the policy combination $(G, t_E) = (0.1, 41)$, so the labour supply effect reduces the welfare gains of moving from the high- to the low-pollution equilibrium.

4.2 Finite lives

In the second extension we take labour supply to be exogenous but assume that individuals face an age-independent probability of death, μ. In particular, we use the Blanchard (1985) model of consumer behaviour. At time t, expected remaining lifetime utility of an individual born at time v $(v \leq t)$ is given by:

$$\mathbb{E}\Lambda (v, t) \equiv \int_t^{\infty} \left[\ln C(v, \tau) + \varepsilon_E \ln \left[\bar{E} - P(\tau) \right] \right] \cdot e^{(\rho+\mu)(t-\tau)} d\tau, \tag{20}$$

where $C(v, \tau)$ is consumption, ρ is the pure rate of time preference, and μ is the instantaneous mortality rate. With a positive mortality rate, future felicity is discounted more heavily than in the representative-agent model because the finitely-lived agent simply may not be alive to enjoy felicity in the future – see Yaari (1965). Following Blanchard (1985) and Yaari (1965), we assume that there exist perfect annuities. The actuarially fair annuity rate of interest is equal to $r(\tau) + \mu$ and rational individuals fully annuitize because it expands their choice set. The agent's budget identity is thus given by:

$$\dot{A}(v, \tau) = [r(\tau) + \mu] A(v, \tau) + w(\tau) - C(v, \tau) - T(\tau), \tag{21}$$

where $A(v, \tau)$ is the stock of financial assets at time τ of an agent born at time v. Newborn agents are born without financial assets, i.e. $A(v, v) = 0$.

An agent born of vintage v chooses time profiles for $C(v, \tau)$ and $A(v, \tau)$ which maximize (20) subject to (21) and a solvency requirement. The solution for consumption in the planning period t amounts to:

$$C(v, t) = (\rho + \mu) \cdot [A(v, t) + H(t)], \tag{22}$$

where expected life-time human wealth at that time, $H(t)$, is given by:

$$H(t) \equiv \int_t^{\infty} [w(\tau) - T(\tau)] \cdot e^{-\int_t^{\tau} [r(s)+\mu] ds} d\tau. \tag{23}$$

The optimal time profile for individual consumption is of the same form as (7):

$$\frac{\dot{C}(v, \tau)}{C(v, \tau)} = r(\tau) - \rho, \qquad \tau \geq t \geq v. \tag{24}$$

In (22) the mortality rate features in the propensity to consume out of total wealth because mortality is yet another reason to be impatient. In (23) the mortality rate features because agents use the annuity rate of interest to discount after-tax non-asset income. Importantly, the annuity rate drops out of the individual consumption growth equation (24) because annuities are perfect; a result first demonstrated by Yaari (1965, p. 147).

We assume that the crude birth rate is equal to the mortality rate so that (i) the aggregate population is constant and can be normalized to unity, and (ii) the relative population size of cohort v at time t ($> v$) is given by $\mu e^{-\mu(t-v)}$. Aggregate variables can thus be calculated as the weighted sum of the values for different generations, e.g. $C(t) \equiv \int_{-\infty}^{t} C(v,t)\mu e^{-\mu(t-v)}dv$ is aggregate consumption. By aggregating (24), we arrive at the *aggregate* consumption growth equation:

$$\frac{\dot{C}(t)}{C(t)} = r\left(t\right) - \rho - \mu(\rho+\mu)\cdot\frac{A(t)}{C(t)} = \frac{\dot{C}(v,t)}{C(v,t)} - \mu\cdot\frac{C(t)-C(t,t)}{C(t)}. \tag{25}$$

Equation (25) has the same form as the Euler equation for individual households (24), except for a correction term capturing the wealth redistribution caused by the turnover of generations. Optimal individual consumption *growth* is the same for all generations since they face the same rate of interest. But the consumption *level* of old generations is higher than that of young generations, reflecting the larger stock of financial assets owned by old generations. Because existing generations are continually being replaced by newborns, who are born without financial wealth, aggregate consumption growth falls short of individual consumption growth. The correction term appearing in (25) thus represents the difference in average consumption, $C(t)$, and consumption by newborns, $C(t,t)$.

A well-known property of the Blanchard-Yaari model concerns the non-neutrality of public debt. In the presence of public debt, the capital market equilibrium condition is given by $A\left(t\right) = K\left(t\right) + B\left(t\right)$ so that consumption growth equation is given by:

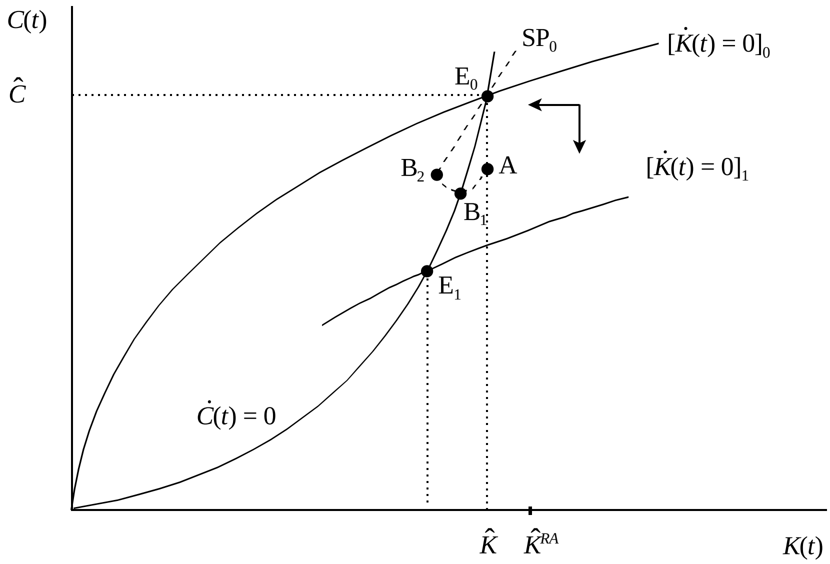
$$\frac{\dot{C}(t)}{C(t)} = r\left(t\right) - \rho - \mu(\rho+\mu)\cdot\frac{K(t)+B(t)}{C(t)}. \tag{26}$$

Equation (26) replaces replaces (T1.1) in Table 1.

The nonlinear system of differential equations characterizing the economic system can now be written as:

$$\begin{aligned}
\dot{C}\left(t\right) &= \left[(1-\varepsilon_L)\Omega_0 K(t)^{-\varepsilon_L} - \rho - \delta\right] C\left(t\right) - \mu(\rho+\mu)\left[K(t)+B(t)\right], \\
\dot{K}(t) &= \Omega_0 K(t)^{1-\varepsilon_L} - C(t) - G\left(t\right) - \delta K(t).
\end{aligned}$$

Figure 7: Consumption-capital dynamics with overlapping generations



The phase diagram for the overlapping generations (OG) model is given in Figure 7. We assume that public debt is equal to zero ($B(t) = 0$). In contrast to the core model, the OG model features an upward sloping $\dot{C}(t) = 0$ line. For points above (below) the $\dot{C}(t) = 0$ line, the generational turnover term is relatively small (large), the interest rate exceeds (falls short of) the rate of time preference, and consumption rises (falls) over time. The dynamics for the capital stock is exactly the same as in the core model, and the OG model features a unique steady state at point E_0.

A temporary abatement policy of the form given in (12) financed with lump-sum taxes ($T(t) = G(t)$) gives rise to the adjustment path $\text{E}_0\text{AB}_1\text{B}_2\text{E}_0$ in Figure 7. The intuition is as follows. The shock shifts the capital equilibrium locus to $[\dot{K}(t) = 0]_1$. At impact the tax increase reduces human wealth for all agents, $H(v, 0)$, which prompts them to reduce consumption. This is the jump from E_0 to A directly below it. At point A the interest rate is unchanged, but the generational turnover term is increased so aggregate consumption gradually falls. Aggregate consumption, however, is too high to maintain the initial capital stock $\hat{K}$ so capital starts to decumulate. At some time t such that $0 < t < t_E$ the economy arrives at point B_1, after which aggregate consumption starts to increase even though the capital stock continues to fall. At time $t = t_E$, the economy arrives at point B_2, the abatement policy is terminated, and the capital equilibrium locus shifts back to $[\dot{K}(t) = 0]_0$. From then on the dynamic forces are such as to increase both consumption and capital as the economy moves from B_2 to E_0.

The solid lines in Figure 8 depict the dynamic adjustment paths for the different variables under lump-sum tax financing. As the comparison between Figures 3 and 8 reveals, the transition paths for the core model and the OG model are qualitatively very similar. The abatement shock cause temporary crowding out of capital, a feature which aids the environmental cleanup. To glean the quantitative significance of this effect we once again calibrate and simulate the model. This time we use the same parameters for δ, ε_L, Ω_0, κ, and γ as in the core model – see Table 2. We assume that the birth/mortality rate is $\mu = 0.015$ and choose ρ such that the initial steady-state interest rate is the same as in the core model ($\hat{r} = 0.04$). This gives $\rho = 0.0374$. The initial steady state is "observationally equivalent" to the steady state for the core model, i.e. $\hat{Y} = 1$, $\hat{K} = 2.7273$, $\hat{I} = 0.1909$, and $\hat{C} = 0.8091$. At these parameter values, a shock of $G = 0.1$ succeeds in moving the ecology to the clean equilibrium provided $t_E = 38$ years. Hence, in the OG model the capital stock effect exerts a stronger effect on the environmental cleanup than in the core model.

Because pure lump-sum tax financing is used ($T(t) = G(t)$ for all t), the distribution of costs and benefits of environmental policy is very uneven in the OG model. In the scenario discussed so far, the costs are borne by all pre-shock generations (whose $v < 0$) and all post-shock generations born before the policy is terminated (for whom $0 \leq v < t_E$). Generations born after t_E do not have to pay any taxes but benefit fully from the environmental cleanup, more so the later they are born. Generalizing the welfare measure given in (15) to the BY model (see Appendix B), we find for shock-time newborns $EV(0,0) = 10.8\%$ and for steady-state newborns $EV(\infty,\infty) = 33.5\%$.

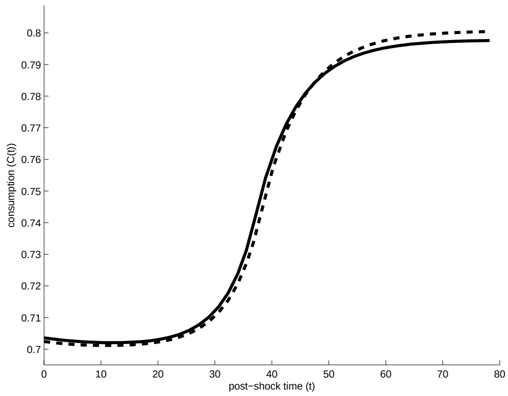
The uneven distribution of costs and benefits can be repaired by the policy maker because public debt is non-neutral in the OG model. Both the economic allocation and the intergenerational welfare implications of the abatement policy depend on the way the government balances its budget. The periodic budget identity can be written as follows:

$$\dot{B}(\tau) = r(\tau)B(\tau) + G(\tau) - T(\tau). \tag{27}$$

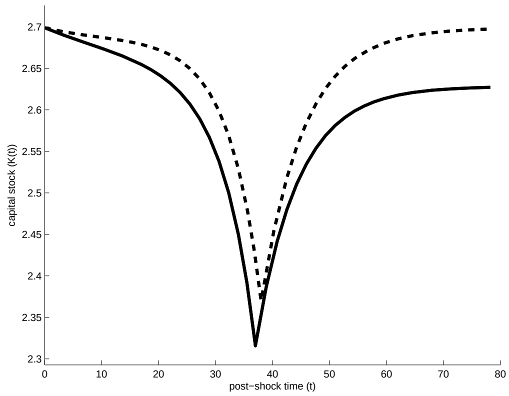
The government can finance its spending by either issuing more debt ($\dot{B}(\tau) \equiv dB(\tau)/d\tau > 0$) or by levying lump-sum taxes ($T(\tau)$). The No Ponzi Game condition ensures that the

Figure 8: Dynamic effects of government abatement: Overlapping generations

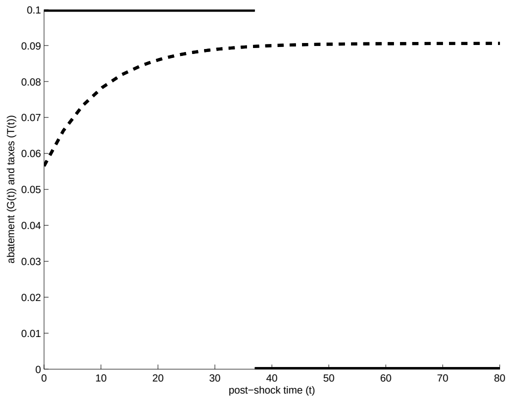
(a) consumption ($C(t)$)



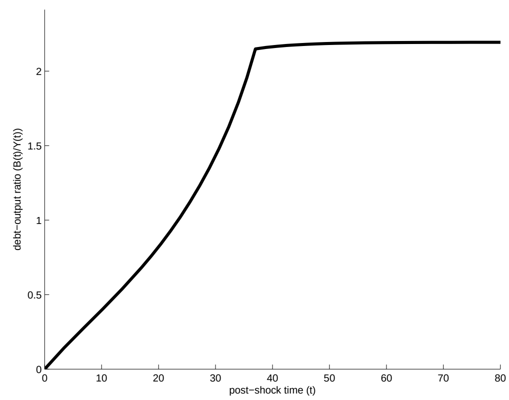
(b) capital stock ($K(t)$)



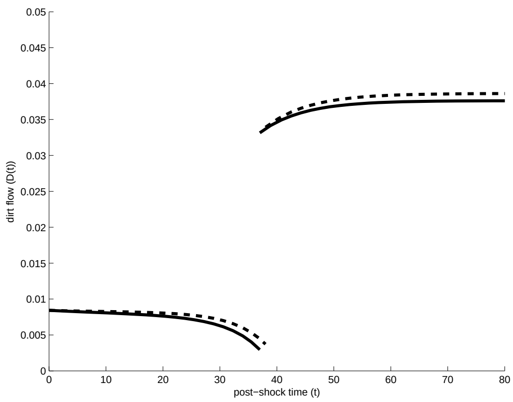
(c) government abatement/taxes ($G(t$ and $T(t)$)



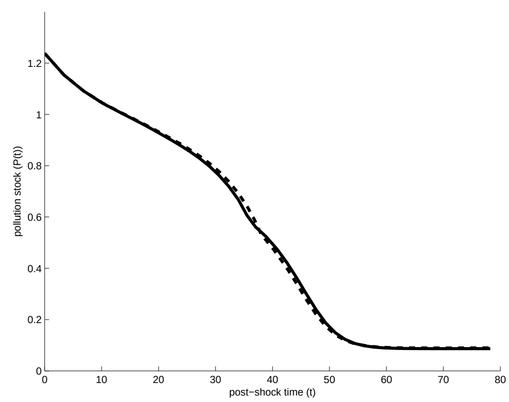
(d) debt-output ratio ($B(t)/Y(t)$)



(e) dirt flow ($D(t)$)



(f) pollution stock ($P(t)$)



government remains solvent:

$$\lim_{\tau \to \infty} B(\tau) \cdot e^{-\int_t^\tau r(s)ds} = 0. \tag{28}$$

The government's intertemporal budget constraint is derived by integrating (27) and using the solvency condition (28):

$$B(t) = \int_t^\infty [T(\tau) - G(\tau)] \cdot e^{-\int_t^\tau r(s)ds} d\tau. \tag{29}$$

If there is a positive debt at time t, it must be covered in a present-value sense by future primary surpluses.

The bond policy that we consider takes the following form. We assume that debt is zero initially, i.e. $B(0) = 0$, and postulate a parametric tax path of the form $T(t) = T_0 + T_1 \left[1 - e^{-\xi t}\right]$ for $t \geq 0$ and $\xi > 0$.[17] Here $T(0) = T_0$ stands for the initial tax, $T(\infty) = T_0 + T_1$ is the long-run tax, and ξ is the speed of debt stabilization. Using the tax function as well as (12) in (29) we obtain the government solvency condition in terms of parameters:

$$\int_0^\infty \left[T_0 + T_1 \left[1 - e^{-\xi t}\right]\right] \cdot e^{-\int_t^\tau r(s)ds} d\tau = G \cdot \int_0^{t_E} e^{-\int_t^\tau r(s)ds} d\tau. \tag{30}$$

An abatement *cum* debt policy consists of the vector (G, t_E, T_0, T_1, ξ) such that (i) t_E is as small as feasible for the given shock, and (ii) equation (30) is satisfied by suitable choice of T_0 and/or T_1.

The dashed lines in Figure 8 illustrate the transition paths for a shock featuring $G = 0.1$, $t_E = 37$, $\xi = 0.1$, $T_0 = 0.0564$, and $T_1 = 0.0342$. By making all generations pay for the abatement policy, the tax path lies in between the two branches of the path for $G(t)$ – compare the solid and dashed lines in panel (c). As is illustrated in panel (d), public debt accumulates at a rapid pace during the active phase of the abatement policy ($0 \leq t < t_E$) but is stabilized rapidly thereafter. The long-run debt-output ratio is equal to $(\hat{B}/\hat{Y})_1 = 2.1721$ whilst the debt-service ratio settles at $(\hat{r}\hat{B}/\hat{Y})_1 = 0.0913$.

The long-run public debt burden causes a reduction in the steady-state capital stock, from $\hat{K}_0 = 2.7273$ to $\hat{K}_1 = 2.6566$, and an increase in the steady-state interest rate, from $\hat{r}_0 = 0.04$ to $r_1 = 0.042$. Interestingly, the ecology settles at a cleaner equilibrium than in the core model because capital is crowded out also in the long-run, i.e. $\hat{P}_1 = 0.0906$ ($< \hat{P}_G = 0.0936$). So even

[17] This form represents an analytically tractable way to model intergenerational redistribution. It is quite standard in the literature – see, e.g., Bovenberg and Heijdra (1998, 2002).

though future newborns are confronted with a higher tax bill and lower wages than they would have been in the absence of debt policy, they do inherit a cleaner environment as a result of this financing method. Using the welfare measures explained in Appendix B, we find for shock-time newborns $EV(0,0) = 13.5\%$ and for steady-state newborns $EV(\infty,\infty) = 11.4\%$. So the debt policy has almost eliminated the difference in welfare gains between the two types of newborns.

5 Conclusions

In this paper we have studied the environmental and macroeconomic effects of temporary public abatement activities in the presence of multiple stable steady-state ecological equilibria. Under shallow-lake dynamics (SLD) the isocline for the stock of pollution possesses two stable branches, featuring, respectively, a high and a low stock of pollution. We focus on the case in which the ecology is initially located on the upper (high-pollution) branch of the pollution isocline and assume that the resilience parameter is such as to render the ecological equilibrium reversibly hysteretic. We show that a suitably designed temporary abatement policy can be used to steer the environment from the high- to the low-pollution equilibrium. Despite the fact that consumption is lower than its pre-shock level during transition, these utility costs are more than compensated for by the increased quality of the environment. In all models considered in this paper, a "cold turkey" abatement policy provides the largest welfare gain, i.e. the highest feasible shock should be conducted for the shortest possible amount of time.

In contrast to much of the existing literature on SLD, we embed the ecological model in a micro-founded macroeconomic model. Since public abatement must be paid for by taxes levied on individuals, the environmental policy induces behavioral responses in that households change their consumption and savings plans. In the core model we demonstrate the qualitative and quantitative importance of this *capital feedback effect* by employing one of the workhorse models of modern macroeconomics – the Ramsey model featuring infinitely lived representative agents and exogenous labour supply. In this setting, the capital feedback effect "simplifies" environmental policy somewhat (compared to the case with a fixed capital stock) because households reduce their saving (and decumulate capital) during the early phase of the abatement policy thus reducing the inflow of dirt even further.

Interestingly, the particular model used to characterize the macroeconomic system has non-trivial implications for the ease with which a successful abatement policy can be conducted. This is because both the sign and magnitude of the capital feedback effect depends critically on features of the macroeconomic model. Compared to the core model with long-lived agents and exogenous labour supply, the model with endogenous labour supply makes environmental policy harder whilst the model with finite lives makes it easier. In the overlapping generations model, bond policy can be used to smooth the benefits from the environmental cleanup across generations.

In this paper we have deliberately restricted attention to a fairly simple environmental policy consisting of a temporary time-invariant increase in public abatement. This has enabled us to zoom in on the core mechanisms that are at work in a macroeconomic environmental model featuring SLD. In future research we wish to use the models constructed here to study optimal environmental policy in the presence of SLD. We anticipate that the benevolent social planner will not only (a) impose corrective taxation in the form of a time-varying Pigouvian dirt tax but also (b) engage in time-varying abatement activities in order to decentralize the first-best social optimum. An interesting by-product of such a social planning exercise is that it allows us to quantify to what extent the restrictive policy studied in the current paper falls short of the first-best social optimum.

Appendix

This appendix provides some technical details on the isocline for the stock of pollution (Appendix A) and on the welfare measure used for the BY model of Section 4.2 (Appendix B). All technical details regarding the economic system can be found in Heijdra and Heijnen (2012).

A Ecological dynamics

We assume that $\frac{1}{2} < \pi < \frac{3\sqrt{3}}{8}$, thus ensuring that nature does not feature irreversible equilibria (because $\pi > \frac{1}{2}$) and that the P isocline is S-shaped – see Mäler *et al.* (2003, p. 606). To see why this is the case, consider Figure 1 depicting the phase diagram for the stock of pollution. The $\dot{P}(t) = 0$ line is obtained from (2) and represents all combinations of $P(t)$ and $D(t)$ such that the stock of pollution is constant over time. With $\frac{1}{2} < \pi < \frac{3\sqrt{3}}{8}$ the $\dot{P}(t) = 0$ line is S-shaped with strictly positive threshold points at $D(t) = D_L$ and $D(t) = D_U$.

To find these threshold points, we set $\dot{P}(t) = 0$ in (2) and solve for D in terms of P:

$$[D =]\ \Phi(P) \equiv \pi P - \frac{P^2}{P^2+1}.$$

The thresholds are the local extrema of $\Phi(P)$, i.e. the P values for which $\Phi'(P) = 0$. The P-coordinates of points B and C in Figure 1 are thus the solutions to:

$$\Phi'(P) = 2\left[\frac{\pi}{2} - \frac{P}{(P^2+1)^2}\right] = 0$$

The zeroes of this function are associated with one local minimum dirt flow and one local maximum dirt flow. These dirt flow thresholds are themselves obtained by substituting these coordinates into (2) and imposing $\dot{P}(t) = 0$.

For $\pi = 1/2$, we find $P_L = 1$, $D_L = \Phi(P_L) = 0$, and $\Phi'(P_L) = 0$, i.e. point C is on the vertical axis and the ecology features *irreversible* equilibria, namely all steady-state points located on the upper branch of the $\dot{P}(t) = 0$ line. Since the flow of dirt cannot become negative, there is no way to get to the lower branch from there.

For $\pi < \frac{3\sqrt{3}}{8}$ we find that the P isocline is no longer S-shaped. Indeed, $\Phi(P)$ is increasing in P with an inflexion point at $P_I = \frac{\sqrt{3}}{3}$. At that point we have that $\Phi'(P_I) = \Phi''(P_I) = 0$.

B Equivalent-variation welfare measure for the BY model

We compute the equivalent variation welfare measure for the BY model as follows. First, we note that welfare for the initial steady-state newborn at point F in Figure 1 is given by:

$$\begin{aligned}
\mathbb{E}\Lambda_F(0,0) &\equiv \int_0^\infty \left[\ln \hat{C}(0,t) + \varepsilon_E \ln\left[\bar{E} - \hat{P}_B\right]\right] \cdot e^{-(\rho+\mu)t} dt, \\
\hat{C}(0,t) &= \hat{C}(0,0) \cdot e^{\hat{r}t},
\end{aligned}$$

where $\hat{C}(0,0) = (\rho+\mu)\hat{H}(0)$ and $\hat{H}(0) = \frac{\hat{w}}{\hat{r}+\mu}$. It follows that:

$$\mathbb{E}\Lambda_F(0,0) \equiv \hat{r} \cdot \int_0^\infty t e^{-(\rho+\mu)t} dt + \frac{1}{\rho+\mu} \frac{\ln \hat{C}(0,0) + \varepsilon_E \ln\left[\bar{E} - \hat{P}_B\right]}{\rho+\mu}. \tag{C.1}$$

With the abatement policy, welfare for a shock-time newborn is given by:

$$\mathbb{E}\Lambda_A(0,0) \equiv \int_0^\infty \left[\ln C(0,t) + \varepsilon_E \ln\left[\bar{E} - P(t)\right]\right] \cdot e^{-(\rho+\mu)t} dt, \tag{C.2}$$

where we must use the actual paths for $C(0,t)$ and $P(t)$ to evaluate this integral. We compute $C'(0,0)$ such that $\mathbb{E}\Lambda_F(0,0)$ evaluated for this consumption level equals $\mathbb{E}\Lambda_A(0,0)$.

The welfare measure is the given by:

$$EV(0,0) \equiv 100 \cdot \frac{C'(0,0) - \hat{C}(0,0)}{\hat{C}(0,0)}. \tag{C.3}$$

For the steady-state newborns we find:

$$\mathbb{E}\Lambda_A(\infty,\infty) \equiv \hat{r}_n \cdot \int_0^\infty te^{-(\rho+\mu)t}dt + \frac{1}{\rho+\mu}\frac{\ln \hat{C}_n(0,0) + \varepsilon_E \ln\left[\bar{E} - \hat{P}_n\right]}{\rho+\mu}, \tag{C.4}$$

where $\hat{C}_n(\infty,\infty) = (\rho+\mu)\frac{\hat{w}_n - \hat{T}_n}{\hat{r}_n+\mu}$ and the subscript n designates the new steady state. (Of course, without bond policy, $\hat{r}_n = \hat{r}$, $\hat{w}_n = \hat{w}$, $\hat{T}_n = 0$, $\hat{C}_n(0,\tau) = \hat{C}(0,\tau)$, and $\hat{P}_n = \hat{P}_G$). The welfare measure $EV(\infty,\infty)$ is obtained from (C.3) by finding $C'(0,0)$ which, upon substitution in (C.1), results in $\mathbb{E}\Lambda_F(0,0) = \mathbb{E}\Lambda_A(\infty,\infty)$.

References

Arrow, K. J., Bolin, B., Costanza, R., Dasgupta, P., Folke, C., Holling, C., Jansson, B.-O., Levin, S., Mäler, K.-G., Perrings, C., and Pimentel, D. (1995). Economic growth, carrying capacity, and the environment. *Science*, 268:520–521.

Biggs, R., Carpenter, S. R., and Brock, W. A. (2009). Turning back from the brink: Detecting an impending regime shift in time to avert it. *Proceedings of the National Academy of Sciences*, 106:826–831.

Blanchard, O.-J. (1985). Debts, deficits, and finite horizons. *Journal of Political Economy*, 93:223–247.

Bovenberg, A. L. and Heijdra, B. J. (1998). Environmental tax policy and intergenerational distribution. *Journal of Public Economics*, 67:1–24.

Bovenberg, A. L. and Heijdra, B. J. (2002). Environmental abatement and intergenerational distribution. *Environmental and Resource Economics*, 23:45–84.

Brock, W. A. and Starrett, D. (2003). Managing systems with non-convex positive feedback. *Environmental and Resource Economics*, 26:575–602.

Crépin, A.-S. (2003). Multiple species boreal forests: What Faustmann missed. *Environmental and Resource Economics*, 26:625–646.

Crépin, A.-S. (2007). Using fast and slow processes to manage resources with thresholds. *Environmental and Resource Economics*, 36:191–213.

Heijdra, B. J. (2009). *Foundations of Modern Macroeconomics*. Oxford University Press, Oxford, second edition.

Heijdra, B. J. and Heijnen, P. (2012). Environmental abatement policy and the macroeconomy in the presence of ecological thesholds: Mathematical appendix. Faculty of Economics and Business, University of Groningen, July.

Heijnen, P. and Wagener, F. (2009). Managing the environment and the economy in the presence of hysteresis and irreversibility. Mimeo, CeNDEF, University of Amsterdam, (March).

Janssen, M. A., Anderies, J. M., and Walker, B. H. (2004). Robust strategies for managing rangelands with multiple stable attractors. *Journal of Environmental Economics and Management*, 47:140–162.

Levin, S. A. and Pacala, S. W. (2003). Ecosystem dynamics. In Mäler, K.-G. and Vincent, J. R., editors, *Handbook of Environmental Economics*, volume 1. North-Holland, Amsterdam.

Mäler, K.-G. and Li, C.-Z. (2010). Measuring sustainability under regime shift uncertainty: A resilience pricing approach. *Environmental and Development Economics*, 15:707–719.

Mäler, K.-G., Xepapadeas, A., and de Zeeuw, A. (2003). The economics of shallow lakes. *Environmental and Resource Economics*, 26:603–624.

Muradian, R. (2001). Ecological thresholds: A survey. *Ecological Economics*, 38:7–24.

Prieur, F. (2009). The environmental Kuznets curve in a world of irreversibility. *Economic Theory*, 40:57–90.

Ranjan, R. and Shortle, J. (2007). The environmental Kutnets curve when the environment exhibits hysteresis. *Ecological Economics*, 64:204–215.

Scheffer, M., Carpenter, S., Foley, J. A., Folke, C., and Walker, B. (2001). Catastrophic shifts in ecosystems. *Nature*, 413:591–596.

Wagener, F. (2009). Shallow lake economics run deep. Discussion Paper TI 2009-033/1, Tinbergen Institute.

Wirl, F. (2004). Sustainable growth, renewable resources and pollution: Thresholds and cycles. *Journal of Economic Dynamics and Control*, 28:1149–1157.

Xepapadeas, A. (2005). Economic growth and the environment. In Mäler, K.-G. and Vincent, J. R., editors, *Handbook of Environmental Economics*, volume 3. North-Holland, Amsterdam.

Yaari, M. E. (1965). Uncertain lifetime, life insurance, and the theory of the consumer. *Review of Economic Studies*, 32:137–150.