The Macroeconomic Dynamics of Demographic Shocks

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September 2004

Abstract

The paper employs an extended Yaari-Blanchard model of overlapping generations to study how the macroeconomy is affected over time by various demographic changes. It is shown that a proportional decline in fertility and death rates has qualitatively similar effects to capital income subsidies; both per capita savings and per capita consumption increase in the new steady state. A drop in the birth rate, while keeping the death rate constant, reduces per capita savings, but increases per capita consumption if the generational turnover effect is dominated by the intertemporal labor supply effect. If the generational turnover effect is sufficiently strong, however, a decline in the birth rate may, contrary to standard results, give rise to an increase in per capita savings. Finally, a fertility rate reduction which leaves unaffected the rate of generational turnover is shown to have effects qualitatively similar to those of a fall in public consumption. Both per capita savings and per capita output decline, but per capita consumption rises.

JEL codes: E12, E63, L16.

Keywords: fertility rate, intertemporal labor supply, overlapping generations, Blanchard model, demographic shocks, transition effects.

1   Introduction

Population aging and its macroeconomic effects have emerged over the last decade as a key issue on the policy agendas of most industrialized countries. During the post-war period, the population share of elderly people has increased dramatically. Following the post-war "baby boom"—during which population growth rates temporarily accelerated—fertility rates have

declined substantially, commonly known as the "baby bust." At the same time, mortality rates have decreased in most industrialized nations, owing to healthier lifestyles and medical advances.[1] Both trends give rise to population aging.

The effects of the post-war demographic transition on old-age dependency ratios (that is, the ratio of the population aged 65 years and older to the population aged 15-64 years) for selected OECD countries are presented in Table 1. The evolution of the old-age dependency ratio shows pronounced population aging for all countries, where it is apparent that populations in European countries and Japan are older than elsewhere. Japan stands out as having an old-age dependency ratio of only 10 percent in 1970, and a projected ratio of more than 70 percent in 2050, a large increase unparalleled across OECD countries. The youth dependency ratio[2] is projected to fall by about one third between 1970 and 2050, reflecting a steady decline in fertility rates, although there are significant differences among OECD countries. Canada and New Zealand experience a decline in the youth dependency ratio of about 26 percentage points, against a fall of only 9 percentage points in Japan.

Demographic changes have profound economic effects, which may span many generations. Particularly, the impending retirement of the baby boom generation is raising a great deal of concern. If a large fraction of the population retires (or passes away), society is expected to save less, leading to a lower rate of capital accumulation and lower living standards.[3] The aim of the paper is to analyze the macroeconomic effects of various demographic changes in a model of a closed economy. Various questions arise. How do changes in the population growth rate affect aggregate savings, consumption, employment and output in the new steady state? Does a drop in fertility, reducing potential labor supply, drive up wages? How are the relevant macroeconomic variables affected along the transition path?

The informal literature on population aging, for example, Group of Ten (1998) and Mc-Morrow and Röger (2003), is voluminous. Many formal contributions employ calibrated life-cycle models—in the tradition of Samuelson (1958), Diamond (1965)[4] and Auerbach and Kotlikoff (1987)—to study numerically the effects of population aging.[5] All of these studies assume exogenous population dynamics. Some authors employing the life-cycle approach—for example, Elmendorf and Sheiner (2000)—have examined the steady-state effects of demo-

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[1]Birth rates (per 100 of the population) in the United States came down from 2.43 in 1950 to 1.45 in 2000. The drop in death rates (per 100 of the population) was less spectacular, declining from 0.95 in 1950 to 0.83 in 2000 (United Nations, 2003).

[2]The youth dependency ratio is defined as the ratio of the population aged 0-14 years to the population aged 15-64 years.

[3]Bloom, Canning and Graham (2003) provide econometric evidence on the positive relationship between life expectancy and the savings rate.

[4]Diamond (1965) assumes that individuals live for two discrete time periods, in which they work and save in the first period and consume out of savings in the second period.

[5]Auerbach and others (1989), Auerbach, Cai and Kotlikoff (1991), Ríos-Rull (2001), and Brooks (2002) employ a Diamond-Samuelson overlapping generations model, which is generalized to many periods. Cutler and others (1990) use a representative-agent model.

Table 1. Dependency Ratios for Selected OECD Countries, 1970-2050 (In Percent)

Country	Youth Dependency Ratios[1]				Old-Age Dependency Ratios[2]			
	1970	2000	2025[3]	2050[3]	1970	2000	2025[3]	2050[3]
Belgium	37	27	25	26	21	26	37	47
Canada	49	28	23	26	13	18	33	44
Denmark	36	27	25	26	19	22	35	42
France	40	29	27	28	21	24	36	46
Germany	37	23	22	27	22	24	38	49
Italy	38	21	18	25	17	27	40	65
Japan	35	21	20	26	10	25	50	72
Netherlands	44	27	25	26	16	20	33	42
New Zealand	53	35	29	27	14	18	29	38
Spain	45	21	18	26	16	24	35	68
Switzerland	37	25	22	24	17	24	45	55
United Kingdom	39	29	25	26	21	24	31	38
United States	46	33	32	29	16	19	28	32

Source: United Nations (2003), World Population Prospects Database. (1) The ratio of the population aged 0-14 to the population aged 15-64. (2) The ratio of the population aged 65 years or over to the population aged 15-64. (3) Medium variant projections.

graphic changes analytically, but have not studied the entire transition path.[6] Our analysis follows a different take. It draws on the overlapping generations framework of Yaari (1965) and Blanchard (1985), which assumes that all agents face a constant probability of death in a world of zero population growth, as well as Buiter (1988), who introduces (exogenous) population growth in the Yaari-Blanchard model.[7] In particular, we extend Buiter's model to include endogenous (intertemporal) labor supply, which allows us to study the labor market effects of various demographic shocks. By explicitly modeling the labor market we can disentangle labor supply responses—and thus the participation decision—from labor demand conditions, thus allowing for a meaningful analysis of intertemporal wage profiles. Moreover, we can allow for voluntary retirement of households by incorporating a wealth effect in labor supply that makes old agents—having accumulated much wealth—work less hours. Our framework is partly related to the model of Heijdra and Ligthart (2000, 2002), which has introduced endogenous labor supply in the Yaari-Blanchard framework with a view to study taxation issues.[8]

A simple graphical apparatus is developed to provide an intuitive account of the long-run and dynamic effects of various demographic changes. The model is versatile because it encompasses results of various seminal works—that of Blanchard (1985), Buiter (1988), and Weil (1989)—by varying the assumptions made on demography and the intertemporal substitution elasticity in labor supply. Moreover, it can be employed to get insight into and extend the results from the population aging literature. Our approach differs from previous theoretical analyses on population dynamics by being able to trace out impulse-responses at business cycle frequencies.[9] In the Samuelson-Diamond framework a typical period lasts 35 years, implying that transitional dynamics can only be studied at low frequencies. Knowledge of the entire transition path is of importance to policy analysis, however, because the short-run effects of demographics shock differ markedly from their long-run effects.

Three demographic scenarios are analyzed analytically. The first shock concerns an unexpected and permanent decrease in the (exogenous) fertility rate (that is, a pure "baby bust"). It is shown that the optimal savings response to declining fertility entails either a decrease or an increase in per capita savings depending on the assumptions made on the elasticity of labor supply and the generational turnover effect, thereby generalizing the results of Elmendorf and Sheiner (2000). Second, we study a proportionate fall in the fertility and death rates so as

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[6]Momota and Futagami (2000), however, study demographic transition in a small open economy using endogenous fertility theory (see Becker and Barro (1988)).

[7]Weil (1988) also allows for population growth, but he assumes infinitely-lived overlapping generations and thus differs from the uncertain lifetimes approach of Yaari-Blanchard.

[8]Bovenberg (1993) has employed the Yaari-Blanchard-Buiter framework to study the effects of a permanent rise in the capital tax in an open economy. Bovenberg and Heijdra (1998) consider a closed economy, but they do not allow for net population growth or endogenous labour supply.

[9]The Laplace transform technique of Judd (1982) is used to solve for the entire transition path of the demographic change.

to yield a stationary population growth rate. Under this scenario, the qualitative results are identical to those of a subsidy on capital; both the per capita capital stock and per capita consumption rise in the new steady state. The final scenario—studying a drop in the fertility rate and a compensating increase in the death rate so as to maintain the rate of generational turnover constant—gives rise to a rise in long-run per capita consumption, although the per capita capital stock falls.

The remainder of the paper is organized as follows. Section 2 sets out the Yaari-Blanchard overlapping generations model extended for endogenous labor supply and exogenous population dynamics. Section 3 solves the model graphically and analyzes the dynamics around the long-run equilibrium. Section 4 employs the graphical framework of Section 3 to qualitatively study various demographic shocks. Section 5 concludes.

2 A Model of Overlapping Generations

2.1 Individual Households and Demographics

As in Blanchard (1985), individual households face an age-invariant probability of death ($\beta \geq 0$). Each household has a time endowment of unity, which is allocated optimally over labor supply and leisure. The utility functional at time t of the representative agent born at time v is denoted by $\Lambda(v,t)$:

$$\Lambda(v,t) \equiv \int_t^\infty \left[\varepsilon_C \log \bar{c}(v,\tau) + (1-\varepsilon_C) \log \left[1 - \bar{l}(v,\tau)\right]\right] e^{(\alpha+\beta)(t-\tau)} d\tau, \tag{1}$$

where $\bar{c}(v,t)$ and $\bar{l}(v,t)$ are, respectively, private consumption and labor supply in period t by an agent born in period v, α is the pure rate of time preference ($\alpha > 0$) that applies across generations and ε_C is the share of consumption in utility.[10] The logarithmic felicity function implies that the intertemporal substitution elasticity for goods consumption is unity and for labor supply is $\left[1 - \bar{l}(v,t)\right]/\bar{l}(v,t)$. The representative agent's dynamic budget identity can be expressed as:

$$\dot{\bar{a}}(v,t) = [r(t) + \beta]\,\bar{a}(v,t) + w(t)\bar{l}(t) - \bar{z}(t) - \bar{c}(v,t), \tag{2}$$

where $\dot{\bar{a}}(v,t) \equiv d\bar{a}(v,t)/dt$, $\bar{a}(v,t)$ are real financial assets, $r(t)$ is the real rate of interest, $w(t)$ is the real wage rate (assumed age-independent for convenience), and $\bar{z}(t)$ are real net lump-sum taxes. The return on financial assets exceeds the rate of interest because, with life-time uncertainty and in the absence of bequest motives, agents conclude actuarially fair contracts with life insurance companies.[11]

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[10]We use the notation introduced by Buiter (1988) by letting lowercase barred variables denote values at the individual household level.

[11]In particular, agents receive an annuity payment from the insurance company proportional to their financial wealth ($\beta\bar{a}(v,t)$) in exchange for transferring their entire estate to the insurance company upon death. Since the contracts are actuarially fair, the annuity rate equals the death rate β.

The individual household chooses time profiles for $\bar{c}(v,t)$ and $\bar{l}(v,t)$ in order to maximize $\Lambda(v,t)$ subject to the budget identity (2) and a No-Ponzi-Game solvency condition, $\lim_{\tau\to\infty} \bar{a}(v,\tau) \exp[-\int_t^\tau [r(s)+\beta]\,ds] = 0$. The optimal solutions for private consumption and labor supply on the interval $t \in [0,\infty)$ are fully characterized by:[12]

$$\bar{c}(v,t) = \varepsilon_C (\alpha + \beta) \left[\bar{a}(v,t) + \bar{h}(t)\right], \tag{3}$$

$$1 - \bar{l}(v,t) = \frac{(1-\varepsilon_C)\bar{c}(v,t)}{\varepsilon_C w(t)}, \tag{4}$$

$$\frac{\dot{\bar{c}}(v,t)}{\bar{c}(v,t)} = r(t) - \alpha, \tag{5}$$

where $\bar{h}(t)$ is expected lifetime human wealth:

$$\bar{h}(t) \equiv \int_t^\infty [w(\tau) - \bar{z}(\tau)] \exp\left[-\int_t^\tau [r(s)+\beta]\,ds\right] d\tau, \tag{6}$$

which is age independent. According to (3) goods consumption in the planning period t is proportional to total wealth, comprising the sum of financial and human wealth. Equation (4) shows that in each period, the marginal rate of substitution between leisure and private consumption is equated to the wage rate. Note that labor supply is a negative function of individual consumption. This wealth effect causes wealthier agents to consume more leisure and thus allows for the proportion of agents opting for "voluntary retirement" to increase with age.[13] As the Euler equation (5) shows, the time profile of individual consumption is governed by the difference between the real interest rate and the rate of pure time preference. Finally, equation (6) implies that human wealth is the after-tax value of the time endowment discounted at the risk-of-death adjusted rate of interest $r + \beta$.

To allow for net population growth or decline, we draw on Buiter (1988) and distinguish between the probability of death β (≥ 0) and the birth rate η (≥ 0). An attractive feature of modelling demographics this way is that it nests two seminal overlapping generations models as special cases. By setting $\eta = \beta$ Blanchard's (1985) model[14] is obtained and by setting $\beta = 0$ Weil's (1989) model is derived.[15] Without international migration, the (net)

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[12]Details of the solution methods and all mathematical derivations can be found in a technical appendix (Heijdra and Ligthart, 2004), which can be downloaded from the authors' website.

[13]Because there is no upper limit on an agent's age, there exist some old (and wealthy) agents who consume more leisure than their unit time endowment allows for (that is, $l(v,t) < 0$ for $v \to -\infty$). Such agents no longer work themselves but rather are net demanders of labor.

[14]Furthermore, the intertemporal labor supply elasticity should equal unity because labor supply is exogenous in Blanchard's model. See the Appendix for further details.

[15]Bovenberg (1993) interprets the special case of $\eta = 0$ and $\beta < 0$ as a Ramsey model with *intra*-dynasty population growth, implying that the Ricardian equivalence proposition still holds. If $\eta > 0$ and $\beta = 0$, there is *extra*-dynasty growth. Due to the birth of disconnected generations (or dynasties) Ricardian equivalence would not hold.

population growth rate, n_N, equals the difference between the birth and death rate:[16]

$$n_N \equiv \frac{\dot{N}(t)}{N(t)} = \eta - \beta \lessgtr 0, \quad \text{if} \quad \eta \lessgtr \beta, \tag{7}$$

where the population size at time t is denoted by $N(t)$. The size of a newborn generation is proportional to the current population $N(v,v) = \eta N(v)$, where $N(v,t)$ is the size at time t of the cohort born at some time v ($t \geq v$). Since the death rate is constant and cohorts are assumed to be large, the size of each existing generation falls exponentially according to:

$$N(v,t) = e^{-\beta(t-v)} N(v,v), \qquad t \geq v. \tag{8}$$

2.2 Aggregate Household Sector

Given the simple demographic structure, aggregate variables can be calculated as the weighted integral of the values for the different generations. Aggregate financial wealth is, for example, defined as $A(t) \equiv \int_{-\infty}^{t} N(v,t)\,\bar{a}(v,t)dv$, where $N(v,t) = \eta e^{\eta v} e^{-\beta t}$ (and aggregate values for $C(t)$ and $L(t)$ are derived in a similar fashion). The main equations describing optimal behavior of the aggregate household sector can be written as:

$$C(t) = \varepsilon_C(\alpha+\beta)\left[A(t) + N(t)\,\bar{h}(t)\right], \tag{9}$$

$$N(t) - L(t) = \frac{(1-\varepsilon_C)C(t)}{\varepsilon_C w(t)}, \tag{10}$$

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \alpha - \left[\frac{\beta C(t) - \eta N(t)\,\bar{c}(t,t)}{C(t)}\right]. \tag{11}$$

Equations (9) and (10) are aggregate versions of (3) and (4), respectively. Equation (11) is the Keynes-Ramsey rule modified for the existence of overlapping generations of finitely-lived agents. It says that aggregate consumption growth differs from individual consumption growth (equation (5)), owing to the distributional effects caused by the turnover of generations. This so-called *generational turnover* effect (cf. Heijdra and Ligthart (2000))—represented by the second term between brackets of (11)—is comprised of two opposing forces. On the one hand, aggregate growth exceeds individual growth because of the birth of *new* generations, who start consuming out of human wealth immediately (represented by $\eta N(t)\,\bar{c}(t,t)$). On the other hand, aggregate consumption growth falls short of individual growth, reflecting that at each instant of time a cross section of the population *dies* and consequently ceases to consume (represented by $\beta C(t)$). For future reference, equation (11) can be rewritten in terms of aggregate variables:[17]

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \alpha + n_N - \eta \varepsilon_C(\alpha+\beta)\left[\frac{A(t)}{C(t)}\right]. \tag{12}$$

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[16]By solving (7) subject to the initial condition $N(0) = 1$, the path for the aggregate population is obtained: $N(t) = e^{n_N t}$.

[17]Using equations (3), (7), and (9) and noting that, in the absence of bequests, newborns possess no financial wealth (so that $\bar{a}(t,t) = 0$).

2.3 Firms

Firms in the final goods sector[18] rent capital, $K(t)$, and labor, $L(t)$, from households to produce a homogeneous good, $Y(t)$, which is either consumed by households or the government or invested by households to augment the physical capital stock. The final goods sector is characterized by perfect competition. Technology is described by a Cobb-Douglas production function:

$$Y(t) = \Psi_Y K(t)^{\varepsilon_K} L(t)^{1-\varepsilon_K}, \quad 0 < \varepsilon_K < 1, \Psi_Y > 0, \tag{13}$$

where Ψ_Y is a general technology index, which is assumed to be constant. Real profits of the representative firm are defined in the usual way:

$$\Pi(t) \equiv (1 - \tau_K(t))[Y(t) - w(t)L(t)] - [r(t) + \delta]K(t), \tag{14}$$

where $r(t) + \delta$ is the effective rental rate of capital, δ is the rate of capital depreciation, and $\tau_K(t)$ is a capital income tax (or capital subsidy if $\tau_K(t) < 0$). The representative producer chooses $K(t)$ and $L(t)$ in order to maximize $\Pi(t)$, taking factor prices as given. The first-order conditions for this static optimization problem are:

$$(1 - \tau_K(t))\frac{\partial Y(t)}{\partial K(t)} = r(t) + \delta, \quad \frac{\partial Y(t)}{\partial L(t)} = w(t). \tag{15}$$

Since technology features constant returns to scale and markets are perfectly competitive, excess profits are zero (that is, $\Pi(t) = 0$). Furthermore, since there are no adjustment costs associated with investment, the value of household share holdings equals the capital stock, that is, $V(t) = K(t)$.

2.4 Government and Market Equilibrium

The government consumes a fixed share of the final good. Abstracting from public debt and labor taxes, the periodic budget restriction of the government can be written as:

$$G(t) = N(t)\bar{z}(t) + \tau_K(t)[Y(t) - w(t)L(t)], \tag{16}$$

where $G(t)$ denotes public consumption and $N(t)\bar{z}(t)$ are total net lump-sum taxes.

Because of the assumption of perfect foresight, agents' behavior depends on current and future prices. Flexible factor prices cause factor markets to clear instantaneously. Financial market equilibrium implies that households' claims on capital equal the physical capital stock (that is, $A(t) = K(t)$). Equilibrium on the goods market implies that:

$$Y(t) = C(t) + G(t) + I(t), \tag{17}$$

where $I(t)$ denotes gross investment:

$$\dot{K}(t) = I(t) - \delta K(t), \tag{18}$$

where $\dot{K}(t) \equiv dK(t)/dt$ is net capital accumulation.

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[18]There are many identical firms and, for convenience, their number is normalized to unity.

Table 2. The Model$^{(\S)}$

(a) Dynamic equations:

$$\dot{k}(t) = y(t) - c(t) - g(t) - (\delta + n_N)\,k(t) \tag{T1.1}$$

$$\dot{c}(t) = [r(t) - \alpha]\,c(t) - \eta\varepsilon_C\,(\alpha + \beta)\,k(t) \tag{T1.2}$$

(b) Static equations:

$$y(t) = \Psi_Y k(t)^{\varepsilon_K}\,l(t)^{1-\varepsilon_K} \tag{T1.3}$$

$$w(t) = (1 - \varepsilon_K)\left(\frac{y(t)}{l(t)}\right) \tag{T1.4}$$

$$\frac{r(t) + \delta}{1 - \tau_K(t)} = \varepsilon_K\left(\frac{y(t)}{k(t)}\right) \tag{T1.5}$$

$$w(t)\,[1 - l(t)] = \left(\frac{1 - \varepsilon_C}{\varepsilon_C}\right)c(t) \tag{T1.6}$$

$$\bar{z}(t) = g(t) - \tau_K(t)\,[y(t) - w(t)l(t)] \tag{T1.7}$$

$(\S)$ *Variables:* $y(t) \equiv Y(t)/N(t)$: per capita output; $c(t) \equiv C(t)/N(t)$: per capita consumption; $g(t) \equiv G(t)/N(t)$: per capita government consumption; $k(t) \equiv K(t)/N(t)$: per capita capital stock; $l(t) \equiv L(t)/N(t)$: per capita labor supply (that is, the macroeconomic participation rate); $\bar{z}(t)$: per capita lump-sum tax; $w(t)$: real wage rate; $r(t)$: real interest rate; and $\tau_K(t)$: capital income tax. *Parameters:* δ: rate of capital depreciation; α: pure rate of time preference; $n_N \equiv \eta - \beta$: net population growth rate; η: birth rate; β: death rate; ε_K: capital share in output; and ε_C: share of consumption in utility.

2.5 Model Summary

In the presence of population growth, the model will give rise to ongoing economic growth also in the steady state.[19] In order to study the steady-state dynamics, we rewrite the model in a stationary format by expressing all growing variables relative to the *population size*, $N(t)$. The key equations of the model are presented in Table 2. Consider first the dynamic equations. Equation (T1.1) describes the evolution of the per capita stock of capital, which is obtained by combining (17) and (18) and dividing by the population size. The second equation, given by (T1.2), shows the optimum time path of per capita consumption. With a positive birth rate ($\eta > 0$), the steady-state rate of interest must exceed the pure rate of time preference, that is, $r > \alpha$.[20] The rising individual consumption profile (see (5)) ensures that in steady state financial wealth is transferred—via the life-insurance companies—from old to young generations (see Blanchard (1985)).

Equations (T1.3)-(T1.7) are essentially static equations. Equation (T1.3) is the intensive-form production function, obtained from (13). The factor demand equations (T1.4)-(T1.5) are derived by rewriting the expressions in (15) in intensive form. Equation (T1.4) represents a downward sloping labor demand curve in the (w, l) space. The per capita labor supply expression (T1.6) results upon rewriting (10), and is referred to as the macroeconomic participation rate. Population growth affects the participation rate both through its effect on the population size and through its effect on aggregate labor supply. Note that the participation rate is a negative function of per capita consumption. The short-run per capita capital supply curve is a vertical schedule—representing a given capital stock—whereas the short-run demand for capital (T1.5) is a standard downward sloping demand curve, owing to diminishing returns to capital accumulation. Finally, the government budget restriction (T1.7) is a reworked version of (16).

3 Graphical Solution

As was shown in the previous section, the dynamic part of the model can be analytically reduced to two variables: the per capita capital stock (a predetermined variable) and the per capita consumption (a forward-looking or jump variable). The model can be graphically summarized by a phase diagram as shown in Figure 1.[21] The figure is drawn while holding constant the g/y share. The $\dot{k} = 0$ line represents all combinations of c and k for which the per capita stock of capital is constant over time. It passes though the origin and is upward sloping provided k falls short of its golden-rule level. For points above (below) the $\dot{k} = 0$ line,

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[19]Growth is exogenous in the steady state, but endogenous during transition. See Sections 3 and 4.

[20]This follows from noting that the steady-state aggregate stock of financial assets (or capital stock per capita) is positive ($k(t) > 0$). If the birth rate is zero (that is, $\eta = 0$), then (T1.2) implies the familiar Ramsey result of $r = \alpha$ in the steady state.

[21]Details of the derivation of the phase diagram are found in Heijdra and Ligthart (2004).
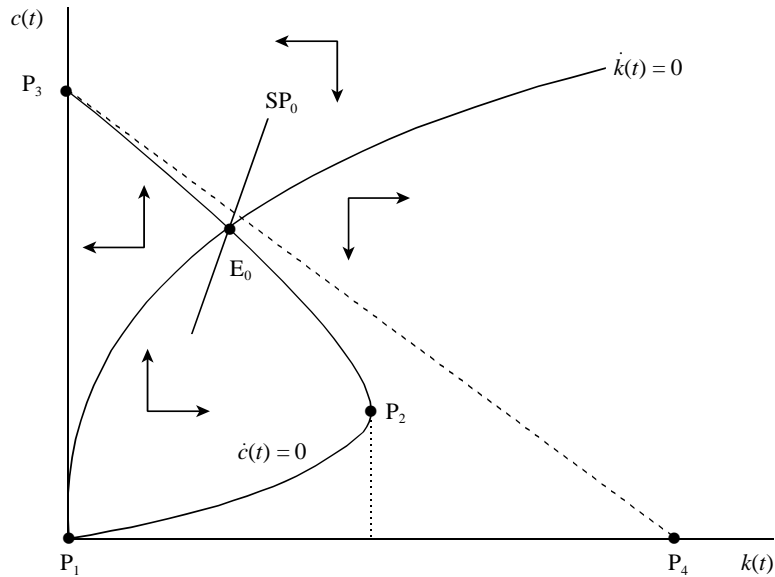
Figure 1: Phase Diagram

employment[22] is too low (high) and consumption is too high (low) so that the capital stock
falls (rises), which is indicated by the horizontal arrows in Figure 1.

The $\dot{c} = 0$ line denotes all (c, k)-combinations for which per capita consumption is constant
over time. The dashed line connecting points P_3 and P_4 in Figure 1 is the $\dot{c} = 0$ line for the
special case of a zero birth and mortality rate (that is, $\eta = \beta = 0$), known in the literature as
the infinite-horizon Ramsey (1928) model.[23] In this case, the steady-state rate of interest is
pinned down by the pure rate of time preference (that is, $r = \alpha$), which implies that w, y/k,
and k/l are all constant in the steady state. Consequently, in view of (T1.6), the $\dot{c} = 0$ line
is linear and negatively sloped.

For a positive birth rate, the $\dot{c} = 0$ curve is given by the solid line connecting points P_1,
P_2, and P_3. The position and slope of the $\dot{c} = 0$ line is determined by two effects working in
opposite directions: (i) the generational turnover effect; and (ii) the aggregate labor supply
effect. The $\dot{c} = 0$ line is almost horizontal near the origin, where labor supply is close to unity
and thus approaches full exogeneity (corresponding to the Blanchard (1985), Buiter (1988)

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[22]In deriving the equilibrium loci, we take into account that equilibrium employment depends on both c and
k. Indeed, by combining labor demand (T1.4), labor supply (T1.6), and the production function (T1.3) we
find that the labor market equilibrium condition can be written as:

$$(f(l) \equiv) \quad \frac{1 - l}{l^{\varepsilon_K}} = \frac{\omega_0 c}{k^{\varepsilon_K}},$$

where ω_0 is a positive constant, $f'(\cdot) < 0$, and $f''(\cdot) > 0$ (for $l \in [0, 1]$). Given k, an increase in c reduces
labor supply and thus lowers equilibrium employment.

[23]Strictly speaking, $\beta = 0$ is not needed to generate $r = \alpha$ in steady state. See (T.1.2) in Table 2 and the
discussion in footnote 15.

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and Weil (1989) models).[24] The $\dot{c} = 0$ line is upward sloping on the line segment P_1P_2, reflecting the dominant generational turnover effect. In contrast, on the line segment P_2P_3, labor supply is fairly elastic, yielding a downward sloping $\dot{c} = 0$ curve that is steeper than the Ramsey $\dot{c} = 0$ line (which is given by the dashed line going through points P_3 and P_4). If the elasticity of intertemporal labor supply approaches infinity (near P_3), the two curves coincide.

The consumption dynamics—illustrated by the vertical arrows in Figure 1—are as follows. For points to the left (right) of the $\dot{c} = 0$ line, consumption rises (falls) over time. To see why, note the following that the interest rate depends on both c and k according to $r(c,k)$, where $\partial r/\partial k < 0$ and $\partial r/\partial c < 0$. The $\dot{c} = 0$ line can thus be written in short-hand notation as $r(c,k) - \alpha = \eta(\alpha + \beta)\varepsilon_C(k/c)$. Given c, a fall (rise) in k leads to an increase (decrease) in the rate of interest and a decrease (increase) in the k/c ratio—representing the generational turnover term—yielding an increase (decrease) in consumption growth.

There is a unique equilibrium at point E_0 and the configuration of arrows in Figure 1 confirms that this equilibrium is a saddle point. See the Appendix for a formal proof. The saddle path associated with E_0 is denoted by SP_0. Although Figure 1 has been drawn under the assumption that the equilibrium occurs along the downward sloping segment P_3P_2 of the $\dot{c} = 0$ line, it cannot be ruled out *a priori* that the intersection occurs somewhere along the upward sloping segment P_1P_2. In Section 5, however, we shall argue that the case illustrated in Figure 1 is empirically the most relevant one.

4   Qualitative Analysis of Small Demographic Shocks

This section studies the effect of demographic shocks on the optimal savings-labor supply response of the household sector and on the investment decisions of firms. Specifically, we analyze the short-run, transition, and long-run macroeconomic effects of stylized demographic scenarios, employing the graphical apparatus developed in the previous section. To keep matters simple, attention is paid to *unanticipated* and *permanent* changes in demographics. The formal proofs underlying the qualitative analysis—obtained by log-linearizing the model around an initial steady state and subsequently perturbing the system—can be found in the Appendix.

Three demographic shocks are considered. The first shock concerns an exogenous drop in fertility, taking as given the mortality rate, which we shall refer to as the *pure baby-bust scenario*. Here, we focus on a drop in the fertility rate rather than a drop in the death rate because the former is in many industrialized countries the more important factor quantitatively. The next two shocks pertain to composite changes as both the fertility and

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[24]The $\dot{c} = 0$ line can only be described parameterically, that is, by varying l in the feasible interval $[0, 1]$. In moving from point P_1 to P_3, l *falls* from 1 to 0; it follows that in P_1, $l = 1$ and the labor-leisure ratio (ω_{LL}) equals zero, while in P_3, $l \to 0$ and $\omega_{LL} \to \infty$.

death rates are changed simultaneously. It is of interest to analyze these cases because it is not a priori evident whether the macroeconomy would be affected at all. One scenario is an exogenous decrease in fertility exactly matched by an increase in longevity (that is, a fall in the death rate), so as to maintain a *constant population growth rate.* Although it is a stylized case, some industrialized countries may at times be experiencing this type of demographic change. Another scenario concerns an exogenous decrease in the birth rate while adjusting the death rate endogenously so as to offset the generational turnover effect of a lower birth rate. This scenario could be of practical relevance to developing countries at war or to post-conflict economies, where drops in birth rates and rises in death rates often occur simultaneously. It will be shown that the latter two scenarios have qualitatively different macroeconomic results, owing to the generational turnover effect.

4.1 Pure Baby Bust

A permanent and unexpected decrease in the fertility rate (that is, $d\eta < 0$), given a constant mortality rate, decreases the population growth rate (that is, $dn_N = d\eta < 0$). As a result, generational turnover decreases, reflecting that deceased generations are replaced by newly born agents at a slower pace.[25] Figure 2 shows the qualitative effects of this so-called pure baby-bust scenario. The sudden drop in fertility shifts the $\dot{k} = 0$ line up and moves the $\dot{c} = 0$ line to the right, shifting the long-run equilibrium from point E_0 to E_1, where per capita consumption has increased. The lines labeled $[\cdot]_0$ represent the equilibrium loci before the demographic shock, whereas the ones labeled $[\cdot]_1$ represent the loci after the shock. Figure 2 depicts the situation where the shift in the $\dot{k} = 0$ line is sufficiently large to generate a new equilibrium to the left of the old equilibrium (see the discussion below).

On impact, consumption jumps up to point A on the new saddle path, reflecting the drop in the short-run interest rate, making present consumption more attractive than future consumption. As a result, per capita labor supply (that is, the macroeconomic participation rate) falls, pushing up short-run wages, and thus benefiting young generations—who mainly consume out of wage income—while depressing interest income of the elderly. During transition, consumption gradually falls, mirroring the smooth rise in the interest rate, which, however, remains below its old steady state value.

Per capita consumption is higher[26] and per capita labor supply is lower in the new long-run equilibrium. The long-run effect on the per capita capital stock (that is, the capital intensity) is ambiguous; it depends on whether the generational turnover effect is dominated by the labor supply effect. If individual consumption growth profiles are fairly flat—and thus the generational turnover effect is weak—and intertemporal labor supply is sufficiently elastic, the

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[25]The model features a constant probability of death, implying that a young person has the same expected remaining lifetime—that is, the inverse of the probability of death—as a very old person.

[26]This partly reflects what Cutler and others (1990) have labeled the "Solow effect." Due to the reduction in the population growth rate a lower amount of savings is required to maintain a given per capita capital stock.
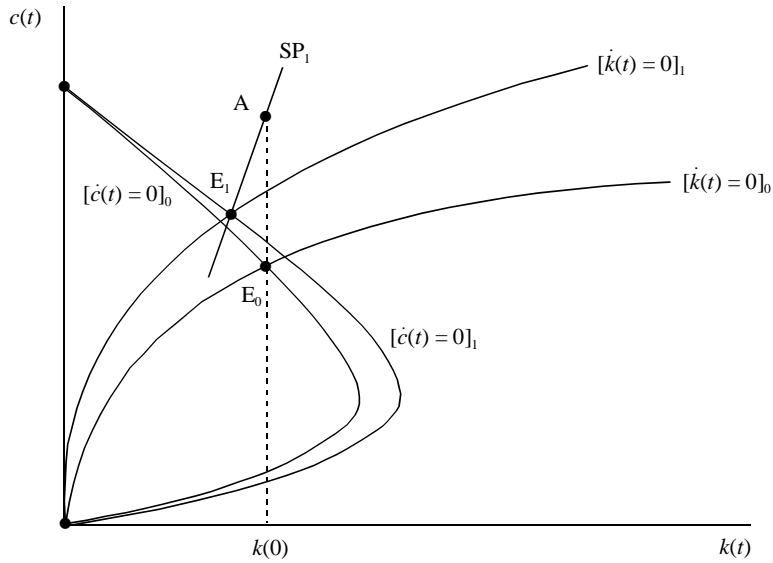
Figure 2: Baby Bust

optimal per capita capital stock falls.[27] In view of the smaller population increment, *aggregate* savings must have fallen by more than *per capita* savings. In the long run, the participation rate is below its old steady-state level, reflecting reduced labor supply (as consumption is higher) and lower labor demand associated with lower per capita assets in the production sector. On a net basis, long-run wages have risen and—from the factor price frontier— interest rates have fallen compared to the old steady state. Accordingly, young existing agents consuming out of human capital benefit, while the elderly lose out. Table 3 compares the impact and long-run effects on the per capita macroeconomic variables.

If individual consumption growth profiles are relatively steep, the per capita capital stock may even increase. Intuitively, the positive savings effect associated with the reduction in generational turnover attenuates the fall in aggregate savings induced by the lower rate of interest. Overlapping generations may thus give rise to diametrically different results than those derived in the infinite-horizon Ramsey model. The assumptions made on the elasticity of labor supply are crucial in this respect.[28] If labor supply is exogenous, the per capita capital stock unambiguously rises as is also shown in the life-cycle model by Elmendorf and Sheiner (2000).[29]

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[27]This requires that the initial birth and death rates are small so that the rightward shift in the $\dot{c} = 0$ line is sufficiently small. With initial birth and death rates close zero—that is, $r \approx \alpha$, thereby approximating the case of infinitely-lived households—the per capita capital stock would unambiguously fall.

[28]In the following, it is assumed that the labor supply effect is sufficiently strong to generate a stable equilibrium on the downward-sloping section of the consumption equilibrium locus (Figure 1).

[29]Elmendorf and Sheiner (2000) employ a Diamond model with exogenous population growth, but they do not work out the comparative statics analytically.

The Appendix shows that the results of Cutler and others (1990), who assume infinitely-lived agents, exogenous population growth and exogenous labor supply, are a special case of our model. In their framework, a fertility drop yields a reduction in *aggregate* steady-state savings, whereas the optimal capital intensity remains—in contrast to our results—unaffected.[30] Allowing for endogenous labor supply reinforces this negative savings effect, yielding a reduction in the optimal capital intensity.

Table 3. Summary of the Qualitative Effects of Various Demographic Shocks[§]

Demographic Shock	Period	y	k	l	c	w	r
Pure Baby Bust	impact	−	0	−	+	+	−
	long run	−	−	−	+	+	−
Stationary Population Growth	impact	+	0	+	−	−	+
	long run	+	+	+	+	+	−
Constant Generational Turnover	impact	−	0	−	+	+	−
	long run	−	−	−	+	+	−

(§) It is assumed that the generational turnover effect is dominated by the labor supply effect.

4.2 Stationary Population Growth

Consider a demographic change which involves simultaneously decreasing the birth rate and mortality rate (that is, $d\eta = d\beta < 0$ and thus $dn_N = 0$) so as to yield a stationary population growth rate. Generational turnover falls, but by less than in the pure baby-bust scenario. Figure 3 shows that this type of demographic shock leaves the $\dot{k} = 0$ line unaffected, but shifts the $\dot{c} = 0$ line to the right, yielding a higher capital intensity. Equiproportionate changes in the birth and death rates thus have a non-neutral effect on the economy.

On impact, the rate of interest rises, making current consumption less attractive compared to future consumption. The latter is represented by a downward jump in consumption from point E_0 to point A on the new saddle path SP_1. The fall in consumption per capita shifts the short-run labor supply curve to the right, so that for a given level of labor demand, per capita employment and per capita output rise and wages fall. The simultaneous decrease in per capita consumption and increase in per capita output crowds in investment during transition.

Consumption gradually increases along the transition path to the new steady state, where per capita capital accumulation and per capita consumption are higher than in the old steady

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[30]Cutler and others (1990) thus effectively set $\beta < 0$ and $\eta = 0$ (see footnote 15), so that $r(k) = \alpha$, explaining why the optimal capital intensity remains unaffected in the steady state.
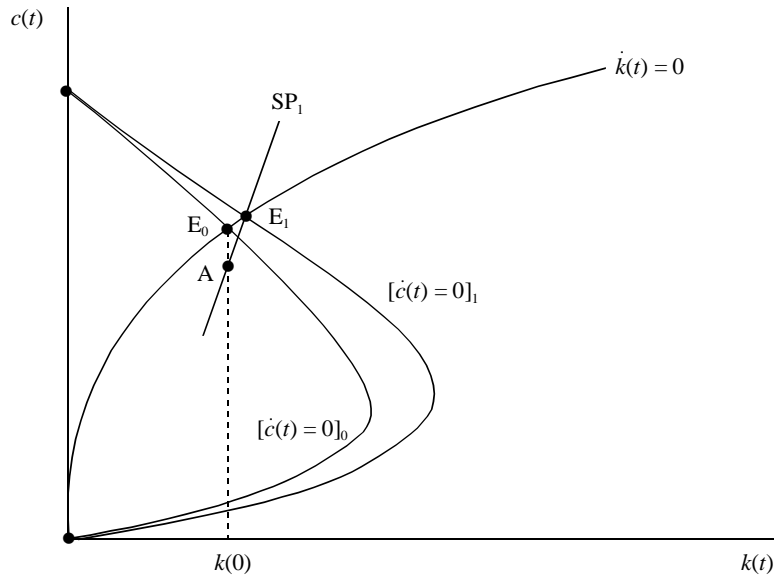
Figure 3: Stationary Population Growth

state. Given the stationarity of the population growth rate, the *aggregate* stock of capital and *aggregate* consumption have increased as well (that is, they are both on a higher path exhibiting the same rate of exponential growth). The rise in the capital stock increases labor demand, but the rise in consumption induces households to supply less labor. On a net basis, equilibrium employment rises. The steady-state interest rate falls and the wage rate rises, reflecting an increase in the long-run capital-labor ratio.

The qualitative effects of a decline in population turnover are identical to those of a capital subsidy (see Appendix).[31] Both per capita output and per capita savings increase. Due to the decline in generational turnover, newly born generations—not owning any financial capital yet—are added to the population at a slower pace, which raises the average asset holdings of the population. A subsidy increases average asset holdings as well, but by raising the return to savings.

4.3 Constant Generational Turnover

Rather than keeping the population growth rate constant, one could also consider a demographic change which leaves unaffected the rate of generational turnover (that is, the second term in (T1.2)). This requires that the death rate has to rise at impact by $d\beta = -(\alpha + \beta)(d\eta/\eta) > 0$ to compensate for the fall in population turnover induced by the reduced rate at which new generations are born (that is, $d\eta < 0$). Accordingly, $dn_N = (\alpha+\beta+\eta)(d\eta/\eta) < 0$, implying a reduction in net population growth, which is larger than under a pure baby bust

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[31]See Heijdra and Ligthart (2000) for an overview of the macroeconomic effects of capital income taxes.
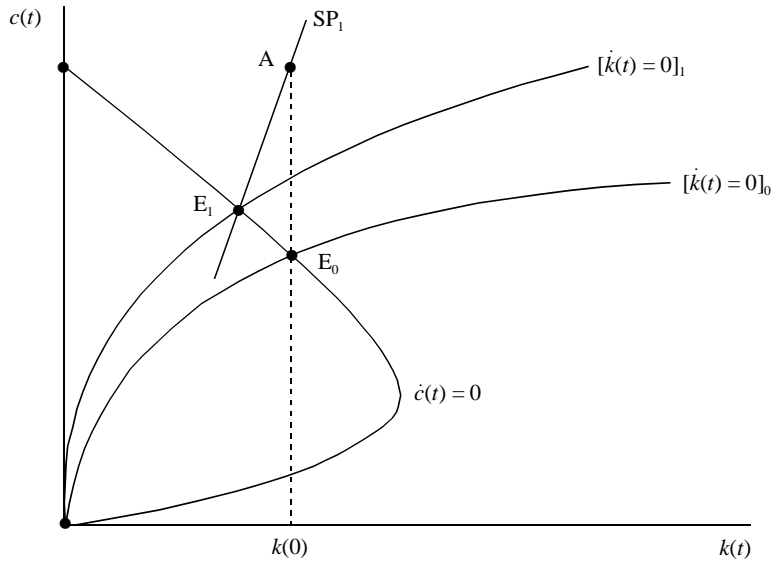
Figure 4: Constant Generational Turnover

scenario.

The effects of the shock can be analyzed with the aid of Figure 4. The $\dot{c} = 0$ line remains unaffected, but the $\dot{k} = 0$ shifts up. The short-run and long-run qualitative effects are equivalent to those of a pure baby bust, although generational turnover is left unaffected. In the new steady state, per capita consumption has increased while the per capita capital stock has fallen. Quantitatively, the two demographic shocks do differ, however, which is not surprising given that the generational turnover effect is a drag on aggregate consumption growth.[32]

As is shown in the Appendix, the demographic scenario of constant generational turnover yields macroeconomic results qualitatively similar to that of a fall in per capita public spending. Intuitively, both decrease labor market participation via the wealth effect in aggregate labor supply, thereby raising wages and reducing the long-run rate of interest. The latter discourages household savings.

5 Concluding Remarks

The paper studies the dynamic macroeconomic effects of demographic shocks employing a Yaari-Blanchard overlapping generations framework extended for endogenous labor supply while allowing for a richer demography. Our theory model provides for a flexible framework, enabling us to reproduce key results from seminal articles and to provide new insights on the

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[32]Using numerical simulations, it can be easily shown that the fall in the per capita capital stock is larger under constant generational turnover for reasonable values of the parameters.

role of demographics in macroeconomics.

The main results are summarized as follows. With overlapping generations, a drop in fertility does not necessarily lead to a reduction in per capita savings and output as is derived in the standard infinitely-lived household model. Per capita savings may increase if the effect of generational turnover is sufficiently strong to dominate the aggregate labor supply effect, but this is not the empirically relevant case.

Endogenizing labor supply in a model with exogenous population growth and infinitely-lived households reinforces the negative aggregate savings effect found by Cutler and others (1990). With endogenous labor supply, aggregate savings fall by more than under exogenous labor supply, giving rise to a fall in per capita savings and per capita output. With exogenous labor supply, however, the capital intensity is left unaffected by the shock.

Depending on the nature of the demographic change, the steady state effects on the macroeconomy differ. A pure baby bust gives rise to a fall in steady-state output, but a rise in per capita consumption. A drop in fertility while keeping generational turnover constant by adjusting the death rate yields results qualitatively similar to those of a pure baby bust. The qualitative effects of a decline in generational turnover at constant population growth are diametrically different. Output per capita increases, but steady-state consumption per capita declines. Policy makers should therefore carefully analyze what changes in demography give rise to observed population dynamics before prescribing a suitable long-run fiscal policy response.

The short-run effects of demographic changes can differ markedly from the long-run effects, not only quantitatively, but also qualitatively. For example, an equiproportionate decline in death and fertility rates yields a decline in per capita consumption on impact, but increases per capita consumption in the new steady state. A pure baby bust scenario, however, gives rise to overshooting in per capita consumption while yielding higher per capita consumption in the new steady state. Notice that long-run wages are pushed up in all three scenarios, and generally rise in the short run too, except in a demographic scenario that keeps the population growth rate constant.

The analysis has abstracted from a pension sector. Furthermore, we have not looked at the welfare effects of demographic changes. This would be particularly relevant in the study of the design of optimal policies to address population aging. These extensions are left for further research.

Appendix: Model Solution

A.1   General Solution Approach

In this appendix we show how the main results mentioned in the text were derived. We log-linearize the model of Table 1 around an initial steady state, using the notational conventions

mentioned at the beginning of Section 2. After some simplifications, the following quasi-reduced form equations can be derived:

$$\tilde{y}(t) = \phi\varepsilon_K\tilde{k}(t) - (\phi - 1)\tilde{c}(t), \tag{A.1}$$

$$\omega_I\tilde{i}(t) = \tilde{y}(t) - \omega_C\tilde{c}(t) - \omega_G\tilde{g}(t), \tag{A.2}$$

$$(1 - \varepsilon_K)\tilde{l}(t) = \tilde{y}(t) - \varepsilon_K\tilde{k}(t), \tag{A.3}$$

$$-\varepsilon_K\left[\tilde{y}(t) - \tilde{k}(t)\right] = (1 - \varepsilon_K)\tilde{w}(t) = -\varepsilon_K\left[\left(\frac{r}{r + \delta}\right)\tilde{r}(t) + \tilde{\tau}_K(t)\right], \tag{A.4}$$

where a tilde ($\tilde{}$) denotes a relative change (for example, $\tilde{y}(t) \equiv dy/y$, except for $\tilde{\tau}_K \equiv d\tau_K/(1 - \tau_K)$), $\omega_C \equiv c/y$ denotes the share of private consumption in real output, $\omega_I \equiv i/y$ is the share of investment in real output, and $\omega_G \equiv g/y$ denotes the share of government consumption in real output.

The parameter ϕ represents the intertemporal labor supply effect, which is defined as:

$$\phi \equiv \frac{1 + \omega_{LL}}{1 + \omega_{LL}\varepsilon_K} \geq 1, \tag{A.5}$$

where ω_{LL} ($\equiv (1-l)/l = (N-L)/L \geq 0$) is the ratio of leisure to labor, which also represents the *aggregate* intertemporal substitution elasticity of labor supply. Notice that $\phi = 1$ if labor supply is exogenous (because $l = 1$ or $N = L$ implies that $\omega_{LL} = 0$). Since $\omega_{LL} \geq 0$ the sign restriction on ϕ is automatically satisfied if $\varepsilon_K \geq 0$. If $\varepsilon_K > 0$, ϕ is a concave function of ω_{LL} with a positive asymptote of $1/\varepsilon_K$ as $\omega_{LL} \to \infty$, and if $\varepsilon_K = 0$, we arrive at $\phi = 1 + \omega_{LL} \geq 1$.

The dynamics of the per capita capital stock and per capita consumption are given by:

$$\dot{\tilde{k}}(t) = \omega_I\left(\frac{y}{k}\right)\left[\tilde{i}(t) - \tilde{k}(t)\right] - \eta\tilde{\eta} + \beta\tilde{\beta}, \tag{A.6}$$

$$\dot{\tilde{c}}(t) = (r - \alpha)\left[\tilde{c}(t) - \tilde{k}(t) - \tilde{\eta} - \left(\frac{\beta}{\alpha + \beta}\right)\tilde{\beta}\right] + r\tilde{r}(t), \tag{A.7}$$

where a variable with a tilde and a dot is the time rate of change (relative to the initial steady state) and $y/k = (r + \delta)/(\varepsilon_K(1 - \tau_K))$. Using equations (A.1)-(A.4) and (A.6)-(A.7) the model can be reduced to a two-dimensional system of first-order differential equations in the per capita capital stock, $\tilde{k}(t)$, and per capita private consumption, $\tilde{c}(t)$. In its most general form, the dynamic system can be written as:

$$\begin{bmatrix} \dot{\tilde{k}}(t) \\ \dot{\tilde{c}}(t) \end{bmatrix} = \Delta \begin{bmatrix} \tilde{k}(t) \\ \tilde{c}(t) \end{bmatrix} - \begin{bmatrix} \gamma_K(t) \\ \gamma_C(t) \end{bmatrix}, \tag{A.8}$$

where Δ denotes the Jacobian matrix (with typical element δ_{ij}, where $i, j = 1, 2$) evaluated at steady state:

$$\Delta \equiv \begin{bmatrix} \frac{y}{k}(\phi\varepsilon_K - \omega_I) & -\frac{y}{k}(\omega_C + \phi - 1) \\ -[(r - \alpha) + (r + \delta)(1 - \phi\varepsilon_K)] & (r - \alpha) - (r + \delta)(\phi - 1) \end{bmatrix}, \tag{A.9}$$

and $\gamma_K(t)$ and $\gamma_C(t)$ are shock terms:

$$\begin{bmatrix} \gamma_K(t) \\ \gamma_C(t) \end{bmatrix} = \begin{bmatrix} \left(\frac{y}{k}\right)\omega_G\tilde{g} + d\eta - d\beta \\ (r-\alpha)(\frac{d\eta}{\eta} + \frac{d\beta}{\alpha+\beta}) + (r+\delta)\tilde{\tau}_K \end{bmatrix}. \tag{A.10}$$

It can be shown that the determinant of the Jacobian matrix is negative, that is,

$$\begin{aligned} |\Delta| &= -\lambda_1\lambda_2 \\ &= -(r+\delta)\left(\frac{y}{k}\right)\left[\left(\frac{r-\alpha}{r+\delta}\right)(\phi(1-\varepsilon_K)-\omega_G) + (\phi-1)\omega_G + \omega_C(1-\phi\varepsilon_K)\right] < 0, \end{aligned} \tag{A.11}$$

where $\phi(1-\varepsilon_K) - \omega_G > 0$, $-\lambda_1 < 0$ is the stable characteristic root, and $\lambda_2 > 0$ is the unstable root. The latter satisfies the inequality $\lambda_2 > r - \alpha + \omega_C(r+\delta)$, which we employ to sign the short run consumption change. Thus, there exists a unique steady state.

The Laplace transform[33] method, as employed in Judd (1982), is used here to solve the model (see Heijdra and Ligthart (2004)). This yields the following long-run effects:

$$\begin{bmatrix} \tilde{k}(\infty) \\ \tilde{c}(\infty) \end{bmatrix} = \frac{\text{adj}(\Delta)}{|\Delta|} \begin{bmatrix} \gamma_K \\ \gamma_C \end{bmatrix}, \tag{A.12}$$

where $t = \infty$ identifies the new steady state that materializes after the demographic shock and $\text{adj}(\Delta)$ is the adjoint matrix of Δ.

A.2 Comparative Statics

A.2.1 Pure Baby Bust

Let us first consider the pure baby bust scenario. Using (A.10) and (A.12) we can derive the long-run effects of an exogenous decrease in the fertility rate (that is, $d\eta < 0$) while keeping the death rate constant (that is, $d\beta = 0$). The effect on the steady-state per capita capital stock is ambiguous:

$$\tilde{k}(\infty) = \left[(r-\alpha)\left(1 + \frac{\omega_C}{\eta}\left(\frac{y}{k}\right)\right) + \left(\frac{r-\alpha}{\eta}\right)\left(\frac{y}{k}\right)(\phi-1) - (r+\delta)(\phi-1)\right]\frac{d\eta}{|\Delta|} \lessgtr 0, \tag{A.13}$$

where $r > \alpha$ due to the overlapping generations structure of the model and $|\Delta|^{-1}d\eta > 0$. If the initial death rate is small—and thus individual consumption growth profiles are fairly flat—and intertemporal labor supply is sufficiently elastic, a drop in fertility depresses the per capita capital stock.

In the infinite horizon Ramsey model, featuring $r = \alpha$ in steady state, the first and second term of (A.13) drop out,[34] giving rise to an unambiguous decline in *per capita* savings and

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[33]The Laplace transform of $x(t)$ is denoted by $\mathcal{L}\{x,s\} \equiv \int_0^\infty x(t)e^{-st}dt$. Intuitively $\mathcal{L}\{x,s\}$ represents the present value of $x(t)$ using s as the discount rate.

[34]Note that $r = \alpha$ for $\eta = 0$ because there is no extra-dynasty growth.

thus a smaller per capita capital stock. If, in addition, labor supply is exogenous (that is, $\phi = 1$), the third term drops out as well, yielding the familiar Ramsey result of a constant capital intensity. Accordingly, *aggregate* savings decline as is also shown by Cutler and others (1990).

With overlapping generations (that is, $r > \alpha$) and exogenous labor supply (that is, $\phi = 1$, so that the second and third term of (A.13) drop out), the per capita capital stock unambiguously rises:

$$\tilde{k}(\infty) = (r - \alpha) \left[1 + \left(\frac{1}{\eta} \right) \left(\frac{y}{k} \right) \omega_C \right] \frac{d\eta}{|\Delta|} > 0. \tag{A.14}$$

Making use of (A.10) and (A.12) again, we can derive the long-run effect of a pure baby bust on per capita consumption:

$$\tilde{c}(\infty) = \left[(r - \alpha) \left(1 + \left(\frac{1}{\eta} \right) \left(\frac{y}{k} \right) (\phi \varepsilon_K - \omega_I) \right) + (r + \delta)(1 - \phi \varepsilon_K) \right] \frac{d\eta}{|\Delta|} > 0, \tag{A.15}$$

which is unambiguously positive. Initial per capita consumption changes according to:

$$\tilde{c}(0) = \left[\frac{\lambda_2 - \delta_{22}}{\delta_{12}} + \frac{r - \alpha}{\eta} \right] \frac{d\eta}{\lambda_2} \lessgtr 0, \tag{A.16}$$

where $\lambda_2 - \delta_{22} > 0$ and $t = 0$ identifies the time of the shock. The first term of (A.16) is negative and the second term positive, giving rise to an ambiguous effect. The initial effect on consumption is positive if the labor supply effect dominates the generational turnover effect. Using equations (A.1)-(A.4) together with (A.12) and (A.15), the steady-state and impact effects on l, i, r, and w can be derived as well.[35]

A.2.2   Stationary Population Growth Rate

The stationary population growth rate scenario implies an equiproportionate fall in the birth and death rate (that is, $d\eta = d\beta < 0$, so that $dn_N = 0$). The per capita capital stock rises in the new steady state:

$$\tilde{k}(\infty) = \frac{(r - \alpha)(\alpha + \beta + \eta)(\omega_C + \phi - 1)(y/k)}{\eta(\alpha + \beta)} \frac{d\eta}{|\Delta|} > 0, \tag{A.17}$$

and the change in steady-state per capita consumption is given by:

$$\tilde{c}(\infty) = \frac{(r - \alpha)(\alpha + \beta + \eta)(\phi \varepsilon_K - \omega_I)(y/k)}{\eta(\alpha + \beta)} \frac{d\eta}{|\Delta|} > 0. \tag{A.18}$$

The impact effect on private consumption is unambiguously negative:

$$\tilde{c}(0) = \mathcal{L}\{\gamma_C, \lambda_2\} = \frac{(r - \alpha)(\alpha + \beta + \eta)}{\eta(\alpha + \beta)\lambda_2} d\eta < 0, \tag{A.19}$$

where use is made of $\gamma_C \equiv \frac{(r-\alpha)(\alpha+\beta+\eta)}{\eta(\alpha+\beta)} d\eta < 0$.

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[35] See Heijdra and Ligthart (2004) for a formal derivation.

A.2.3 Constant Generational Turnover

If generational turnover is kept constant, the birth rate falls by less than the death rate, that is, $d\eta = -\eta d\beta/(\alpha + \beta) < 0$, where $d\beta > 0$, so that $dn_N = (\alpha + \beta + \eta)(d\eta/\eta) < 0$. The long-run effect on the per capita capital stock is negative if the labor supply effect dominates the generational turnover effect:

$$\tilde{k}(\infty) = (\alpha + \beta + \eta)\left[r - \alpha - (r + \delta)(\phi - 1)\right]\frac{\tilde{\eta}}{|\Delta|} < 0. \tag{A.20}$$

The long-run change in per capita consumption is given by:

$$\tilde{c}(\infty) = (\alpha + \beta + \eta)\left[r - \alpha + (r + \delta)(1 - \phi\epsilon_K)\right]\frac{\tilde{\eta}}{|\Delta|} > 0, \tag{A.21}$$

and private consumption jumps up on impact:

$$\tilde{c}(0) = \frac{(\lambda_2 - \delta_{22})(\alpha + \beta + \eta)d\eta}{\eta\delta_{12}\lambda_2} > 0, \tag{A.22}$$

since $\delta_{12} < 0$ and $d\eta < 0$.

A.2.4 A Fall in Public Spending

A fall in public spending (that is, $\tilde{g} < 0$) has qualitatively similar effects to a fall in the birth rate while keeping the rate of generational turnover constant. The steady state effect on the per capita capital stock is negative and is given by:

$$\tilde{k}(\infty) = \omega_G\left(\frac{y}{k}\right)\left[r - \alpha - (r + \delta)(\phi - 1)\right]\frac{\tilde{g}}{|\Delta|} < 0, \tag{A.23}$$

if the labor supply effect is sufficiently strong. The steady-state effect on private consumption is given by

$$\tilde{c}(\infty) = \omega_G\left(\frac{y}{k}\right)\left[r - \alpha + (r + \delta)(1 - \phi\epsilon_K)\right]\frac{\tilde{g}}{|\Delta|} > 0, \tag{A.24}$$

and per capita private consumption rises initially:

$$\tilde{c}(0) = \frac{(\lambda_2 - \delta_{22})\omega_G\left(\frac{y}{k}\right)\tilde{g}}{\delta_{12}\lambda_2} > 0. \tag{A.25}$$

A.2.5 A Rise in Capital Income Subsidies

A rise in the capital income subsidy (which is represented by $\tilde{\tau}_K < 0$ and $\tau_K < 0$) yields an increase in the per capita capital stock:

$$\tilde{k}(\infty) = (r + \delta)(\omega_C + \phi - 1)(y/k)\frac{\tilde{\tau}_K}{|\Delta|} > 0, \tag{A.26}$$

and a rise in per capita consumption in the new steady state:

$$\tilde{c}(\infty) = (r + \delta)(\phi \epsilon_K - \omega_I)(y/k)\frac{\tilde{\tau}_K}{|\Delta|} > 0. \tag{A.27}$$

while per capita private consumption falls initially:

$$\tilde{c}(0) = \frac{(r + \delta)\tilde{\tau}_K}{\lambda_2} < 0. \tag{A.28}$$

References

Auerbach, A.J., and L.J. Kotlikoff, 1987, *Dynamic Fiscal Policy*, Cambridge University Press, Cambridge.

Auerbach, A.J., J. Cai, and L.J. Kotlikoff, 1991, "U.S. Demographics and Saving: Predictions of Three Savings Models," *Carnegie-Rochester Conference Series on Public Policy*, Vol. 34, pp. 135-156.

Auerbach, A.J., L.J. Kotlikoff, R.P. Hagemann, and G. Nicoletti, 1989, "The Economic Dynamics of an Aging Population: The Case of Four OECD Countries," *OECD Economic Studies*, Vol. 12, pp. 97-130.

Becker, G.S., and R.J. Barro, 1988, "A Reformulation of the Economic Theory of Fertility," *Quarterly Journal of Economics*, Vol. 108, pp. 1-25.

Blanchard, O.-J., 1985, "Debts, Deficits, and Finite Horizons," *Journal of Political Economy*, Vol. 93, pp. 223-247.

Bloom, D.E., D. Canning, and B. Graham, 2003, "Longevity and Life Cycle Savings," *Scandinavian Journal of Economics*, Vol. 105, pp. 319-338.

Bovenberg, A.L., 1993, "Investment Promoting Policies in Open Economies: The Importance of Intergenerational and International Distributional Effects," *Journal of Public Economics*, Vol. 51, pp. 3-54.

Bovenberg, A.L., and B.J. Heijdra, 1998, "Environmental Tax Policy and Intergenerational Distribution," *Journal of Public Economics*, Vol. 67, pp. 1-24.

Brooks, R., 2002, "Asset Market Effects of the Baby Boom and Social Security Reform," *American Economic Review*, Vol. 92, pp. 402-406.

Buiter, W.H., 1988, "Death, Birth, Productivity Growth and Debt Neutrality," *Economic Journal*, Vol. 98, pp. 279-293.

Cutler, D., J. Poterba, L. Sheimer, and L. Summers, 1990, "An Aging Society: Opportunity or Challenge?" *Brookings Papers on Economic Activity*, Vol. 1, pp. 1-73.

Diamond, P.A., 1965, "National Debt in the Neoclassical Growth Model," *American Economic Review*, Vol. 55, pp. 279-293.

Elmendorf, D.W., and L.M. Sheiner, 2000, "Should America Save for its Old Age? Fiscal Policy, Population Aging and National Savings," *Journal of Economic Perspectives*, Vol. 14, pp. 57-74.

Group of Ten, 1998, *The Macroeconomic and Financial Implications of Aging Populations,* Report of a Working Party of the Group of Ten Deputies on the Aging of Populations. http://www.bis.org/publ/gten04.pdf.

Heijdra, B.J., and J.E. Ligthart, 2000, "The Dynamic Macroeconomic Effects of Tax Policy in an Overlapping Generations Model," *Oxford Economic Papers*, Vol. 52, pp. 677-701.

Heijdra, B.J., and J.E. Ligthart, 2002, "Tax Policy, the Macroeconomy, and Intergenerational Distribution," *IMF Staff Papers*, Vol. 49, pp. 106-127.

Heijdra, B.J., and J.E. Ligthart, 2004, "The Macroeconomic Dynamics of Demographic Shocks: Mathematical Appendix," mimeo, Groningen University.

Judd, K.L., 1982, "An Alternative to Steady State Comparisons in Perfect Foresight Models," *Economics Letters,* Vol. 10, pp. 55-59.

McMorrow, K., and W. Röger, 2003, "Economic and Financial Market Consequences of Ageing Populations," European Commission Economic Paper No. 182, European Commission, Brussels.

Momota, A., and K. Futagami, 2000, "Demographic Transition Pattern in a Small Country," *Economics Letters*, Vol. 67, pp. 231-237.

Ríos-Rull, J.-V., 2001, "Population Changes and Capital Accumulation: The Aging of the Baby Boom," *The B.E. Journals in Macroeconomics*, No. 1, Article 7.

Samuelson, P.A., 1958, "An Exact Consumption-Loan Model of Interest With or Without the Social Contrivance of Money," *Journal of Political Economy*, Vol. 66, pp. 467-482.

United Nations, 2003, *World Population Prospects: The 2002 Revision Population Database,* http://www.un.org/esa/population/unpop.htm.

Weil, P., 1989, "Overlapping Families of Infinite-Lived Agents," *Journal of Public Economics*, Vol. 38, pp. 183-198.

Yaari, M.E., 1965, "Uncertain Lifetime, Life Insurance and the Theory of the Consumer," *Review of Economic Studies*, Vol. 32, pp. 137-150.